

# Square Difference Prime Labeling for Some Snake Graphs

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## Abstract

A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. The graph for which every edge  $(uv)$ , the labels assigned to  $u$  and  $v$  are whole numbers and for each vertex of degree at least 2, the gcd of the labels of the incident edges is 1. Here we characterize some snake graphs for square difference prime labeling.

**Keywords:** Graph labeling, prime labeling, square difference, prime graphs, snake graphs.

## 1. INTRODUCTION

All graphs in this paper are finite and undirected. The symbol  $V(G)$  and  $E(G)$  denotes the vertex set and edge set of a graph  $G$ . The graph whose cardinality of the vertex set is called the order of  $G$ , denoted by  $p$  and the cardinality of the edge set is called the size of the graph  $G$ , denoted by  $q$ . A graph with  $p$  vertices and  $q$  edges is called a  $(p,q)$ - graph.

A graph labeling is an assignment of integers to the vertices or edges. Some basic notations and definitions are taken from [1],[2],[3] and [4] . Some basic concepts are taken from Frank Harary [1]. In this paper we investigated square difference prime labeling of some snake graphs.

## 2. MAIN RESULTS

**Definition 2.1** Let  $G = (V(G), E(G))$  be a graph with  $p$  vertices and  $q$  edges . Define a bijection

$f : V(G) \rightarrow \{0, 1, 2, \dots, p-1\}$  by  $f(v_i) = i - 1$  , for every  $i$  from 1 to  $p$  and define a 1-1 mapping  $f_{sdp}^* : E(G) \rightarrow$  set of natural numbers  $N$  by  $f_{sdp}^*(uv) = |f(u)^2 - f(v)^2|$ . The induced function  $f_{sdp}^*$  is said to be a square difference prime labeling, if for each vertex of degree at least 2, the g c d of the labels of the incident edges is 1.

**Definition 2.2** A graph which admits square difference prime labeling is called a square difference prime graph.

**Definition 2.3** The triangular snake  $T_n$  is obtained from the path  $P_n$  by replacing each edge of the path by a triangle  $C_3$ .

**Definition 2.4** An alternate triangular snake  $A(T_n)$  is obtained from a path  $u_1, u_2, \dots, u_n$  by joining  $u_i, u_{i+1}$  (alternately) to a new vertex  $v_i$ .

**Definition 2.5** A double triangular snake  $D(T_n)$  consist of two triangular snakes that have a common path.

**Theorem: 2.1** An alternate triangular snake  $A(T_n)$  admits square difference prime labeling, when  $n$  is a positive even integer and the triangle starts from the first vertex.

**Proof :** Let  $G = A(T_n)$  and let  $v_1, v_2, \dots, v_{3n/2}$  are the vertices of  $G$ .

Here  $|V(G)| = \frac{3n}{2}$  and  $|E(G)| = 2n - 1$ .

Define a function  $f : V \rightarrow \{0,1,2,\dots,\frac{3n}{2}-1\}$  by

$$f(v_i) = i-1, \quad i = 1,2,\dots,\frac{3n}{2}$$

For the vertex labeling  $f$ , the induced edge labeling  $f_{sdp}^*$  is defined as follows

$$f_{sdp}^*(v_i v_{i+1}) = 2i-1, \quad i = 1,2,3,\dots,\frac{3n}{2}-1$$

$$f_{sdp}^*(v_{3i-2} v_{3i}) = 12i-8, \quad i = 1,2,3,\dots,\frac{n}{2}$$

According to this pattern  $A(T_n)$ , admits square difference prime labeling.

**Example 2.1**  $G = A(T_6)$

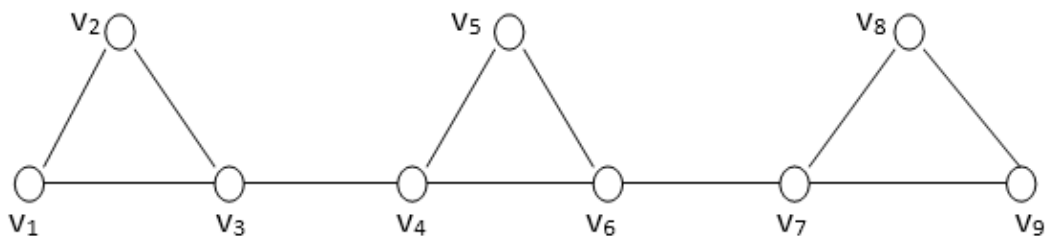


Fig - i

**Theorem: 2.2** An alternate triangular snake  $A(T_n)$  admits square difference prime labeling, when  $n$  is a positive even integer and the triangle starts from the second vertex.

**Proof :** Let  $G = A(T_n)$  and let  $v_1, v_2, \dots, v_{3n-2/2}$  are the vertices of  $G$ .

Here  $|V(G)| = \frac{3n-2}{2}$  and  $|E(G)| = 2n - 3$ .

Define a function  $f : V \rightarrow \{0,1,2,\dots,\frac{3n-2}{2}-1\}$  by

$$f(v_i) = i-1, \quad i = 1,2,\dots,\frac{3n-2}{2}$$

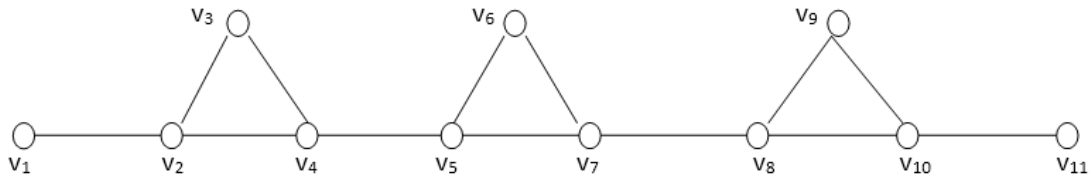
For the vertex labeling  $f$ , the induced edge labeling  $f_{sdp}^*$  is defined as follows

$$f_{sdp}^*(v_i v_{i+1}) = 2i-1, \quad i = 1,2,3,\dots,\frac{3n-4}{2}$$

$$f_{sdp}^*(v_{3i-1} v_{3i+1}) = 12i-4, \quad i = 1,2,3,\dots,\frac{n-2}{2}$$

According to this pattern  $A(T_n)$ , admits square difference prime labeling.

**Example 2.2**  $G = A(T_8)$



**Fig - ii**

**Theorem: 2.3** An alternate triangular snake  $A(T_n)$  admits square difference prime labeling, when  $n$  is a positive odd integer and the triangle starts from the first vertex.

**Proof :** Let  $G = A(T_n)$  and let  $v_1, v_2, \dots, v_{3n-1/2}$  are the vertices of  $G$ .

Here  $|V(G)| = \frac{3n-1}{2}$  and  $|E(G)| = 2n - 2$ .

Define a function  $f : V \rightarrow \{0, 1, 2, \dots, \frac{3n-1}{2} - 1\}$  by

$$f(v_i) = i-1, \quad i = 1, 2, \dots, \frac{3n-1}{2}$$

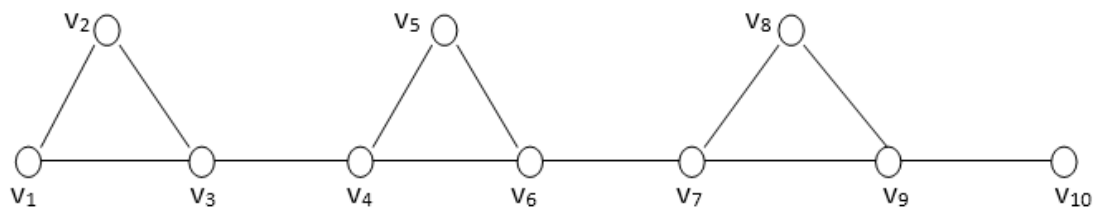
For the vertex labeling  $f$ , the induced edge labeling  $f_{sdp}^*$  is defined as follows

$$f_{sdp}^*(v_i v_{i+1}) = 2i-1, \quad i = 1, 2, 3, \dots, \frac{3n-3}{2}$$

$$f_{sdp}^*(v_{3i-2} v_{3i}) = 12i-8, \quad i = 1, 2, 3, \dots, \frac{n-1}{2}$$

According to this pattern  $A(T_n)$ , admits square difference prime labeling.

**Example 2.3**  $G = A(T_7)$



**Fig - iii**

**Theorem: 2.4** An alternate triangular snake  $A(T_n)$  admits square difference prime labeling, when  $n$  is a positive odd integer and the triangle starts from the second vertex.

**Proof :** Let  $G = A(T_n)$  and let  $v_1, v_2, \dots, v_{3n-1/2}$  are the vertices of  $G$ .

Here  $|V(G)| = \frac{3n-1}{2}$  and  $|E(G)| = 2n - 2$ .

Define a function  $f : V \rightarrow \{0, 1, 2, \dots, \frac{3n-3}{2}\}$  by

$$f(v_i) = i-1, \quad i = 1, 2, \dots, \frac{3n-1}{2}$$

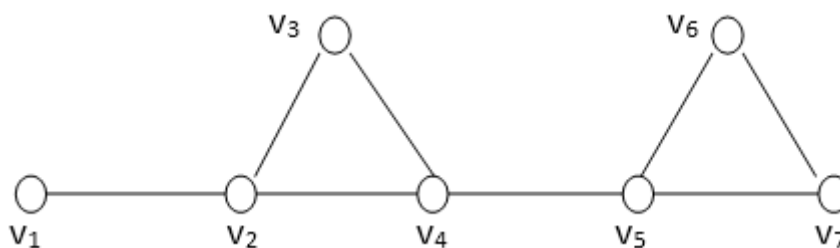
For the vertex labeling  $f$ , the induced edge labeling  $f_{sdp}^*$  is defined as follows

$$f_{sdp}^*(v_i v_{i+1}) = 2i-1, \quad i = 1, 2, 3, \dots, \frac{3n-3}{2}$$

$$f_{sdp}^*(v_{3i-1} v_{3i+1}) = 12i-4, \quad i = 1, 2, 3, \dots, \frac{n-1}{2}$$

According to this pattern  $A(T_n)$ , admits square difference prime labeling.

**Example 2.4**  $G = A(T_5)$



**Fig -iv**

**Theorem: 2.5** The triangular snake  $S_{3,n}$  admits square difference prime labeling.

**Proof :** Let  $G = S_{3,n}$  and let  $v_1, v_2, \dots, v_{2n-1}$  are the vertices of  $G$ .

Here  $|V(G)| = 2n-1$  and  $|E(G)| = 3n-3$

Define a function  $f : V \rightarrow \{0, 1, 2, \dots, 2n - 2\}$  by

$$f(v_i) = i-1, \quad i = 1, 2, \dots, 2n-1$$

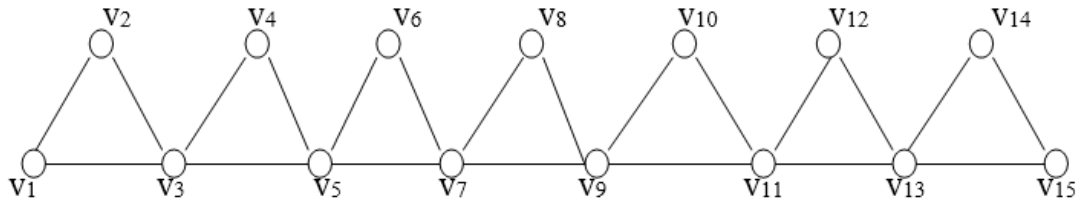
For the vertex labeling  $f$ , the induced edge labeling  $f_{sdp}^*$  is defined as follows

$$f_{sdp}^*(v_i v_{i+1}) = 2i-1, \quad i = 1,2,3,\dots,2n-2$$

$$f_{sdp}^*(v_{2i-1} v_{2i+1}) = 8i-4, \quad i = 1,2,3,\dots, n-1$$

According to this pattern  $S_{3,n}$  admits square difference prime labeling.

**Example : 2.5**  $G = S_{3,8}$



**Fig - v**

**Theorem: 2.6** The double triangular snake  $D(T_n)$  admits square difference prime labeling.

**Proof :** Let  $G = D(T_n)$  and let  $v_1, v_2, \dots, v_{3n-2}$  are the vertices of  $G$ .

Here  $|V(G)| = 3n-2$  and  $|E(G)| = 5n-5$

Define a function  $f : V \rightarrow \{0,1,2,\dots, 3n - 3\}$  by

$$f(v_i) = i-1, \quad i = 1,2,\dots,3n-2$$

For the vertex labeling  $f$ , the induced edge labeling  $f_{sdp}^*$  is defined as follows

$$f_{sdp}^*(v_{3i-2} v_{3i+1}) = 18i-9, \quad i = 1,2,3,\dots,n-1$$

$$f_{sdp}^*(v_{3i-2} v_{3i}) = 12i-8, \quad i = 1,2,3,\dots,n-1$$

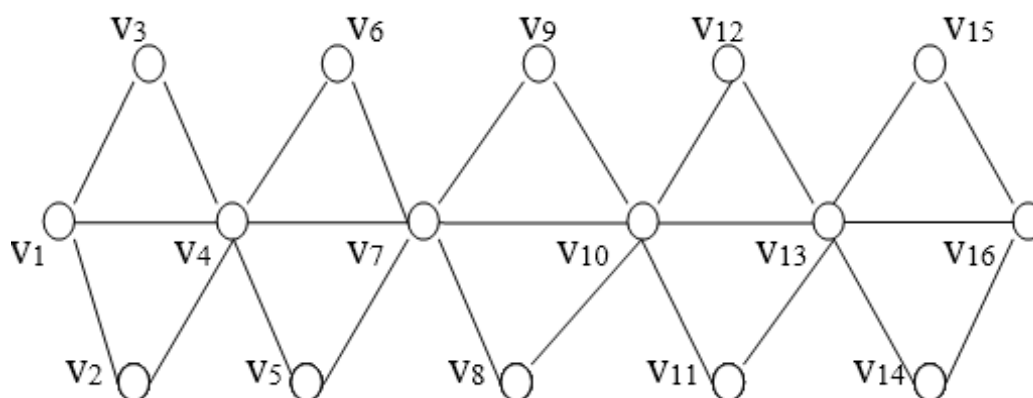
$$f_{sdp}^*(v_{3i-2} v_{3i-1}) = 6i-5, \quad i = 1,2,3,\dots,n-1$$

$$f_{sdp}^*(v_{3i} v_{3i+1}) = 6i-1, \quad i = 1,2,3,\dots,n-1$$

$$f_{sdp}^*(v_{3i-1} v_{3i+1}) = 12i-4, \quad i = 1,2,3,\dots,n-1$$

According to this pattern  $D(T_n)$  admits square difference prime labeling.

**Example :2.6**  $G = D(T_6)$



**Fig -vi**

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