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An Algorithm for Minimal and Minimum Distance - 2 Dominating Sets of Graph

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Abstract

A set D of vertices in a graph G = (V, E) is a distance - 2 dominating set if every vertex in V-D is within distance 2 of at least one vertex in D. The distance - 2 domination number $\gamma_{\leq 2}(G)$ of G equals the minimum cardinality of a distance - 2 dominating set in G. A distance - 2 dominating set D is called a minimal distance - 2 dominating set if no proper subset of D is a distance - 2 dominating set. In this paper, we present an algorithm for finding a minimal and minimum distance - 2 dominating sets of graph.

Keywords: Dominating set, minimal dominating set, minimum dominating set, distance -2 dominating set, minimal distance -2 dominating set, and minimum distance -2 dominating set.

I. INTRODUCTION

There are many origins to the theory of domination. Historically, the first domination type problems came from chess. In the 1850's, several chess players were interested in the minimum number of queens such that every square on the chess board either contains a queen or attacked by a queen. In 1862, the chess master C. F. de Jaenisch wrote a treatise [8] on the applications of mathematical analysis to chess in which he considered the number of queens necessary to attack every square on a n x n chess board. Apart from chess, domination in graphs has applications to several other fields. There are more than a hundred models of dominating and related types of sets in graphs. Each type of domination sprouted, powered and new results came into fruition resulting in many applications in various fields.

All graphs considered here are simple, finite and undirected. In this paper, the terms and notations used may be found in Haynes [3] or Harary [2]. A non empty set D$\subseteq$V (G) is said to be **a dominating set** of G if every vertex not in D is adjacent to at least one vertex in D. A dominating set D is called a minimal dominating set if no proper subset of D is a dominating set. **The domination number** $\gamma(G)$ is the minimum cardinality taken over all the minimal dominating sets of G.

Definition 1.1

A set D of vertices in a graph G = (V, E) is a distance - 2 dominating set if every vertex in V-D is within distance 2 of atleast one vertex in D. The distance-2 domination number $\gamma_{\leq 2}(G)$ of G equals the minimum cardinality of a distance 2- dominating set in G.

A distance - 2 dominating set D is called a minimal distance - 2 dominating set if no proper subset of D is a distance - 2 dominating set.

Example1.2

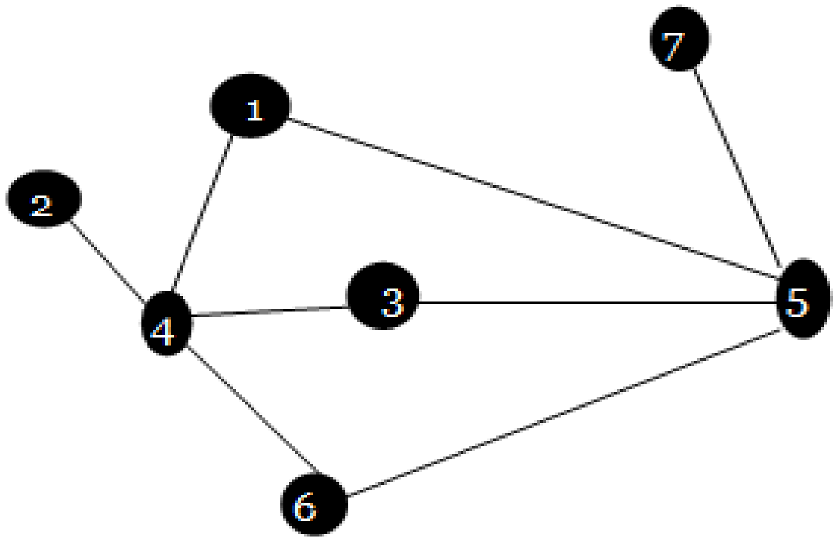

Figure 1: Minimum Distance -2 dominating graph

Here minimum distance -2 dominating set is {1}, {3}, {6}, $\gamma_{\leq 2}(G) = 1$ and minimal distance 2 dominating sets are {2, 7}, {5, 2}.

2. PROPOSED ALGORITHM FOR MINIMAL DISTANCE -2 DOMINATING SET

The following algorithm gives the minimal distance -2 dominating set of a graph.

Working methodologies are described in the following steps.

Step 1: All the vertices in V are initialized to white color.

Step 2: We select any one vertex in V, Change its color to Red and sends a notification to all its neighbors within a distance two. On receiving this notification, the white color neighbor vertices within a distance two are turn into Green color.

Step 3: Now we select any one white color vertex in V which is adjacent to any Green color vertex.

Step 4: Repeat the above process (Step 2- 3) until there are no more white color vertex in the graph.

Step 5: Now, all the Red color vertices in the graph form a minimal distance -2 dominating set.

Example 2.1

Consider the following graph with twenty vertices in V.

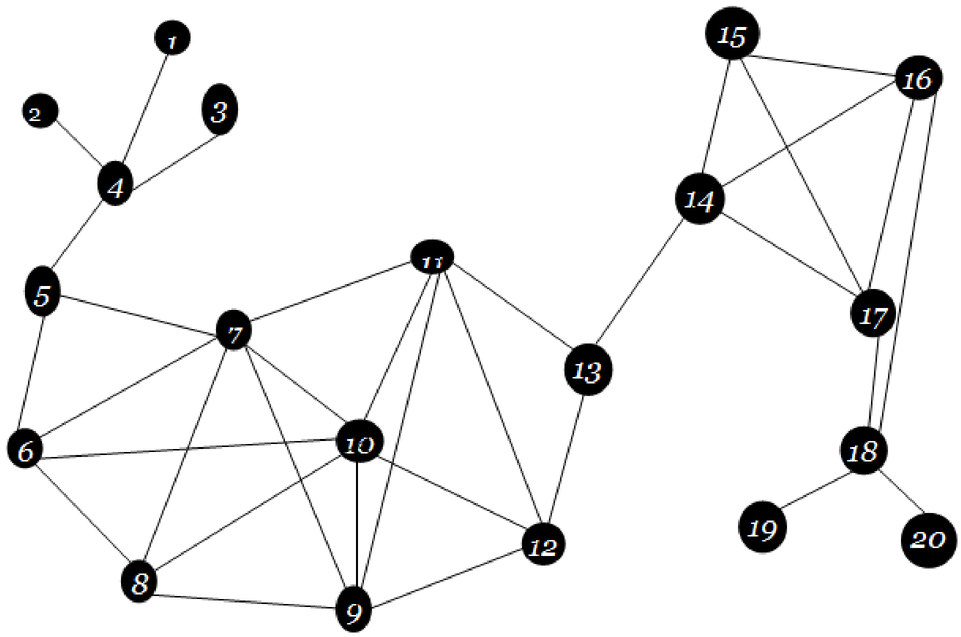

Figure 2: Initialization of vertices

Step 1: All twenty vertices are initialized to white color as shown in figure 2.

Step 2: We select any one vertex in V (In this case we select vertex 1) is changed to Red color and its white color neighbor which is within the distance two are changed to Green color (Vertices 2,3,4, and 5 are Green colored) as shown in figure 3.

Step 3: Now we select any one white color vertex (In case vertex 7 is selected) in V which is adjacent to any Green color vertex (vertex 5).

Step 4: Now the vertex 7 is changed to Red color and its white color neighbor which is within the distance two are changed to Green color (Vertices 6,8,9,10,11,12 and 13 are Green colored) as shown in figure 4. Again we select any one white color vertex (In case vertex 14 is selected) in V which is adjacent to any Green color vertex (vertex 13). Next the vertex 14 is changed to Red color and its white color neighbor which is within the distance two are changed to Green color (Vertices 15,16,17 and 18 are Green colored) as shown in figure 5. At last two vertices (19 and 20) are pendent white color vertex are changed into Red color vertices. Now there is no more white color vertex in the graph.

Step 5: Now, all the Red color vertices in the graph form a minimal distance -2 dominating set. From the graph in figure 6, it can observe that the set {1, 7, 14, 19, 20} forms a minimal distance -2 dominating set.

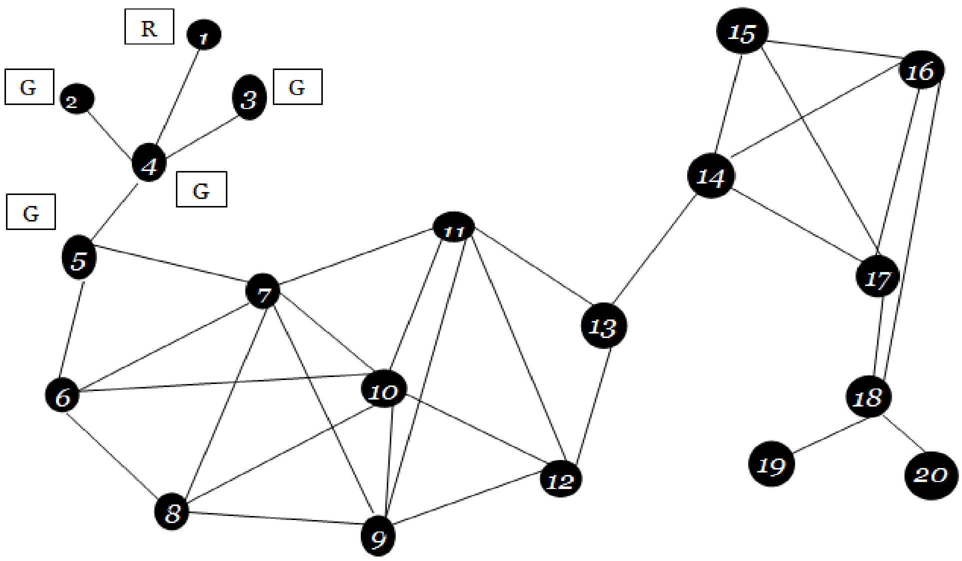

Figure 3: Minimal distance 2 dominating set

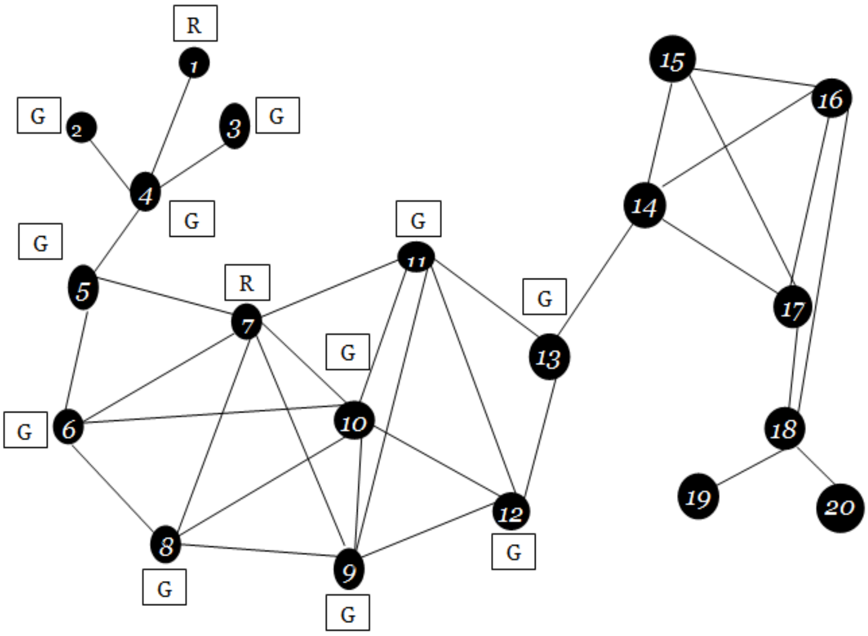

Figure 4: Minimal distance 2 dominating set

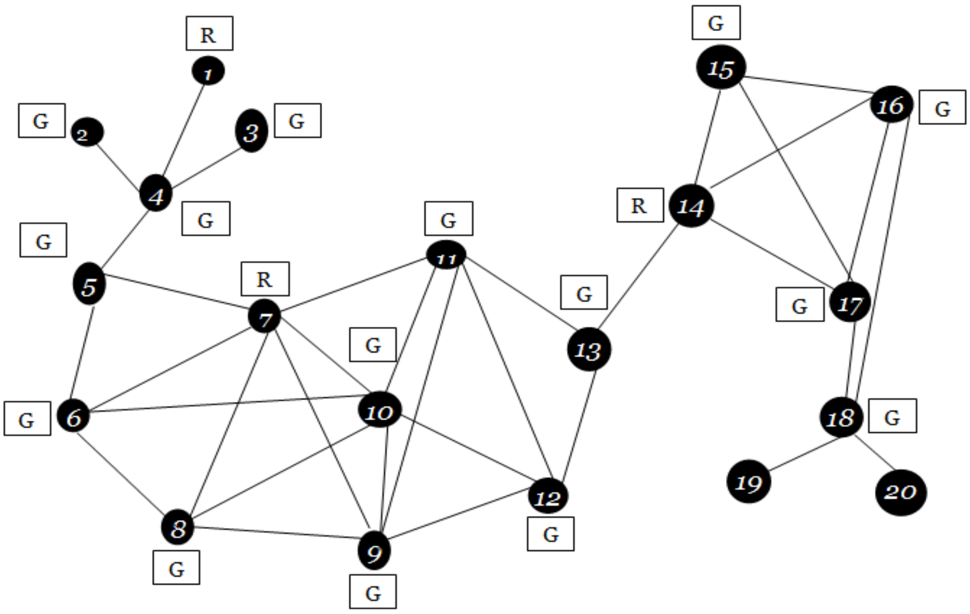

Figure 5: Minimal distance 2 dominating set

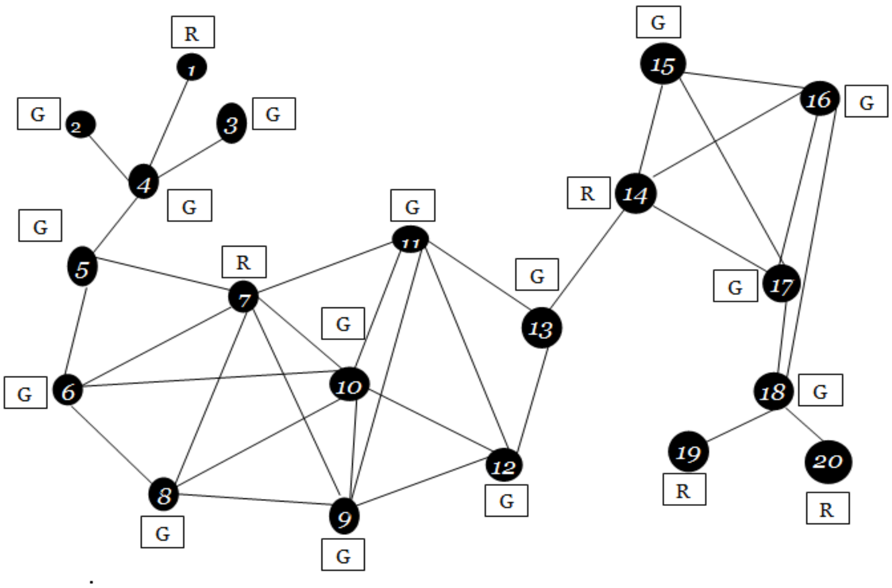

Figure 6: Minimal distance 2 dominating set

3. PROPOSED ALGORITHM FOR MINIMUM DISTANCE -2 DOMINATING SET

The following algorithm gives the minimum distance -2 dominating set of a graph.

This algorithm works with the help of following steps.

Step 1: All the vertices in V are initialized to white color.

Step 2: We select a vertex in V which has maximum degree, (Here we have two vertices, we can choose arbitrarily any one) change its color to Red color and sends a notification to all its neighbors within a distance two. On receiving this notification, the white color neighbor vertices within a distance two turn into Green color.

Step 3: Now we select any one white color vertex in V

Case 1: If the white color vertex has maximum degree, (In case if we have any vertices are equal maximum degree, we select any one) and not adjacent to any Green color vertex in the remaining vertices of V.

Case 2: If the Green color vertex which is exactly at the distance two and has more than one pendent vertex then change the Green color vertex into Red color vertex.

Step 4: Repeat the above process (Step 2- 3) until there are no more white color vertex in the graph.

Step 5: Now, all the Red color vertices in the graph form a minimum distance -2 dominating set.

Example 3.1

Consider the following graph with twenty vertices in V. Now, by applying the vertex sorting procedure i,e., arranging the vertices in the decreasing order of their degrees, we can deduce V={7,10,9,11,4,6,8,12,14,16,17,18,5,13,15,1,2,3,19,20}.

Step 1: All twenty vertices are initialized to white color as shown in figure 2.

Step 2: We select a vertex 7 in V (Here the maximum degree is 6) which has maximum degree, (In case if we have two vertices 7, 10 are equal maximum degree 6, so we select vertex 7) change its color to Red and sends a notification to all its neighbors within a distance two. On receiving this notification, the white color neighbor vertices within a distance two are turn into Green color (vertices 4,5,6,8,9,10,11,12 and 13) as shown in figure 7.

Step 3: Now we select any one white color vertex in V

Case 1: If we select the white color vertex 17 which has maximum degree 4 (Here the vertices14, 16 and 18 have same degree, so we select any one) and not adjacent to any Green color vertex in the remaining vertices of V.

Step 4: Repeat the above process, the vertex 17 is change its color to Red and sends a notification to all its neighbors within a distance two. On receiving this notification, the white color neighbor vertices within a distance two are turn into Green color (vertices 14,15,16,18,19 and 20) as shown in figure 8.

Case 2: If the Green color vertex 4 which is exactly at the distance two and has more than one pendent vertex (pendent vertex 1, 2, 3) then change the Green color vertex into Red color vertex as shown in figure (h).

Step 4: Now there are no more white color vertex in the graph.

Step 5: Now, all the Red color vertices in the graph form a minimum distance -2 dominating set. i.e the minimum distance -2 dominating set is{4,7,17}.

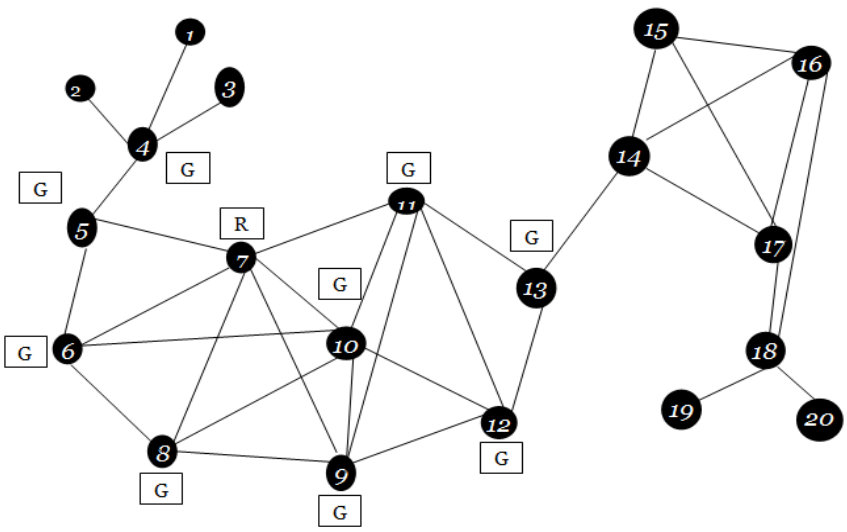

Figure 7: Distance II dominating set graph

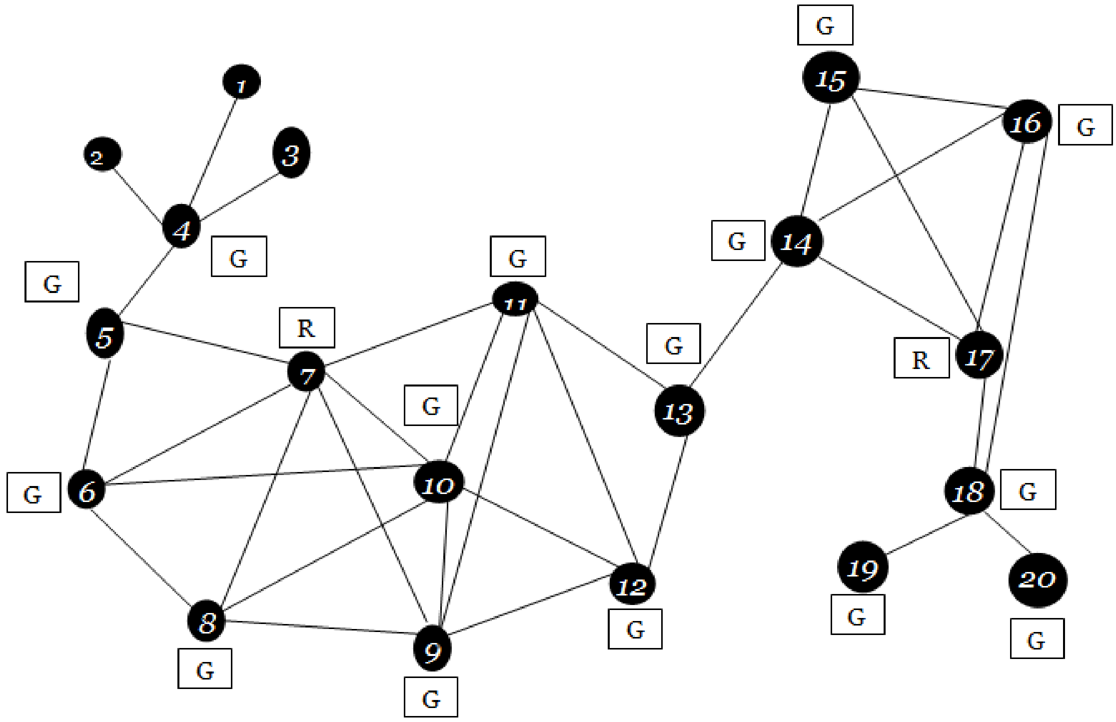

Figure 8: Minimal distance 2 dominating set

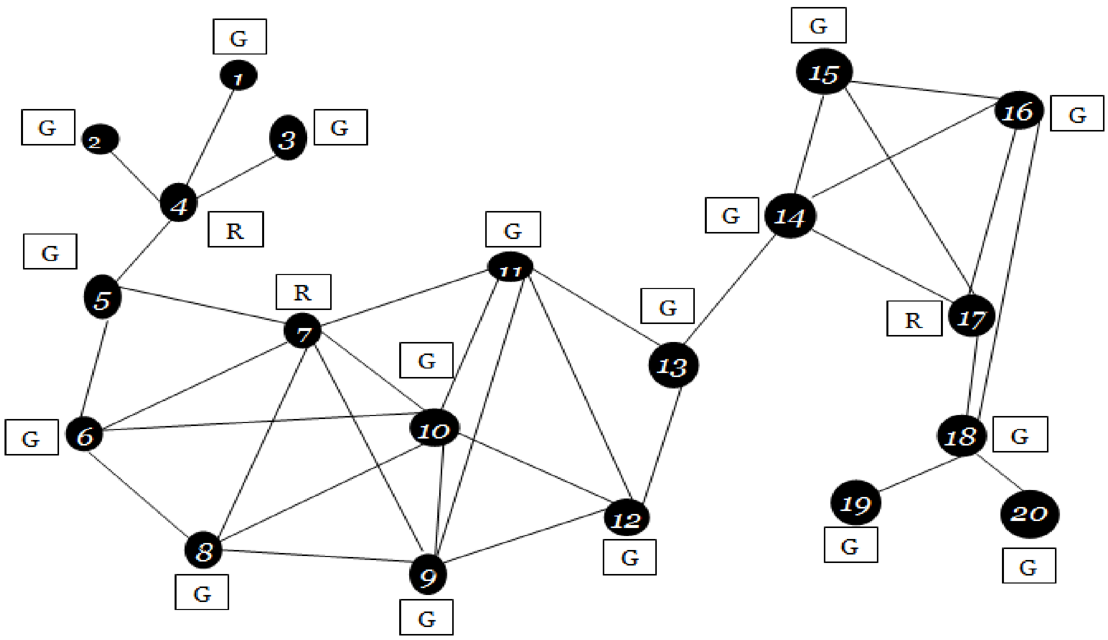

Figure 9: Minimal distance 2 dominating set

4. CONCLUSION

In this paper, A set D of vertices in a graph G = (V, E) is a distance - 2 dominating set if every vertex in V-D is within distance 2 of at least one vertex in D. The distance -2 domination number $\gamma_{\leq 2}(G)$ of G equals the minimum cardinality of a distance - 2 dominating set in G. A distance - 2 dominating set D is called a minimal distance - 2 dominating set if no proper subset of D is a distance - 2 dominating set. We present an algorithm for finding a minimal and minimum distance - 2 dominating sets of graph. The future works will extend the proposed algorithm to an inverse distance – 2 dominating set of a graph.

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