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Even Vertex Odd Mean Labeling of Some Graphs

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Abstract

A graph with p vertices and q edges is said to have an even vertex odd mean labeling if there exists an injective function $f:V(G)\rightarrow\{0, 2, 4, \ldots 2q-2,2q\}$ such that the induced map $f^*: E(G)\rightarrow \{1, 3, 5, \ldots 2q-1\}$ defined by $f^*(uv)=\frac{f(u)+f(v)}{2}$ is a bijection. A graph that admits an even vertex odd mean labeling is called an even vertex odd mean graph. Here we investigate the even vertex odd mean behavior of $G_n^{(+)}(n\geq 3)$, slandding ladder $SL_n(n\geq3)$, $VD(P_n)$, $TW(n)$, double triangular snake $D(Q_n)$, alternative double quadrilateral snake $A(D(Q_n))$, $Q_n \odot K_1$

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1. INTRODUCTION

Through out this paper, by a graph, we mean a finite undirected simple graph. Let G(V,E) be a graph with p vertices and q edges. For notation and terminology, we follow [6]

The graceful labeling of graphs was first introduced by Rosa in 1967. The concept of mean labeling was introduced and studied by Somasundaram and Ponraj [8]. Further some more results on mean graphs are discussed in [4,5]. A graph G is said to be a mean graph if there exists an injective function

$f:V(G) \rightarrow \{0, 1, 2, \ldots q\}$ such that the induced map

$f^*:E(G)\rightarrow\{1, 2, \ldots q\}$ defined by $f^* (uv)= \left| \frac{f(u)+f(v)}{2} \right|$ is a bijection.

In [7], Manickam and Marudai introduced odd mean labeling of a graph. A graph G is said to be odd mean if there exists an injective function $f:V(G)\rightarrow\{0, 1, \ldots 2q-1\}$ defined by $f^*(uv) = \left| \frac{f(u)+f(v)}{2} \right|$ is a bijection. The concept of even mean labeling was introduced and studied by Gayathri and Gopi [3]. A graph G is said to be even mean if there exists an injective function $f: V(G)\rightarrow\{0,1,\ldots 2q\}$ such that the induced map $f^*:E(G)\rightarrow\{2,4,\ldots 2q\}$ defined by $f^*(uv) = \left| \frac{f(u)+f(v)}{2} \right|$ is a bijection.

A graph G is said to have an even vertex odd mean labeling if there exists an injective function $f:V(G)\rightarrow\{0, 2, \ldots 2q-2, 2q\}$ such that the induced map $f^*:E(G)\rightarrow\{1, 3, \ldots 2q-1\}$ defined by $f^*(uv) = \frac{f(u)+f(v)}{2}$ is a bijection. A graph that admits an even vertex odd mean labeling is called even vertex odd mean graph [1, 9].

In this paper, we study the even vertex odd meanness of $G_n^{(+)} (n \geq 3)$, slandding ladder $SL_n (n\geq3)$, $VD(P_n)$, $TW(n)$, double triangular snake $D(Q_n)$, alternative double quadrilateral snake $A(D(Q_n))$, $Q_n \odot K_1$

2. MAIN RESULTS

DEFINITION 2.1

Let G be a mean tree with V(G)= {v_1, v_2, …v_n} and Let G' be a copy of G with {v_1', v_2', …,v_n'}. Then the graph $G_n^{(+)}$ is obtained by joining the vertex v_i with v_i' by an edge for all $1 \le i \le n$.

THEOREM 2.2

The graph $G_n^{(+)}(n \ge 3)$ is a even vertex odd mean graph for any n.

PROOF

Let {v_i, u_i, v_i', u_i'} be the vertices and {b_i, c_i, b_i', c_i', $1 \le i \le n$, a_i, a_i', $1 \le i \le n-1$} be the edges which are denoted as in figure 1.1

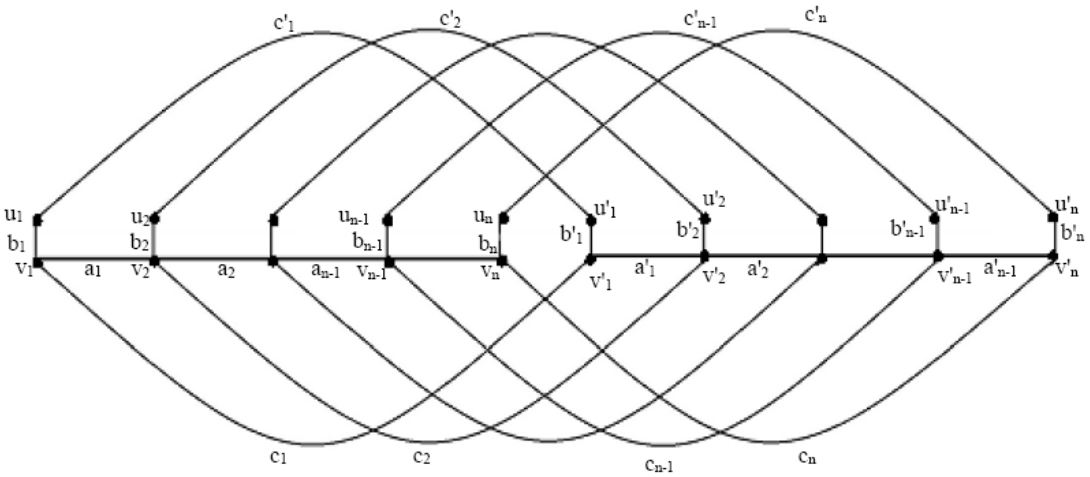

Figure 1.1: Ordinary labeling of $G_n^{(+)}$

First we label the vertices as follows:

Define $f: V \rightarrow \{0, 2, \ldots 2q\}$ by

For $1 \le i \le n$

$$f(v_i) = \begin{cases} 4(i-1) & \text{i is odd} \\ 4i-2 & \text{i is even} \end{cases} \qquad f(v'_i) = \begin{cases} 8n+4i-6 & \text{i is odd} \\ 8n+4i-4 & \text{i is even} \end{cases}$$

$$f(u_i) = \begin{cases} 4i-2 & \text{i is odd} \\ 4i-4 & \text{i is even} \end{cases} \qquad f(u'_i) = \begin{cases} 8n+4(i-1) & \text{i is odd} \\ 8n+4i-6 & \text{i is even} \end{cases}$$

Then the induced edge labels are:

For $1 \le i \le n-1$

$f^*(a_i)=4i-1$; $f^*(a'_i) = 8n+4i-3$

For $1 \le i \le n$

$f^*(b_i)=4i-3$; $f^*(b'_i) = 8n+4i-3$

$$f^*(c_i)=\begin{cases}4n+4i-5 & i \text{ is odd}\\ 4n+4i-3 & i \text{ is even}\end{cases} \qquad f^*(c'_i)=\begin{cases}4n+4i-3 & i \text{ is odd}\\ 4n+4i-5 & i \text{ is even}\end{cases}$$

Therefore, $f^*(E)=\{1, 3, \ldots 2q-1\}$. So, f is a even vertex odd mean labeling and hence, the graph $G_n^{(+)}$ $(n \ge 3)$ is a even vertex odd mean graph for any n.

Even vertex odd mean labeling of $G_4^{(+)}{}_H$ is shown in Figure 1.2

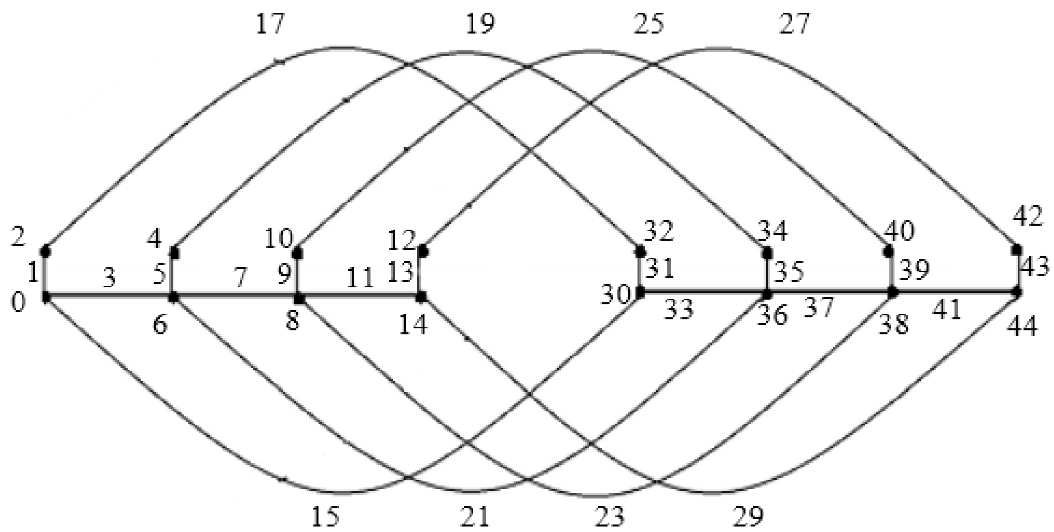

Figure 1.2 Even vertex odd mean labeling of $G_4^{(+)}$

THEOREM 2.3

The slandding ladder SLn($n \ge 3$) is a even vertex odd mean graph.

PROOF :

Let $\{u_i, v_i, 1 \le i \le n\}$ be the vertices and $\{a_i, b_i, c_i, 1 \le i \le n-1\}$ be the edges which are denoted as in figure.1.3

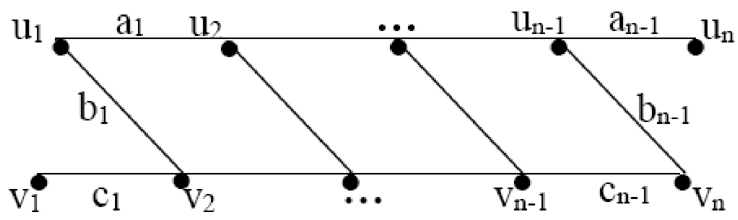

Figure 1.3: Ordinary labeling of SL_n

First we label the vertices as follows:

Define $f:V \to \{0, 2, \ldots 2q\}$

For $1 \le i \le n$

$f(u_i) = 2(i-1)$

$f(v_i) = 4n+2i-6$

Then the induced edge labels are

For $1 \le i \le n-1$

$f^*(a_i) = 2i-1$

$f^*(b_i) = 2n+2i-3$

$f^*(c_i) = 4n+2i-5$

Therefore, $f^*(E) = \{1, 3, \ldots, 2q-1\}$. So, f is a even vertex odd mean labeling and hence, the slandding ladder SL_n $(n \ge 3)$ is a even vertex odd mean graph.

Even vertex odd mean labeling of SL_5 is shown in figure 1.4

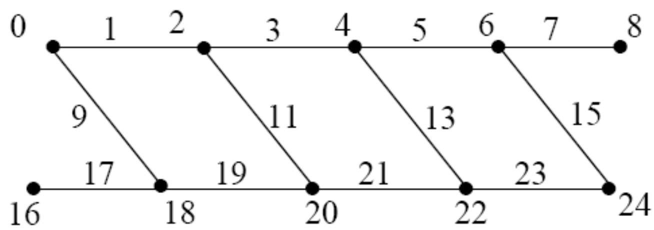

Figure 1.4: Even vertex odd mean labeling of SL_5

THEOREM 2.4

The graph VD(Pn) $(n \geq 4)$ is a even vertex odd mean graph.

PROOF :

Let $\{u', u_i, 1 \leq i \leq n\}$ be the vertices and $\{e'_1, e'_2, e_i, 1 \leq i \leq n-1\}$ be the edges which are denoted as in figure. 1.5

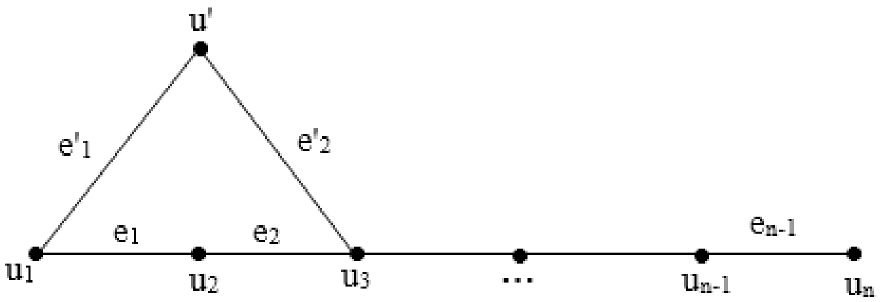

Figure 1.5: Ordinary labeling of $VD(P_n)$

First, we label the vertices as follows:

$f(u') = 6$

$f(u_1) = 0$

$f(u_2) = 2$

For $3 \leq i \leq n$

$f(u_i) = 2(i+1)$

Then the induced edge labels are:

$f^*(e'_1) = 3$; $f^*(e'_2) = 7$

$f^*(e_1) = 1$; $f^*(e_2) = 5$

For $3 \leq i \leq n-1$

$f^*(e_i) = 2i+3$

Therefore, $f^*(E)=\{1, 3, \ldots 2q-1\}$. So, f is a even vertex odd mean labeling and hence, the graph $VD(P_n)$ $(n \geq 4)$ is a even vertex odd mean graph.

Even vertex odd mean labeling of $VD(P_6)$ is shown in figure 1.6.

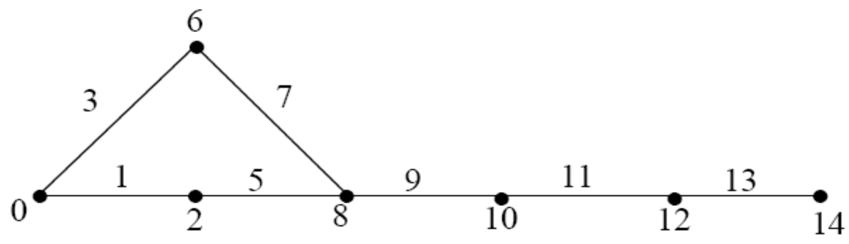

Figure 1.6: Even Vertex Odd Mean Labeling labeling of VD (P_6)

THEOREM 2.5

The twig graph TW(n) ($n \geq 4$) is a even vertex odd mean graph for n is even.

PROOF :

Let $\{u_i, 1 \leq i \leq n, v_i, w_i, 1 \leq i \leq n-2\}$ be the vertices and $\{e_i, 1 \leq i \leq n-1, a_i, b_i, 1 \leq i \leq n-2\}$ be the edges which are denoted as in figure. 1.7

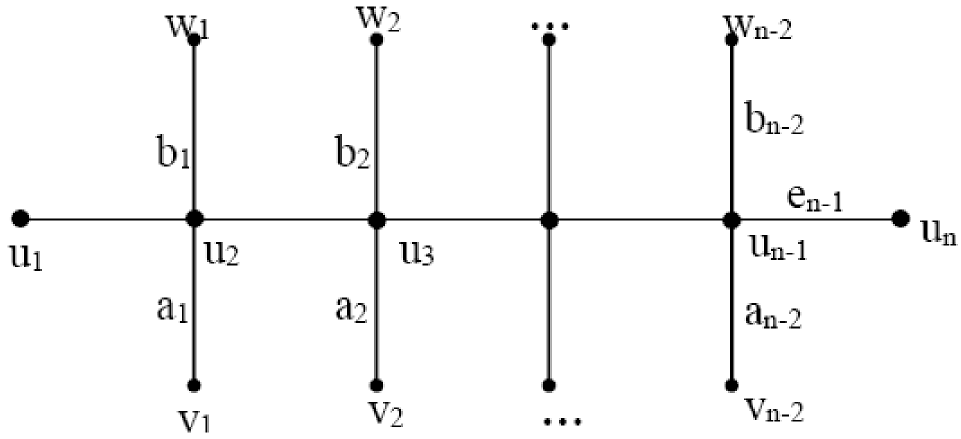

Figure 1.7 : Ordinary Labeling of TW(n)

First we label the vertices as follows:

Define $f: V \rightarrow \{0, 2, \ldots, 2q\}$

For $1 \leq i \leq n$

$$f(u_i) = \begin{cases} 6(i-1) & \text{i is odd} \\ 6i-10 & \text{i is even} \end{cases}$$

For $1 \le i \le n-2$

$$f(v_i) = \begin{cases} 6i+2 & \text{i is odd} \\ 6i-2 & \text{i is even} \end{cases}$$

$$f(w_i) = \begin{cases} 6i-2 & \text{i is odd} \\ 6(i-1) & \text{i is even} \end{cases}$$

Even vertex odd mean labeling of TW(6) is shown in Figure 1.8

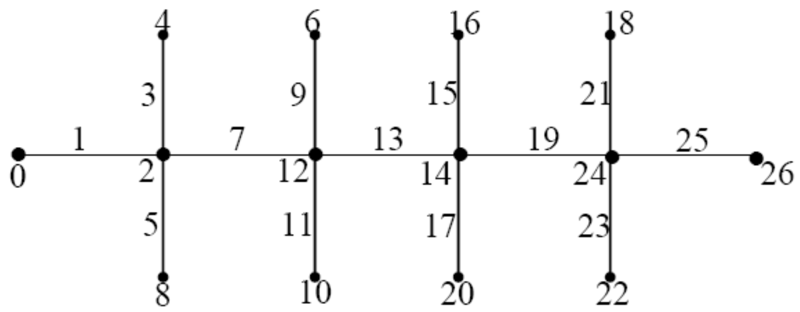

Figure 1.8: Even Vertex odd mean labeling of TW(6)

Then the induced edge labeling are:

For $1 \le i \le n-1$

$f^*(ei)=6i-5$

For $1 \le i \le n-2$

$f^*(a_i) = 6i-1$

$f^*(b_i) = 6i-3$

Therefore, $f^*(E)=\{1, 3, \ldots 2q-1\}$. So, f is a even vertex odd mean labeling and hence, the twig graph TW(n) ($n \ge 4$) is a even vertex odd mean graph for n is even.

THEOREM 2.6

The double triangular snake $D(Q_n)$ is a even vertex odd mean graph for any n.

PROOF:

Let $\{u_i, 1 \leq i \leq n, v_i, w_i, 1 \leq i \leq 2n\text{-}2\}$ be the vertices and $\{e_i, 1 \leq i \leq n\text{-}1, a_i, b_i, 1 \leq i \leq 2n\text{-}2, a'_i, b'_i, 1 \leq i \leq n\text{-}1\}$ be the edges which are denoted as in figure 1.9

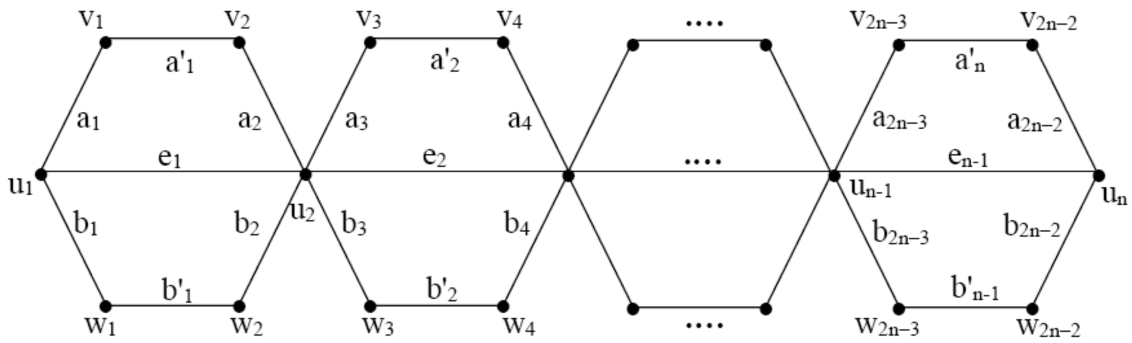

Figure 1.9: Ordinary labeling of D(Qn)

First we label the vertices as follows

Define

$F : V \rightarrow \{0, 2, \ldots, 2q\}$

For $1 \leq i \leq n$

$f(u_i) = 14\,(i\text{-}1)$

For $1 \leq i \leq 2n\text{-}2$

$$f(v_i) = \begin{cases} 7i - 5 & i \text{ is odd} \\ 7i - 6 & i \text{ is even} \end{cases} \qquad f(w_i) = \begin{cases} 7i - 1 & i \text{ is odd} \\ 7i - 2 & i \text{ is even} \end{cases}$$

Then the induced edge labels are :

For $1 \leq i \leq n\text{-}1$

$f^*(ei) = 14\,i - 7$

$f^*(a_i') = 14\,i - 9$; $f^*(b'_i) = 14\,i - 5$

Even Vertex odd mean labeling of $D(Q_3)$ is shown in figure 1.10

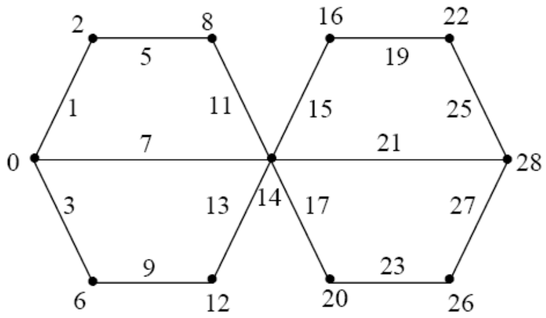

Figure 1.10: Even Vertex odd mean labeling of $D(Q_3)$

For $1 \le i \le 2n-2$

$$f^*(a_i) = \begin{cases} 7i-6 & i \text{ is odd} \\ 7i-3 & i \text{ is even} \end{cases}$$

$$f^*(b_i) = \begin{cases} 7i-4 & i \text{ is odd} \\ 7i-1 & i \text{ is even} \end{cases}$$

Therefore $f^*(E)=\{1, 3, \ldots, 2q-1\}$. So, f is a even vertex odd mean labeling and hence, the double triangular snake D(Qn) is a even vertex odd mean graph for any n.

DEFINITION 2.7

The alternative double quadrilateral snake $A(D(Q_n))$ consists of two alternative quadrilateral snake that have a common path.

THEOREM 2.8

The alternative double quadrilateral snake $A(D(Q_n))$ is a even vertex odd mean graph for any n.

PROOF:

Let $\{u_i, 1 \le i \le n, v_i, w_i, 1 \le i \le n\}$ be the vertices and $\{e_i, 1 \le i \le n-1, a_i, b_i, 1 \le i \le n, a'_i, b'_i, 1 \le i \le n-2\}$ be the edges.

Case (i)

If the double quadrilateral snake starts from u_1

First we label the vertices as follows :

Define

$f : V \rightarrow \{0,2,\ldots,2q\}$

For $1 \leq i \leq n$

$$f(u_i) = \begin{cases} 8(i-1) & i \text{ is odd} \\ 8i-2 & i \text{ is even} \end{cases}$$

$$f(w_i) = \begin{cases} 8i-2 & i \text{ is odd} \\ 8i-4 & i \text{ is even} \end{cases}$$

$$f(v_i) = \begin{cases} 8i-6 & i \text{ is odd} \\ 8(i-1) & i \text{ is even} \end{cases}$$

Then the induced edge labels are :

For $1 \leq i \leq n-1$

$f^*(e_i) = 8i - 1$

For $1 \leq i \leq n$

$$f^*(a_i) = \begin{cases} 8i-7 & i \text{ is odd} \\ 8i-5 & i \text{ is even} \end{cases}$$

$$f^*(b_i) = \begin{cases} 8i-5 & i \text{ is odd} \\ 8i-3 & i \text{ is even} \end{cases}$$

For $1 \leq i \leq n-2$

$f^*(a'_i) = 16i - 11$

$f^*(b'_i) = 16i - 7$

Therefore, $f^*(E) = \{1, 3, \ldots, 2q-1\}$. So f is a even vertex odd mean labeling and hence, the alternative double quadrilateral snake A(D(Qn)) is a even vertex odd mean graph for any n.

Case (ii)

If the double quadrilateral snake starts from u_2

First we label the vertices as follows:

Define $f : V \rightarrow \{0, 2, \ldots, 2q\}$

For $1 \leq i \leq n$

$$f(u_i) = \begin{cases} 8(i-1) & i \text{ is odd} \\ 8i-4 & i \text{ is even} \end{cases}$$

$$f^*(w_i) = \begin{cases} 8i & i \text{ is odd} \\ 8i-2 & i \text{ is even} \end{cases}$$

$$f(v_i) = \begin{cases} 8i-4 & i \text{ is odd} \\ 8i-6 & i \text{ is even} \end{cases}$$

Then the induced edge labels are :

For $1 \leq i \leq n-1$

$f^*(e_i) = 8i - 7$

For $1 \leq i \leq n$

$$f^*(a_i) = \begin{cases} 8i-5 & i \text{ is odd} \\ 8i-3 & i \text{ is even} \end{cases}$$

$$f(b_i) = \begin{cases} 8i-3 & i \text{ is odd} \\ 8i-1 & i \text{ is even} \end{cases}$$

For $1 \leq i \leq n-2$

$f^*(a'_i) = 16i - 9$ $\quad\quad$ $f^*(b'_i) = 16i - 5$

Therefore, $f^*(E) = \{1, 3, \ldots, 2q-1\}$. So f is a even vertex odd mean labeling and hence, the alternative double quadrilateral snake $A(D(Q_n))$ is a even vertex odd mean graph for any n.

Even vertex odd mean labeling of $A(D(Q_4))$ and $A(D(Q_b))$ is shown in figure 1.11 and figure 1.12 respectively.

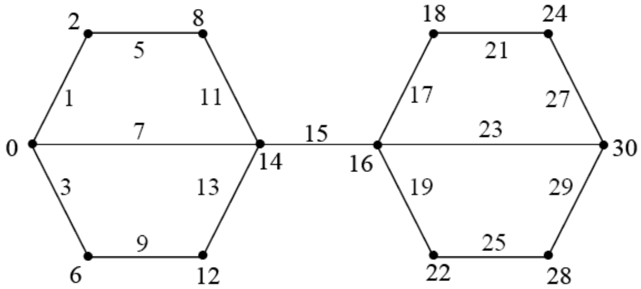

Figure 1.11: Even Vertex odd mean labeling of $A(D(Q_4))$

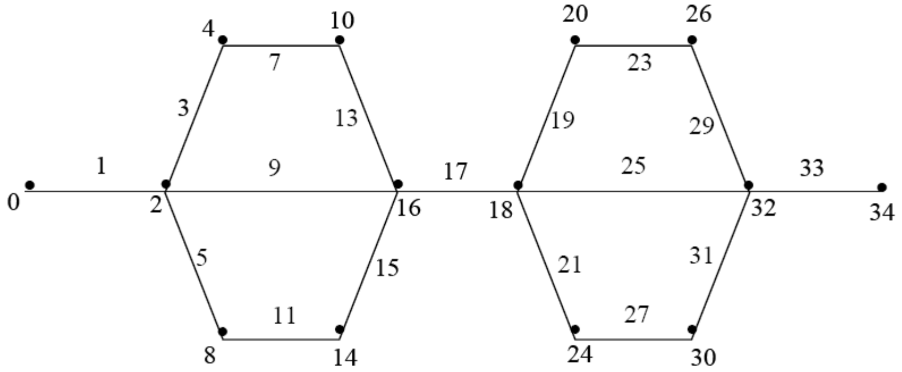

Figure 1.12: Even Vertex odd mean labeling of $A(D(Q_6))$

THEOREM 2.9

The graph $Q_n \odot K$, is a even vertex odd mean graph for any n.

PROOF

Let $\{u_i, u'_i, 1 \le i \le n, v_i, w_i, 1 \le i \le 2n\text{-}2\}$ be the vertices and $\{e_i, e'_i, a'_i, 1 \le i \le n\text{-}1, a_i, b_i, 1 \le i \le 2n\text{-}2\}$ be the edges which are denoted as in figure 1.13.

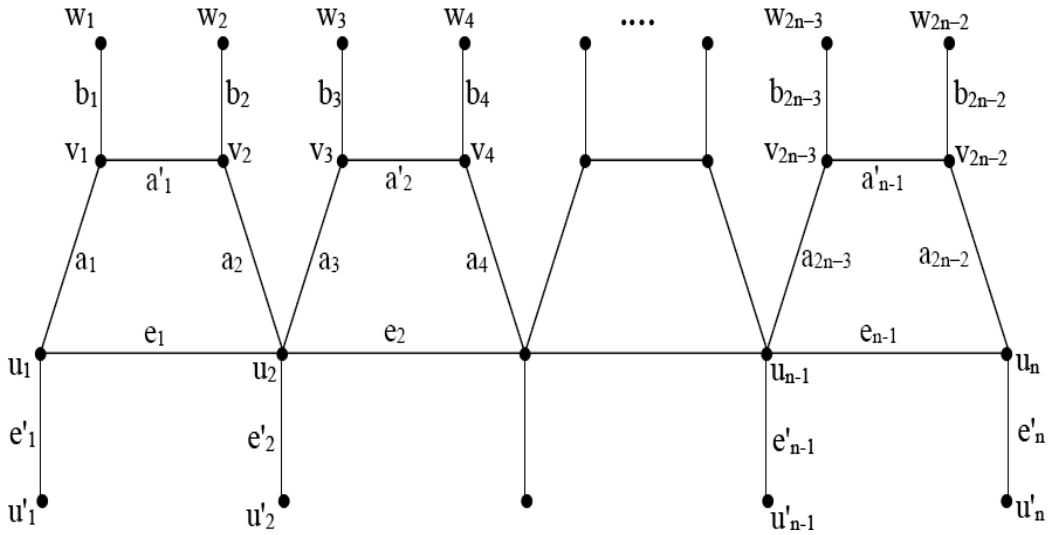

Figure 1.13: Ordinary labeling of $Q_n \odot K_1$

First we label the vertices as follows

Define $f : V \to \{0, 2, \ldots, 2q\}$

$$f(u_1) = 4$$

For $2 \leq i \leq n$

$f(u_i) = 14i$

$f(u'_i) = 6$

For $2 \leq i \leq n$

$f(u'_i) = 14i - 12$

$f(v_1) = 2$

For $2 \leq i \leq 2n-2$

$$f(v_i) = \begin{cases} 7i-1 & i \text{ is odd} \\ 7i-2 & i \text{ is even} \end{cases}$$

$f(w_1) = 0$

For $2 \leq i \leq 2n-2$

$$f(w_i) = \begin{cases} 7i-3 & i \text{ is odd} \\ 7i-4 & i \text{ is even} \end{cases}$$

Then the induced edge labels are

$f^*(e_1) = 9$

For $2 \leq i \leq n-1$

$f^*(e_i) = 14i - 7$

$f^*(e'_1) = 5$

For $2 \leq i \leq n$

$f^*(e'_1) = 14i - 1`3$

For $2 \leq i \leq 2n-2$

$$f^*(a_i) = \begin{cases} 7i-4 & i \text{ is odd} \\ 7i-1 & i \text{ is even} \end{cases}$$

$f^*(b_1) = 1$

For $2 \leq i \leq 2n-2$

$$f^*(b_i) = \begin{cases} 7i-2 & i \text{ is odd} \\ 7i-3 & i \text{ is even} \end{cases}$$

$f^*(a'_1) = 7$

For $2 \leq i \leq n-1$

$f^*(a'_i) = 14i - 5$

Therefore, $f^*(E) = \{1,3,\ldots,2q-1\}$. So f is a even vertex odd mean labeling and hence, the graph $Q_n \odot K_1$ is a even vertex odd mean graph for any n even vertex odd mean labeling of $Q_6 \odot K_1$ is shown in figure 1.14

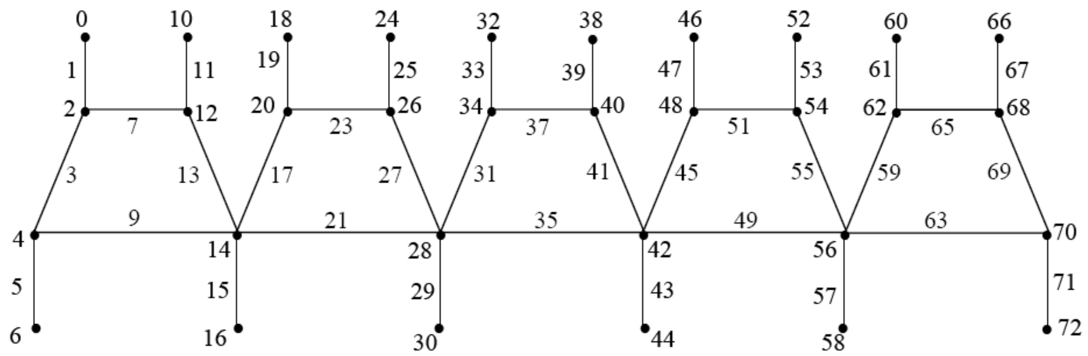

Figure 1.14: Even Vertex odd mean labeling of $Q_6 \odot K_1$

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