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Local Stability of Prey-Predator with Holling type IV Functional Response

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Abstract

In this paper a prey-predator model involving Holling type I and Holling type IV functional responses is proposed and analyzed. The local stability analysis of the system is carried out. Finally, the numerical simulation is used to study the global dynamical behavior of the system.

Keywords: Holling type IV functional response, equilibrium points, stability.

1. INTRODUCTION

Variety of the mathematical models for interacting species incorporating different factors to suit the varied requirements are available in literature, a successful model is one that meets the objectives, explains what is currently happening and predicts what will happen in future. The first major attempt to predict the evolution and existence of species mathematically is due to the American physical chemist Lotka (1925) and independently by the italian mathematician Volterra (1926), see ref. [8], which constitute the main them of the deterministic theory of population-dynamics in theoretical biology even today.

On the other hand, ecology relates to the study of living beings in relation to their living styles. Research in the area of the theoretical ecology was initiated by Lotka (1925) and by Volterra (1926). Since then many mathematicians and ecologists contributed to the growth of this area of knowledge. Consequently, several mathematical models deal with the dynamics of prey predator models involving different types of functional responses have been proposed and studied, see for example [1,2,3,5,6,7] and the references therein.

2. MATHEMATICAL MODEL FORMULATION

Consider the simple prey-predator system with Holling type IV functional response which can be written as:

$$\begin{aligned}
\frac{dx}{dt} &= (a - bx)x - \frac{\alpha_1 \gamma x y}{x^2 + \gamma x + \gamma\beta} - \alpha_2 x z \\
\frac{dy}{dt} &= \frac{e_1 \alpha_1 \gamma x y}{x^2 + \gamma x + \gamma\beta} - h_1 y - \delta_1 y z \\
\frac{dz}{dt} &= e_2 \alpha_2 x z - h_2 z - \delta_2 y z
\end{aligned} \tag{1}$$

Here $x(t)$ be the density of prey species at time t, $y(t)$ and $z(t)$ represent are the density of two predator species at time t respectively. While the parameters $a > 0$ is the intrinsic growth rate of the prey population; $b > 0$ is the strength of intra-specific competition among the prey species; the parameter $\beta > 0$ can be interpreted as the half-saturation constant in the absence of any inhibitory effect; the parameters $\gamma > 0$ is a direct measure of the predator immunity from the prey; the predator consumer consume their food according to Holling type IV of functional response, where $\alpha_i, i = 1,2$ are the predation rate on the predator; $e_i, i = 1,2$ are the conversion rate of predation into higher level species; while there is a competition interaction between $y(t)$ and $z(t)$ for light and space with competition rates $\delta_i, i = 1,2$. Finally $h_i, i = 1,2$ are the death rates of the predator population. The initial condition for system (1) may be taken as any point in the region $R_+^2 = \{(x, y, z) : x \geq 0, y \geq 0, z \geq 0\}$. Obviously, the interaction functions in the right hand side of system (1) are continuously differentiable functions on R_+^3, hence they are Lipschitizian. Therefore the solution of system (1) exists and is unique. Further, all the solutions of system (1) with non-negative initial condition are uniformly bounded as shown in the following theorem.

Theorem (1): All the solutions of system (1) which initiate in R_+^3 are uniformly bounded.

Proof. Let $((x(t), y(t), z(t))$ be any solution of the system (1) with non-negative initial condition (x_0, y_0, z_0). According to the first equation of system (1) we have

$$\frac{dx}{dt} \le (a - bx)x$$

Then by solving this differential inequality we obtain that

$$x(t) \le \frac{ax_0}{ae^{-at} + (1 - e^{-at})bx_0}$$

Thus $\lim_{t\to\infty} Sup\, x(t) \le M$ where $M = \max\left\{\frac{a}{b}, x_0\right\}$. Define the function:

$$W(x, y, z) = x + \frac{1}{e_1} y + \frac{1}{e_2} z$$

So the time derivative of $W(t)$ along the solution of the system (1)

$$\frac{dW}{dt} = \frac{dx}{dt} + \frac{1}{e_1}\frac{dy}{dt} + \frac{1}{e_2}\frac{dz}{dt}$$

$$\frac{dW}{dt} \le (a+1)x - (x + \frac{h_1}{e_1} y + \frac{h_2}{e_2} z)$$

$$\frac{dW}{dt} + \omega W \le m$$

Where $m = a + 1$ and $\omega = \min\{1, h_1, h_2\}$by solving the above linear differential inequality we get

$$\lim_{t\to\infty} Sup.\, W(t) \le \tfrac{m}{\omega} \to W(t) \le \frac{m}{\omega}, t > 0$$

Hence, all the solutions of system (1) are uniformly bounded, and then the proof is complete. ■

3. EXISTENCE OF EQUILIBRIUM POINTS

The system (1) have at most three non-negative equilibrium points, two of them namely $E_0 = (0,0,0)$, $E_x = (\frac{a}{b}, 0, 0)$ always exist. While the existence of other equilibrium points is shown in the following:

The second predator free equilibrium point $E_{xy} = (\hat{x}, \hat{y}, 0)$,

$$a - bx - \frac{\alpha_1 \gamma y}{x^2 + \gamma x + \gamma\beta} = 0 \tag{2a}$$

$$\frac{e_1 \alpha_1 \gamma x}{x^2 + \gamma x + \gamma\beta} - h_1 = 0 \tag{2b}$$

From (2a) we have

$$\hat{y} = (a - b\hat{x}) \frac{\hat{x}^2 + \gamma\hat{x} + \gamma\beta}{\alpha_1 \gamma}$$

Clearly, $\hat{y} > 0$ if the following condition holds

$$a > b\hat{x}$$

while $\hat{x}$, represents the positive root to the following equation

$$f(x) = A_2 x^2 + A_1 x + A_0 \tag{3}$$

Where

$$A_2 = -h_1,\ A_1 = \gamma(e_1\alpha_1 - h_1),\ A_0 = -h_1\gamma\beta$$

So by using Descartes rule of signs, Eq. (3) has either no positive root and hence there is no equilibrium point or two positive roots depending on the following condition holds:

$$e_1 \alpha_1 > h_1$$

The first predator free equilibrium point $E_{xz} = (\bar{x}, 0, \bar{z})$,

$$a - bx - \alpha_2 z = 0 \tag{4a}$$

$$e_2 \alpha_2 x - h_2 = 0 \tag{4b}$$

Where

$$\bar{x} = \frac{h_2}{e_2 \alpha_2}\ ,\ \bar{z} = \frac{1}{e_2 \alpha_2^2}(e_2 \alpha_2 a - b h_2)$$

Clearly, $\bar{z} > 0$ if the following condition holds

$$e_2 \alpha_2 a > b h_2 \tag{5}$$

Finally, the coexistence equilibrium point $E_{xyz} = (x^*, y^*, z^*)$ exists in $Int.R_+^3$,

$$a - bx - \frac{\alpha_1 \gamma y}{x^2 + \gamma x + \gamma\beta} - \alpha_2 z = 0 \tag{6a}$$

$$\frac{e_1 \alpha_1 \gamma x}{x^2 + \gamma x + \gamma\beta} - h_1 - \delta_1 z = 0 \tag{6b}$$

$$e_2 \alpha_2 x - h_2 - \delta_2 y = 0 \tag{6c}$$

From (6b) we have

$$z^* = \frac{1}{\delta_1}\left(\frac{e_1 \alpha_1 \gamma x^*}{x^{*2} + \gamma x^* + \gamma\beta} - h_1 \right) \tag{7}$$

From (4c) we have

$$y^* = \frac{1}{\delta_2}\left(e_2 \alpha_2 x^* - h_2 \right) \tag{8}$$

while x^*, represents the positive root to the following equation

$$f(x) = A_5 x^5 + A_4 x^4 + A_3 x^3 + A_2 x^2 + A_1 x + A_0 \tag{9}$$

Where

$$A_5 = -b\delta_1\delta_2$$

$$A_4 = \delta_1\delta_2(a - 2b\gamma)$$

$$A_3 = \delta_1\delta_2\gamma[2a - b(2\beta + \gamma)]$$

$$A_2 = \delta_1\delta_2\gamma[(2a\beta + \gamma) - 2b\gamma\beta] + \alpha_2\delta_2 h_1$$

$$A_1 = \gamma[\delta_1\delta_2\gamma\beta(2a - b\beta) - \alpha_1\alpha_2(e_2\delta_1 + e_1\delta_2) + h_1\alpha_2\delta_2]$$

$$A_0 = \gamma[\delta_2\beta(a\delta_1\gamma\beta + h_1\alpha_2) + \delta_1\alpha_1 h_2]$$

So by using Descartes rule of signs, Eq. (9) has a unique positive root say x^* provided that one set of the following sets of conditions hold:

$$A_2 > 0, A_1 > 0 \tag{10a}$$

$$A_2 < 0, A_1 > 0 \tag{10b}$$

$$A_2 < 0, A_1 < 0 \tag{10c}$$

Therefore, by substituting x^* in Eqs. (7), (8), system (1) has a unique equilibrium point in the $Int.R_+^3$ by $E_{xyz} = (x^*, y^*, z^*)$, provided that

$$\frac{e_1\alpha_1\gamma x^*}{{x^*}^2 + \gamma x^* + \gamma\beta} > h_1 \tag{11a}$$

$$e_2\alpha_2 x^* > h_2 \tag{11b}$$

4. THE STABILITY ANALYSIS

In this section the stability analysis of the above mentioned equilibrium points of system (1) are investigated analytically.

The Jacobian matrix of system (1) at the equilibrium point $E_0 = (0,0,0)$ can be written as

$$J_0 = J(E_0) = \begin{bmatrix} a & 0 & 0 \\ 0 & -h_1 & 0 \\ 0 & 0 & -h_2 \end{bmatrix}$$

$$\lambda_{01} = a > 0,\ \lambda_{02} = -h_1 < 0,\ \lambda_{02} = -h_2 < 0$$

Therefore, the equilibrium point E_0 is a saddle point.

The Jacobian matrix of system (1) at the equilibrium point $E_x = (\frac{a}{b}, 0, 0)$ can be written as

$$J_x = J(E_x) = \begin{bmatrix} -a & \frac{-\alpha_1\gamma ab}{a^2 + \gamma b(a + \beta b)} & \frac{-\alpha_2 a}{b} \\ 0 & \frac{e_1\alpha_1\gamma ab}{a^2 + \gamma b(a + \beta b)} - h_1 & 0 \\ 0 & 0 & \frac{e_2\alpha_2 a - h_2 b}{b} \end{bmatrix}$$

Hence, the eigenvalues of J_x are:

$$\lambda_{x_1} = -a < 0,\ \lambda_{x_2} = \frac{e_1\alpha_1\gamma ab}{a^2+\gamma b(a+\beta b)} - h_1, \lambda_{x_3} = \frac{e_2\alpha_2 a - h_2 b}{b}$$

Therefore, E_x is locally asymptotically stable if and only if

$$\frac{e_1\alpha_1\gamma ab}{a^2+\gamma b(a+\beta b)} < h_1 \qquad (12a)$$

$$e_2\alpha_2 a < h_2 b \qquad (12b)$$

While E_x is saddle point provided that

$$\frac{e_1\alpha_1\gamma ab}{a^2+\gamma b(a+\beta b)} > h_1 \qquad (12c)$$

$$e_2\alpha_2 a > h_2 b \qquad (12d)$$

The Jacobian matrix of system (1) at the second predator free equilibrium point $E_{xy} = (\hat{x}, \hat{y}, 0)$ can be written as

$$J_{xy} = J(E_{xy}) = \begin{bmatrix} \left(-b + \frac{\alpha_1\gamma\hat{y}(2\hat{x}+\gamma\beta)}{\hat{R}^2}\right)\hat{x} & \frac{-\alpha_1\gamma\hat{x}}{\hat{R}} & -\alpha_2\hat{x} \\ \frac{e_1\alpha_1\gamma\hat{y}(\gamma\beta-\hat{x}^2)}{\hat{R}^2} & 0 & -\delta_1\hat{y} \\ 0 & 0 & e_2\alpha_2\hat{x} - h_2 - \delta_2\hat{y} \end{bmatrix}$$

Where $\hat{R} = \hat{x}^2 + \gamma\hat{x} + \gamma\beta$ Clearly, the eigenvalues of J_{xy} are given by:

$$\lambda_{xy1} + \lambda_{xy2} = \left(-b + \frac{\alpha_1\gamma\hat{y}(2\hat{x}+\gamma\beta)}{\hat{R}^2}\right)\hat{x}$$

$$\lambda_{xy1}.\lambda_{xy2} = \frac{e_1\alpha_1^2\gamma^2\hat{x}\hat{y}(\gamma\beta-\hat{x}^2)}{\hat{R}^3}$$

$$\lambda_{xy3} = e_2\alpha_2\hat{x} - h_2 - \delta_2\hat{y}$$

Therefore, E_{xy} is locally asymptotically stable if and only if

$$b > \frac{\alpha_1\gamma\hat{y}(2\hat{x}+\gamma\beta)}{\hat{R}^2} \qquad (13a)$$

$$\gamma\beta > \hat{x}^2 \tag{13b}$$

$$e_2\alpha_2\hat{x} < h_2 + \delta_2\hat{y} \tag{13c}$$

However, E_{xy} will be unstable point in the R_+^3 if we reversed any one of the above conditions.

The Jacobian matrix of system (1) at the second predator free equilibrium point $E_{xz} = (\bar{x}, 0, \bar{z})$ can be written as:

$$J_{xz} = J(E_{xz}) = \begin{bmatrix} \left(-b + \frac{\alpha_1\gamma\bar{y}(2\bar{x}+\gamma\beta)}{\bar{R}^2}\right)\bar{x} & \frac{-\alpha_1\gamma\bar{x}}{\bar{R}} & -\alpha_2\bar{x} \\ 0 & \frac{e_1\alpha_1\gamma\bar{x}}{\bar{x}^2+\gamma\bar{x}+\gamma\beta} - h_1 - \delta_1\bar{z} & 0 \\ e_2\alpha_2\bar{z} & -\delta_2\bar{z} & 0 \end{bmatrix}$$

Where $\bar{R} = \bar{x}^2 + \gamma\bar{x} + \gamma\beta$ Clearly, the eigenvalues of J_{xz} are given by:

$$\lambda_{xz1} + \lambda_{xz3} = \left(-b + \frac{\alpha_1\gamma\bar{y}(2\bar{x}+\gamma\beta)}{\bar{R}^2}\right)\bar{x}$$

$$\lambda_{xz1}.\lambda_{xz3} = e_2\alpha_2^2\bar{x}\bar{z}$$

$$\lambda_{xz2} = \frac{e_1\alpha_1\gamma\bar{x}}{\bar{x}^2+\gamma\bar{x}+\gamma\beta} - h_1 - \delta_1\bar{z}$$

Therefore, E_{xz} is locally asymptotically stable if and only if

$$b > \frac{\alpha_1\gamma\bar{y}(2\bar{x}+\gamma\beta)}{\bar{R}^2} \tag{14a}$$

$$\frac{e_1\alpha_1\gamma\bar{x}}{\bar{x}^2+\gamma\bar{x}+\gamma\beta} < h_1 + \delta_1\bar{z} \tag{14b}$$

However, E_{xz} will be unstable point in the R_+^3 if we reversed any one of the above conditions.

Theorem (2): Assume that the positive equilibrium point $E_{xyz} = (x^*, y^*, z^*)$ of system (1) exists in $Int.R_+^3$. Then it is locally asymptotically stable provided that the following conditions hold:

$$b > \frac{\alpha_1 \gamma y^* (2x^* + \gamma\beta)}{R^{*2}} \tag{15a}$$

$$e_2 \alpha_1 \gamma \delta_1 R^* > e_1 \alpha_1 \gamma \delta_2 (\gamma\beta - x^{*2}) \tag{15b}$$

Proof: It is easy to verify that, the linearized system of system (1) can be written as

$$\frac{dX}{dT} = \frac{dU}{dT} = J(E_{xyz})U$$

here $X = (x, y, z)^t$ and $U = (u_1, u_2, u_3)^t$ where $u_1 = x - x^*, u_2 = y - y^*$ and $u_3 = z - z^*$ Moreover,

$$J_{xyz} = J(E_{xyz}) = \begin{bmatrix} \left(-b + \frac{\alpha_1 \gamma y^* (2x^* + \gamma\beta)}{R^{*2}}\right) x^* & \frac{-\alpha_1 \gamma x^*}{R^*} & -\alpha_2 x^* \\ \frac{e_1 \alpha_1 \gamma y^* (\gamma\beta - x^{*2})}{R^{*2}} & 0 & -\delta_1 y^* \\ e_2 \alpha_2 z^* & -\delta_2 z^* & 0 \end{bmatrix}$$

Now consider the following positive definite function

$$V = \frac{e_2 u_1^2}{2\delta_2 x^*} + \frac{u_2^2}{2\delta_1 y^*} + \frac{u_3^2}{2\delta_2 z^*}$$

It is clearly that $V : R_+^3 \to R$ and is a continuously differentiable function so that function so that $V(x^*, y^*, z^*) = 0$ and $V(x^*, y^*, z^*) > 0$ otherwise. So by differentiating V with respect to time t , gives

$$\frac{dV}{dt} = \frac{e_2 u_1}{\delta_2 x^*} \cdot \frac{du_1}{dt} + \frac{u_2}{\delta_1 y^*} \cdot \frac{du_2}{dt} + \frac{u_3}{\delta_2 z^*} \cdot \frac{du_3}{dt}$$

Substituting the values of $\frac{du_1}{dt}$, $\frac{du_2}{dt}$ and $\frac{du_3}{dt}$ in the above equation, and after doing some algebraic manipulation; we get that:

$$\frac{dV}{dt} = \frac{e_2}{\delta_2}\left(-b + \frac{\alpha_1\gamma y^*(2x^* + \gamma\beta)}{R^{*2}}\right)u_1^2 - \left(\frac{e_2\alpha_1\gamma}{\delta_2 R^*} - \frac{e_1\alpha_1\gamma(\gamma\beta - x^{*2})}{\delta_1 R^{*2}}\right)u_1u_2 - \left(\frac{e_2\alpha_2}{\delta_2} - \frac{e_2\alpha_2}{\delta_2}\right)u_1u_3 - \left(\frac{\delta_1}{\delta_1} - \frac{\delta_2}{\delta_2}\right)u_2u_3$$

Now it is easy to verify that the above set of conditions (15a)-(15b) guarantee the quadratic terms given below:

$$\frac{dV}{dt} = \frac{e_2}{\delta_2}\left(-b + \frac{\alpha_1\gamma y^*(2x^* + \gamma\beta)}{R^{*2}}\right)u_1^2 - \left(\frac{e_2\alpha_1\gamma}{\delta_2 R^*} - \frac{e_1\alpha_1\gamma(\gamma\beta - x^{*2})}{\delta_1 R^{*2}}\right)u_1u_2$$

So, $\frac{dV}{dt}$ is a negative definite, and hence V is a Lyapunov function. Thus, E_{xyz} is a local asymptotically stable and the proof is complete. ■

In the following the persistence of the system (1) is studied. It well known that the system is said to be persists if and only if each species is persist. Mathematically, this is means that, system (1) is persists if the solution of the system with positive initial condition does not have omega limit sets on the boundary planes of its domain. However, biologically means that, all the species are survivor. In the following theorem the persistence condition of the system (1) is established using the Gard and Hallam technique [4].

Theorem (3): Assume that there are no periodic dynamics in the boundary planes xy and xz respectively. Further, if in addition to conditions (12c), (12d) the following conditions are hold.

$$e_2\alpha_2\hat{x} > h_2 + \delta_2\hat{y} \quad (16a)$$

$$\frac{e_1\alpha_1\gamma\bar{x}}{\bar{x}^2 + \gamma\bar{x} + \gamma\beta} > h_1 + \delta_1\bar{z} \quad (16b)$$

Proof: consider the following function, $\sigma(x,y,z) = x^{p_1} y^{p_2} z^{p_3}$ where $p_i, i = 1,2,3$ undetermined positive constants. Obviously, $\sigma(x,y,z)$ is C^1 positive function defined on R_+^3, and $\sigma(x,y,z) \to 0$, if $x \to 0$ or $y \to 0$ or $z \to 0$. Now since

$$\Psi(x,y,z) = \frac{\sigma'(x,y,z)}{\sigma(x,y,z)} = p_1 \frac{x'}{x} + p_2 \frac{y'}{y} + p_3 \frac{z'}{z}$$

Therefore

$$\Psi(x,y,z) = p_1\left[a - bx - \frac{\alpha_1 \gamma y}{x^2 + \gamma x + \gamma\beta} - \alpha_2 z\right] + p_2\left[\frac{e_1\alpha_1\gamma x}{x^2 + \gamma x + \gamma\beta} - h_1 - \delta_1 z\right] + p_3\left[e_2\alpha_2 x - h_2 - \delta_2 y\right]$$

Now, since it is assumed that there are no periodic attractors in the boundary planes, then the only possible omega limit sets of the system (1) are the equilibrium points E_0, E_x, E_{xy} *and* E_{xz}. Thus according to the Gard technique [4] the proof is follows and the system is uniformly persists if we can proof that $\Psi(.) > 0$ at each of these points. Since

$$\Psi(E_0) = ap_1 - h_1 p_2 - h_2 p_3$$

$$\Psi(E_x) = p_2\left[\frac{e_1\alpha_1\gamma ab}{a^2 + \gamma b(a + \beta b)} - h_1\right] + p_3\left[\frac{e_2\alpha_2 a}{b} - h_2\right] > 0$$

$$\Psi(E_{xy}) = p_3\left[e_2\alpha_2\hat{x} - h_2 - \delta_2\hat{y}\right]$$

$$\Psi(E_{xz}) = p_2\left[\frac{e_1\alpha_1\gamma\bar{x}}{\bar{x}^2 + \gamma\bar{x} + \gamma\beta} - h_1 - \delta_1\bar{z}\right]$$

Obviously, $\Psi(E_0) > 0$ for the value of $p_1 > 0$ sufficiently large than $p_i; i = 2,3$. $\Psi(E_x) > 0$ for any positive constants $p_i; i = 2,3$ provided that conditions (12c)and (12d) hold. However, $\Psi(E_{xy})$ and $\Psi(E_{xz})$ are positive provided that the conditions (16) and (16) are satisfied respectively. Then strictly positive solution of system (1) do not have omega limit set and hence, system (1) is uniformly persistence.

■

5. NUMERICAL SIMULATION

In this section the global dynamics of system (1) is investigated numerically. The system is solved numerically for different sets of parameters values and for different sets of initial conditions, and then the attracting sets and their time series are drown.

For the following set of parameters

$$\begin{aligned} &a = 0.25, b = 0.2, \alpha_1 = 1, \alpha_2 = 0.45, \gamma = 0.75, \beta = 2, \ e_1 = 0.35, \\ &\quad e_2 = 0.35, \delta_1 = 0.01, \delta_2 = 0.01, h_1 = 0.03, h_2 = 0.03 \end{aligned} \tag{17}$$

The attracting sets along with their time series of system (1) are drown in Fig (1). Note that from now onward, in the time series figures, we will use the following representation: **blue color** represents the trajectory of the prey, **green color** represents the trajectory of the first predator and the **red color** represents the trajectory of the second predator.

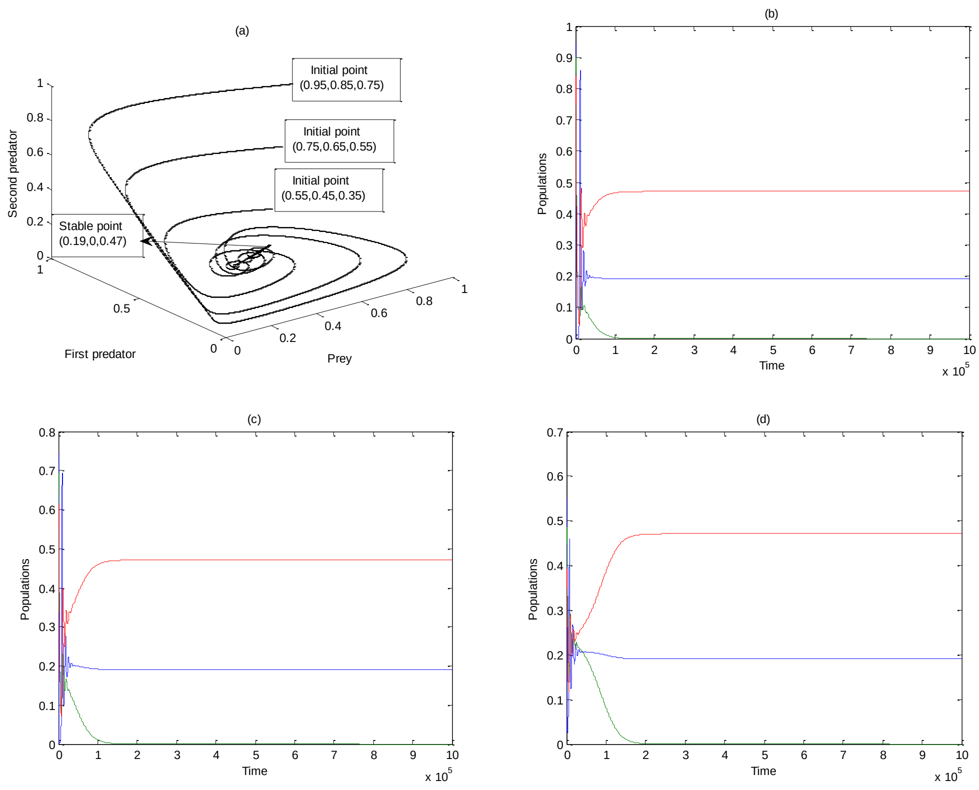

Figure (1): The phase plot of system (1). (a) The solution of system (1) approaches asymptotically to $E_{xz} = (0.19, 0, 0.47)$ initiated at different initial points. (b) Time series of the attractor in (a) initiated at (0.95, 0.85, 0.75). (c) Time series of the attractor in (a) initiated at (0.75,0.65,0.55). (d) Time series of the attractor in (a) initiated at (0.55,0.45,0.35).

Obviously, these figure show that, the system (1) approaches to the globally asymptotically to $E_{xz} = (0.19, 0, 0.47)$ in the $Int.R_{+}^{3}$ starting from different sets of initial conditions. However, for the set of parameters values (17) with $\alpha_2 = 0.41$, system (1) approaches to the globally asymptotically to E_{xy} in the $Int.R_{+}^{3}$ starting from different sets of initial conditions, see **Figure (2).**

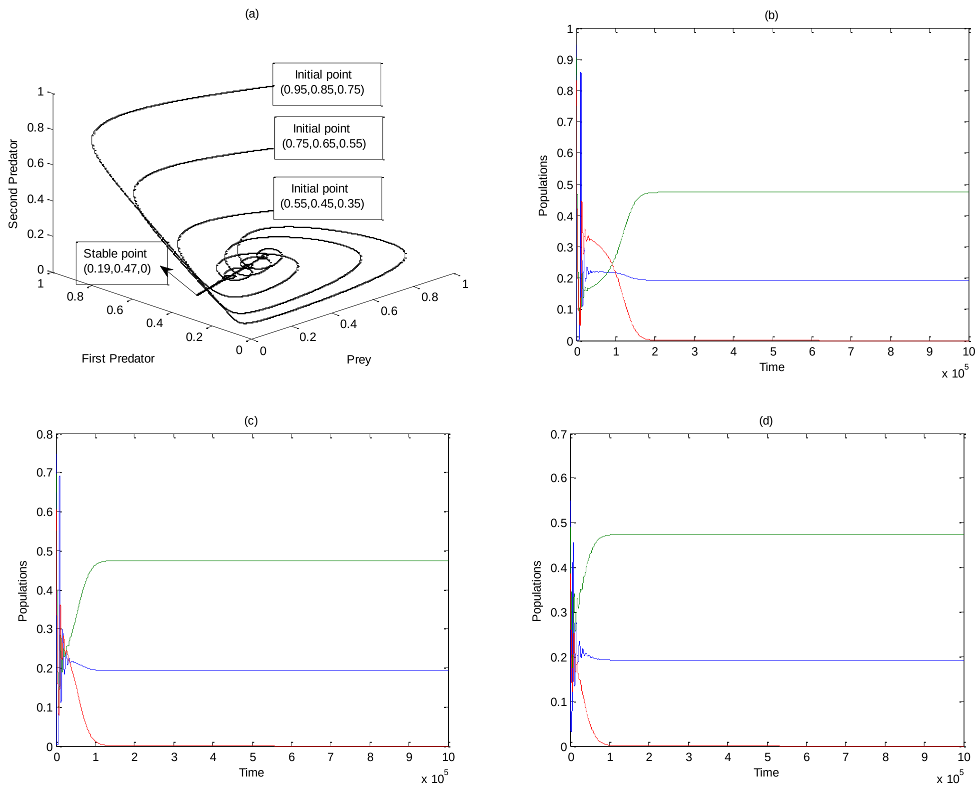

Figure 2: The phase plot of system (1) with $\alpha_2 = 0.41$. (a) The solution of system (1) approaches asymptotically to $E_{xy} = (0.19, 0.47, 0)$ initiated at different initial points. (b) Time series of the attractor in (a) initiated at (0.95,0.85,0.75). (c) Time series of the attractor in (a) initiated at (0.75,0.65,0.55). (d) Time series of the attractor in (a) initiated at (0.55,0.45,0.35).

6. CONCLUSIONS AND DISCUSSION

In this paper, a mathematical model consisting of Holling type I and Holling type IV prey predator model has proposed and analyzed. The model consists of three nonlinear autonomous differential equations that describe the dynamics of three different population namely prey x , first predator y, second predator z. The boundedness of the system (1) has been discussed. The dynamical behavior of system (1) has been investigated locally. To understand the effect of varying parameter on the global dynamics of system (1) and to confirm our obtained analytical results, system (1) has been solved numerically and the following results are obtained:

1. For the set of hypothetical parameters values given Eq. (17), the system (1) approaches asymptotically to E_{xz}.
2. Finally, the predation rate decreases keeping other parameters as in Eq. (17) then the second predator will faces extinction and the solution of system (1) approaches asymptotically to the equilibrium point E_{xy}. However, increasing α_2 causes extinction in the first predator and the solution of system (1) approaches to the equilibrium point E_{xz} .

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