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Weyl's Theorem for quasi N-class A_k operators

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Abstract

In this paper we studied quasi N-class A_k operator, where k is positive integer, which coincides with quasi N-class A operator for $k = 1$. We prove that if T is quasi N-class A_k operator then T is finite ascent, Using matrix representation, we prove that T is an isoloid and Weyl's theorem holds for T and $f(T)$, where f is an analytic function in a neighborhood of the spectrum of T. We also show that quasi N-class A_k operator are closed.

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1. Introduction

Let $T \in B(H)$ be the Banach algebra of all bounded linear operators on a non-zero complex Hilbert space H. By an operator T, We mean an element form $B(H)$. If T lies in $B(H)$, then T^* denotes the adjoint of T in $B(H)$. An operator T is called paranormal if $\|T^2x\| \geq \|Tx\|^2$ for every unite vector $x \in H$. An operator T belongs to class A, if

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$\left|T^2\right| \geq |T|^2$. An operator T is called n-perinormal for positive integer n such that $n \geq 2$, if $T^{*n}T^n \geq \left(T^*T\right)^n$.An operator T is called k-paranornormal for positive integer k, if $\left\|T^{k+1}x\right\| \geq \|Tx\|^{k+1}$ for every unit vector in $x \in H$. For $0 < p < 1$, an operator T is said to be p-hyponormal if $\left(T^*T\right) \geq \left(TT^*\right)^p$ if $p = 1$, T is called hyponormal. An operator T is called log-hyponormal if T is invertible and $log\left(T^*T\right) \geq log\left(TT^*\right)$. An operator T is said to be class $A(k)$ for $k > 0$, if $\left(T^*|T|^{2k}T\right)^{\frac{1}{k+1}} \geq |T|^2$. An operator T is called normaloid if $r(T) = \|T\|$, where $r(T) = sup\{|\lambda| : \lambda \in \sigma(T)\}$ and isoloid if every isoloid point of $\sigma(T)$ is an eigen values of T. We defined an operator $T \in B(H)$ as N-class A_k if $|T|^2 \leq N\left(\left|T^{k+1}\right|\right)^{\frac{2}{k+1}}$ for a positive integer k. If $k = 1$, then N-class A_k coincides with N-class A operator. We have shown that p-hyponormal operators and log-hyponormal operators are class A_k operators, for every positive integer k and class A_k operators are k-paranormal operators.

In this paper, we introduced a new class of operators called quasi N-class A_k operators for each positive integer k, which is superclasses of class A_k operators and prove that weyl's holds for quasi N-class A_k operators.

Class $A \subset$ class $A_k \subset$ N-class $A_k \subset$ quasi N-class A_k.

2. Definition and examples

In this section quasi N-class A_k operators are defined with an example. It is shown that powers and inverse of an invertible class A operator are quasi class A_k for all positive integer k.

Definition 2.1. An operator $T \in B(H)$ is defined to be of quasi N-class A_k if

$$T^*\left[|T|^2 - N\left(\left|T^{k+1}\right|\right)^{\frac{2}{k+1}}\right]T \leq 0$$

for some positive integer k, if $k = 1$, Then quasi N-class A_k operator coincides with quasi N-class A operator.

Example 2.2. Suppose taht H is the direct sum of a denumerable number of copies of two dimensional Hilbert space $R \times R$ and A and B two positive operators on $R \times R$. For any fixed positive integer n, define an operator $T = T_{A,B,n}$ on H as follows:

$$T\left(x_1, x_2, x_3, \ldots, x_n\right) = \left(0, A\left(x_1\right), A\left(x_2\right), \ldots A\left(x_n\right), B\left(x_{n+1}\right) \ldots\right)$$

Its adjoint T^* is given by

$$T^*\left(x_1, x_2, x_3, \ldots, x_n\right) = \left(0, A\left(x_1\right), A\left(x_2\right), \ldots, A\left(x_n\right), B\left(x_{n+1}\right) \ldots\right)$$

For $n \geq k$, $T = T_{A,B,n}$ is qusi N-class A_k if and only if A and B satisfies

$$NA\left(A^{k+1-i}B^{2i}A^{k+1-i}\right)^{\frac{2}{k+1}}A \geq A^4$$

for $i = 1, 2, \ldots, k$. If $A = \begin{bmatrix} \frac{1}{4} & 0 \\ 0 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$, then $T = T_{A,B,n}$ is of quasi N-class A_2.

Since $S \geq 0$ implies $T^*ST \geq 0$, the following result is trivial. The convers is true, if T is invertible.

Thoeorem 2.3. If T belongs to N-class A_k, for some positive integer $k \geq 1$, then T belongs to quasi N-class A_k.

Thoeorem 2.4. If T belongs to N-class A_k operator for some positive integer $k \geq 1$, then T^* is N-class A_k operator.

Form Theorem 2.3 and 2.4, we get the following results.

Thoeorem 2.5. Let T be an invertible N-class A_k operator,
then,

1. T is qusi N-class A_k operator for every positive integer k.
2. qusi N -class$A_1 \subseteq$ qusi N -class A_2 qusi N-class $A_3 \subseteq$.......
3. For all positive integer n, T^n is quasi N-class A_k operator for every integer k.
4. T^{-1} is quasi N-class A_k operator for every positive integer k.

Thoeorem 2.6. [5] If $A, B \in B(H)$ satisfy $A \geq 0$ and $\|B\| \leq 1$, then $\left(B^*AB\right)^\delta \geq B^*A^\delta B$ for all $\delta \in (0, 1]$.

Thoeorem 2.7. [12] If A is a positive operator, then the following inequalities hold for all $x \in H$

1. $\left\langle A^r x, x\right\rangle \leq \langle Ax, x\rangle^r \|x\|^2 (1 - r)$ for all $0 < r \leq 1$.
2. $\left\langle A^r x, x\right\rangle \geq \langle Ax, x\rangle^r \|x\|^2 (1 - r)$ for $r > 1$.

3. Matrix Representation

Matrix representation of an operator is used to study various properties of an operator. $T = \begin{bmatrix} A & S \\ 0 & 0 \end{bmatrix}$ for class A operator with respect to direct sum of closure of range of T and kernel of T^*. We extened this to qausi N-class A_k operator.

Thoeorem 3.1. Let T be a qausi N-class A_k operator for a positive integer k with no dense range and T has the following representation: $T = \begin{bmatrix} T_1 & T_2 \\ 0 & T_3 \end{bmatrix}$ on $H = \overline{ran\,(T)} \oplus$

$ran\left(T^{*}\right)$, then T_1 is N-class A_k operator on $\overline{ran\,(T)}$ and $T_3 = 0$ furthermore $\sigma(T) = \sigma(T_1) \cup \{0\}$.

Proof. Consider the matrix representation of T with respect to decomposition $H = \overline{ran\,(T)} \oplus ker\,(T)$ $T = \begin{bmatrix} A & S \\ 0 & 0 \end{bmatrix}$ let P be the orthogonal projection of H onto $ran\,(T)$ then $T_1 = PTP = TP$ since T is N-class A_k operator we have,

$$P\left[N\left(|T|^{k+1}\right)^{\frac{2}{k+1}} - |T|^2\right]P \geq 0$$

$$\begin{aligned} P\left[N\left(|T|^{k+1}\right)^{\frac{2}{k+1}}\right]P &= P\left[N\left(\left(T^{*}T\right)^{k+1}\right)^{\frac{1}{k+1}}\right]P \\ &= P\left[N\left(T^{*k+1}T^{k+1}\right)^{\frac{1}{k+1}}\right]P \\ &\leq N\left[PT^{*k+1}T^{k+1}P\right]^{\frac{1}{k+1}} \\ &= \begin{bmatrix} \left|T_1^{k+1}\right|^{\frac{2}{k+1}} & 0 \\ 0 & 0 \end{bmatrix} \end{aligned} \tag{3.1}$$

$$\begin{aligned} &= \begin{bmatrix} \left|T_1^{k+1}\right|^{\frac{2}{k+1}} & 0 \\ 0 & 0 \end{bmatrix} \\ &\geq P\left(\left|T_1^{k+1}\right|\right)^{\frac{2}{k+1}} P \\ &\geq |T|^2 \\ &= \begin{bmatrix} |T_1|^2 & 0 \\ 0 & 0 \end{bmatrix} \end{aligned} \tag{3.2}$$

Hence On $\overline{ran\,(T)}$. Also for any $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \in H$

$$\begin{aligned} \langle T_3x_2, x_3\rangle &= \langle T(I-P)x, (I-P)x\rangle \\ &= \left\langle (I-P)x, T^{*}(I-P)x\right\rangle \\ &= 0 \\ T_3 &= 0. \end{aligned} \tag{3.3}$$

since $\sigma(T_1) \cup \sigma(T_2) = \sigma(T) \cup \tau$, where τ is the union of the holds in $\sigma(T)$ which happen to be subset of $\sigma(T_1) \cap \sigma(T_2)$,and $\sigma(T_1) \cap \sigma(T_2)$ has no interior points therefore we have $\sigma(T) = \sigma(T_1) \cup \{0\}$. ∎

Since A_k operators are isoloid, The following results follows immediately.

Corollary 3.2. Let $T \in B(H)$ be quasi N-class A_k operator for a positive integer k and T not have dense range. If T has the following representation $T = \begin{bmatrix} T_1 & T_2 \\ 0 & 0 \end{bmatrix}$ on $H = \overline{ran(T)} \oplus kerT^*$, then T_1 is isoloid.

4. Some Properties of quasi N-class A_k operator

In this section, quasi N-class A_k operators $ker(T - \lambda I) \subseteq ker(T - \lambda I)^*$ for all $\lambda \in C$ and quasi N-class A_k operators have finite ascent.

Theorem 4.1. If T is quasi N-class A_k operator for some positive integer k then T is an isoloid.

Proof. $T = \begin{bmatrix} T_1 & T_2 \\ 0 & 0 \end{bmatrix}$ on $H = \overline{ran(T)} \oplus kerT^*$ and λ_0 be an isolated point of $\sigma(T)$. Then by Theorem 3.1, either $\lambda_0 = 0$ or $0 \neq \lambda_0 \in iso\sigma(T_1)$. Since T_1 is isoloid, if $\lambda_0 \in iso\sigma(T_1)$, then $\lambda_0 \in \sigma_p(T_1)$ and hence $\lambda_0 \in \sigma_p(T)$.

On the contrary, if $\lambda_0 = 0$ and $\lambda_0 \notin \sigma_p(T_1)$, then T_1 is invertible and for any $x \neq 0$ in H. $T\left(-T_1^{-1}T_2x \oplus x\right) = 0$. Hence $-T_1^{-1}T_2x \oplus x \in kerT$ and $\lambda_0 \in \sigma_p(T_1)$. Hence in both cases, λ_0 is an eigenvalue of T. Therefore T is isoloid. ∎

Theorem 4.2. If T is quasi N-class A_k operator for a positive integer k and $(T-\lambda)x = 0$ for $\lambda \neq 0$ and $x \in H$ then $(T - \lambda)^* x = \{0\}$.

Proof. Using Holder-Mc Charthy and Schwarz's inequalities we get,

$$\left\| \left|T^{k+1}\right| Tx \right\|^2 = \left\langle T^{k+1}(Tx), T^{k+1}(Tx)\right\rangle = |\lambda|^2 \|Tx\|^2$$

$$\begin{aligned}
|\lambda|^2 \|Tx\|^2 &= \|Tx\|^2 \\
&= \langle Tx, Tx \rangle \\
&= \left\langle T^*Tx, x \right\rangle \\
&= \left\langle \left(T^*T\right)x, x \right\rangle \\
&= \left\langle |T|^2 x, Tx \right\rangle \\
&= \left\langle N\left(\left|T^{k+1}\right|\right)^{\frac{2}{k+1}} x, x \right\rangle \\
&\leq N\left\langle T^{k+1}x, T^{k+1}x\right\rangle^{\frac{2}{k+1}} \|x\|^{2\left(1-\frac{2}{k+1}\right)} \\
&\leq N\left\langle T^{k+1}x, T^{k+1}x\right\rangle^{\frac{2}{k+1}} \|x\|^{2\left(\frac{k-1}{k+1}\right)} \\
&= N|\lambda|^2 \|Tx\|^2 .
\end{aligned} \tag{4.4}$$

Hence

$$|\lambda|^2 \langle Tx, Tx\rangle = \langle T^*Tx, x\rangle$$
$$= \left\langle N\left(\left|T^{k+1}\right|\right)^{\frac{2}{k+1}} x, x\right\rangle. \tag{4.5}$$
$$|\lambda|^2 \langle Tx, Tx\rangle = \langle (T^*T)Tx, Tx\rangle$$
$$= \left\langle N\left(\left|T^{k+1}\right|\right)^{\frac{2}{k+1}} Tx, Tx\right\rangle. \tag{4.6}$$

since $NT^*\left(\left|T^{k+1}\right|\right)^{\frac{2}{k+1}} Tx$ and Tx are linearly independent. therefore

$$NT^*\left(\left|T^{k+1}\right|\right)^{\frac{2}{k+1}} Tx = |\lambda|^2 Tx \tag{4.7}$$

$$\left\|\left(NT^*\left(\left(\left|T^{k+1}\right|\right)^{\frac{2}{k+1}} - |T|^2\right)T\right)^{\frac{1}{2}} x\right\|^2 = 0$$

$$\left(T^{*2}T^2\right)x = NT^*\left(\left|T^{k+1}\right|\right)^{\frac{2}{k+1}} Tx$$
$$= |\lambda|^2 Tx \tag{4.8}$$

Hence $(T-\lambda)^* x = \{0\}$. ■

Theorem 4.3. If T is quasi N-class A_k operator for a positive integer k, then T satisfies

$$\left\|T^2x\right\|^{k+1} \leq N\left\|T^{k+2}x\right\|\|Tx\|^k$$

Proof. Using Holder-McCharthy inequality Theoerem 2.7 for each $x \in H$.

$$0 \leq \left\langle T^*\left(N\left|T^{k+1}\right|^{\frac{2}{k+1}} - |T|^2\right)Tx, x\right\rangle$$
$$\leq \left\langle T^*N\left|T^{k+1}\right|^{\frac{2}{k+1}} Tx, x\right\rangle - \left\langle T^*|T|^2 Tx, x\right\rangle$$
$$\leq \left\langle N\left|T^{k+1}\right|^{\frac{2}{k+1}} Tx, Tx\right\rangle - \left\langle |T|^2 Tx, Tx\right\rangle$$
$$\leq \left\langle N\left|T^{k+1}\right|^2 Tx, Tx\right\rangle^{\frac{1}{k+1}} \|Tx\|^{\frac{2k}{k+1}} - \left\langle |T|^2 Tx, Tx\right\rangle$$
$$\leq \left\langle NT^{*k+1}T^{k+1}Tx, Tx\right\rangle^{\frac{1}{k+1}} \|Tx\|^{\frac{2k}{k+1}} - \left\langle T^*TTx, Tx\right\rangle$$
$$\leq \left\langle NT^{k+1}Tx, T^{k+1}Tx\right\rangle^{\frac{1}{k+1}} \|Tx\|^{\frac{2k}{k+1}} - \langle TTx, TTx\rangle$$
$$\left\|T^2x\right\|^{k+1} \leq N\left\|T^{k+2}x\right\|\|Tx\|^k. \tag{4.9}$$

For all $x \in H$. ■

5. Weyl's theorem for quasi N-class A_k operator

In this section it is shown that weyl's theorem holds for quasi N-class A_k operators, quasi N-class A_k operators have index less than or equal to zero, spectral mapping theorem for weyl's spectrum holds and also that weyl's theorem holds for any function of quasi N-class A_k operators, which is analytic in a neighborhood of the spectrum of quasi N-class A_k operators.

Theorem 5.1. If T is quasi N-class A_k operator for a positive integer k, then T is finite ascent.

Proof. If $\lambda \neq 0$, by Theorem 4.2, $ker(T-\lambda) \subseteq ker(T-\lambda)^*$. Hence if $x \in ker(T-\lambda)^2$, then $\|(T-\lambda)x\|^2 = \langle (T-\lambda)^*(T-\lambda)x, x \rangle = 0$ which implies $x \in ker(T-\lambda)$. Therefore $ker(T-\lambda)^2 = ker(T-\lambda)$.

If $\lambda = 0$, by let $0 \neq x \in kerT^{k+2}$. Then by Theorem 4.3 $x \in kerT^2 \subset kerT^{k+1}$. Therefore $kerT^{k+2} = kerT^{k+1}$. Hence T is finite asecent. ∎

Proposition 5.2. [7] For given operators $A, B, C \in B(H)$, there is equality $\omega(A) \cup \omega(B) = \omega(M_c) \cup_{\tau}$, where $M_c = \begin{bmatrix} A & C \\ 0 & B \end{bmatrix}$ and τ is the union of certain holes in $\omega(M_c)$ which happen to be a subset of $\omega(A) \cap \omega(B)$.

Proposition 5.3. [2] If T is a N-class A_k operator for a positive integer k, then $f(\omega(T)) = \omega(f(T))$ for every $f \in H(\sigma(T))$.

Proposition 5.4. [7] Suppose $A \in B(H)$ and $B \in B(H)$ are isoloid. If weyl's theorem holds for A and B, and if $\omega(A) \cap \omega(B)$ has no interior points, then weyl's theorem holds for $\begin{bmatrix} A & 0 \\ 0 & B \end{bmatrix}$.

Proposition 5.5. [8] If either $SP(A)$ or $SP(B)$ has no Pseudhoholes and if A is an isoloid operator for which weyl's theorem holds then for every $C \in B(K, H)$,weyl's theorem holds for $\begin{bmatrix} A & 0 \\ 0 & B \end{bmatrix}$ = weyl's theorem holds for $\begin{bmatrix} A & C \\ 0 & B \end{bmatrix}$.

Proposition 5.6. [9] If $T \in B(H)$, then the following are equivalent

1. $ind(T-\lambda I)ind(T-\lambda\mu) \geq 0$ for each pair $\lambda, \mu \in C - \sigma_e(T)$
2. $f(\omega(T)) = \omega(f(T))$ for every $f \in H(\sigma(T))$.

Proposition 5.7. [10]If $T \in B(H)$ is isoloid, then $f(\sigma(T) - \pi_{00}(T)) = \sigma(f((T)) - \pi_{00} f(T))$, for every $f \in H(\sigma(T))$.

Theorem 5.8. If T is quasi N-class A_k operator for some positive integer k, then weyl's theorem holds for T.

Proof. By theorem 3.1 $T = \begin{bmatrix} T_1 & T_2 \\ 0 & T_3 \end{bmatrix}$ on $H = \overline{ran\ (T)} \oplus ker\ (T^*)$ with $ker\ (T_1) = \{0\}$ then T_1 is quasi N-class A_k operator on $\overline{ran\ (T)}$ and T_3. Then by corollary 3.2 and by [2] (Theorem 3.3), $x \in kerT^2 \subset kerT^{k+1}$. Hence $kerT^{k+2} = kerT^{k+1}$. Hence T is finite ascent. ■

Theorem 5.9. If T is quasi N-class A_k operator some positive integer k, then $ind(T - \lambda I) \leq 0$ for all complex numbers λ.

Proof. If T is of finite ascent by lemme[], by [], proposition[], $ind(T - \lambda) \neq 0$ for all complex number λ. ■

Lemma 5.10. If T quasi N-class A_k operator for some positive k, then $f(\sigma\ (T) - \pi_{00}(T)) = \sigma\ (f\ ((T)) - \pi_{00} f(T))$, for every $f \in H\ (\sigma(T))$.

By Lemma 5.9 and Proposition 5.6, the following result is trivial.

Lemma 5.11. If T is a quasi N-class A_k operator for some positive integer k, then $f\ (\omega\ (T)) = \omega\ (f\ (T))$ for every $f \in H\ (\sigma\ (T))$.

Theorem 5.12. If $T \in B(H)$ is quasi N-classA_k for a positive integer k, then weyl's theorem holds for $f(T)$ for every $f \in H\ (\sigma\ (T))$.

Proof. By Theorem 5.10, Theorem 5.8 and Lemma 5.11, for every $f \in H\ (\sigma\ (T))$, $f\ (\sigma\ (T) - \pi_{00}(T)) = \sigma\ (f\ ((T)) - \pi_{00} f(T)) = f\ (\omega\ (T)) = \omega(f\ (T))$, Hence weyl's theorem holds for $f(T)$, for every $f \in H\ (\sigma\ (T))$. ■

6. Weyl's theorem for Algebrically quasi N-class A_k operator

In this section it is shown that the restriction of quasiN - class A_k operators to an invariant subspace is also powers of N-class A_k operator, In this operators have finite asecent and SVEP. If $\lambda \neq 0$ is an isolated point in the spectrum of a powers of N-class A_k operator, then the Riesz idempotent operator E_λ with respect to λ is self adjoint and satisfies $E_\lambda H = ker(T - \lambda) = ker(T - \lambda)^*$.An operator T is called doubly power bounded if $sup\left\{\left\|T^n\right\| : n \in N\right\} < \infty$. Hence nnormaloid are doubly power bounded. Laursen [7] has shown that the secptrum of an doubly power bounded operator equals one if and only if it is an identity operator. Also Duggal has shown that k- paranormal operators are normaloid. Since quasi N-classA_k operators are k - paranormal it follows that powers of N-class A_k operators are normaloids and hence we have the follwing results.

Theorem 6.1. [5] Suppose that $T \in B(H)$ is polaroid

1. If T^* has SVEP, then property(b), property(w), weyl's theorem and a-weyl's theorem holds for T.

2. If T has SVEP, then property(b), property(w), weyl's theorem and a-weyl's theorem hold for T^*.

Theorem 6.2. [7] Suppose $T \in L(X)$, then the following equivalent hold:

1. If T^* has SVEP, then property(w) holds for T if and only if weyl's theorem holds for T, and this is the case if and only if a-weyl's theorem holds for T.

2. If T has SVEP, then property(w) holds for T^* if and only if weyl's theorem holds for T^*, and this is the case if and only if a-weyl's theorem holds for T^*.

Theorem 6.3. If $T \in L(X)$ and T^* has SVEP, then $\sigma_{ea}(T) = w(T)$ and $\sigma(T) = \sigma_a(T)$.

Theorem 6.4. If an operator $T \in L(X)$ has property(H) then T has SVEp and $p(\lambda - T) \leq 1$ for all $\lambda \in C$. Furtheremore both T and T^* are reguloid.

Theorem 6.5. If X is a complex banach space, $T \in L(X)$ and $\mu \in iso\sigma(T)$ with spectral properties E_λ then $E_\lambda(T) = H_0(T - \mu I)$.

Theorem 6.6. If T is class A_k operator for a psitive integer k then $(T - \lambda)$ has finite ascent for every $\lambda \in C$.

Theorem 6.7. If T is quasi N-class A_k operator for some positive integer k and $\lambda \neq 0$ is an isolated point in $\sigma(T)$, then $ker(T - \lambda) = ker(T - \lambda)^*$.

Theorem 6.8. If T is qasi N - class A_k operator for a psitive integer k and for $\lambda \in C$, $\sigma(T) = \{\lambda\}$ then $T = \lambda$.

Proof. If $\lambda = 0$, then $T = 0$, Since powers of N-class A_k operator is normaloid. Assume that $\lambda \neq 0$. Then $T_1 = \frac{1}{\lambda} T$ is an invertible normaloid operator with $\sigma(T_1) = \{1\}$. Therefore $T_1 = 1$. ■

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