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Two Machines Flow Shop Scheduling with Single Transport Facility and Job-Block Criteria under Fuzzy Environment

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Abstract

This paper pertains to a two machine flowshop scheduling involving independent setup time, transportation time and job block criteria in fuzzy environment. Further, a single transporting agent which carries the jobs from one machine to another is also considered. The objective of the paper is to minimize the makespan, idle time of the machines whenever the processing time, setup time each are under fuzzy environment and are represented by triangular membership function. The scheduling problems with minimization of makespan as one of objective is NP-hard, so exact optimization techniques are impractical. A heuristic algorithm based on some mathematical theorems, to find the optimal sequence of jobs processing with minimum makespan is discussed.

Keywords: Fuzzy membership function, Fuzzy schedule, Average high ranking, Transporting agent, Idle time, Flow time, Job Block.

I. INTRODUCTION

Scheduling is a management process that is used on a usual basis in many mechanized and service industries. It deals with the distribution of resources to everyday jobs over given time period with an objective to optimize one or more objectives. It is also important in transportation, circulation settings and in other types of overhaul industries. Over the last fifty years, a batch of research efforts has been paying attention in deterministic scheduling. But there are situations in which many of these

efforts do not refer to realism due to certain complication and vagueness. In real world, the complexity generally arises from vagueness in the form of uncertainty. The probability theory has been an age old and effective tool to handle uncertainty, but it can be applied only to situations whose characteristics are based on random processes. Uncertainty may arise due to partial information about the problem, or due to information which is not fully reliable, or due to inherent imprecision in language, or due to receipt of information from more than one source etc. Fuzzy logic is an excellent mathematical tool to handle the uncertainty arising due to vagueness. From this prospective, the concept of fuzzy environment is introduced in the theory of scheduling. Fuzzy logic is a method to formalize the human capacity of imprecise reasoning. Such reasoning represents the human ability to reason approximately and judge under uncertainty. Most of the manufacturing system operate in fuzzy environment and hence the optimal or near optimal schedules with respect to the estimated data may become obsolete when they are released to the shop floor. Some examples of such real time events include machine failure, failure of transport facility, electricity breakdown, change in processing time, arrival of some urgent job etc. McCahon and lee [8] discussed the job sequencing with fuzzy processing time. Ishibuchi and Lee [11] addressed the formulation of fuzzy flowshop scheduling problem with fuzzy processing time. Hong and Chuang [13] developed a triangular Johnson algorithm. Martin and Roberto [14] developed fuzzy scheduling with application to real time systems. Some of the noteworthy approaches are due to Johnson [1], Bagga [3], Baker [4], Yager [6], MacCahon [9], MacCarthy and Llu [10], Shukla and Chen [12], Cowling and Johanson [15], Sanuja and Xueyan [16], Singh et al. [17,18].

The present paper is an attempt to extend the study made by Gupta, Sharma and Aggarwal [19] by introducing the concept of job block criteria. The idea of job block has a practical significance to create a balance between the cost of providing priority in service and cost of providing service with non priority customers, .i.e. how much is to be charged extra from the priority customer(s) as compared to non priority customer(s). If J_1 and J_2 are two jobs, then ordered pair (J_1, J_2) is called a job block and designated by a single job β. Further it is assumed that no more jobs can be processed in between J_1 and J_2 on all the machines and job J_2 can be processed before J_1 due to technological constraint.

The rest of paper is organized as follows: the second section deals with some practical situations in which the problem discussed find its applications. Third section is dedicated to the fuzzy set theory. The section four describes various notations used in the paper and the problem formulation in which the problem is formulated. In fifth section, mathematical theorems are established to get the optimal sequence of jobs processing with minimum makespan. In sixth section, a heuristic algorithm on the basis of theorem discussed in previous section is developed to get a sequence of jobs with minimum total flow time. In seventh section, a numerical illustration is carried out to show the performance and efficiency of the algorithm discussed. Finally, we provide a conclusion at the end of this paper.

II. PRACTICAL SITUATION

Scheduling, as a decision making process, plays an important role in most manufacturing and production systems as well as in most information processing environment. Many applied and experimental situations exist in our day-to-day working in factories and industrial production concerns where the different jobs are to be processed on various different machines. When the machines on which jobs are to be processed are planted at different places, the transportation time which includes loading time, moving time and unloading time etc. has a significant role in production concern. For example, in computer systems, the output of a job on one processor may require a communication time to become the input of a succeeding job on another processor. In manufacturing systems, we can consider the case of robotic cells that are found in manufacturing systems for semiconductors or textiles and in which an automated guided vehicle carries displacement jobs. Also, in electroplating workshop where pieces are coated with a metal, the displacement of pieces is done mainly by transporter moving horizontally on a rail. Setup includes work to prepare the machine, process or bench for product parts or the cycle. For example, the machine manufacturing dyes of different types has to be set with a proper type specification before the production start. The concept of job block is significant for the situation in which certain ordering of jobs are prescribed either by technological constraints or by some external imposed policy.

III. BASIC CONCEPTS OF FUZZY SET THEORY

Fuzzy sets first introduced by Zadeh [2] as a mathematical way of representing impreciseness or vagueness in everyday life. The real world is complex; complexity in the world generally arises from uncertainty. In actual situation, scheduling is an enduring process where the existence of real time information frequently forces the review and modification of pre-established schedules. There are three key reasons to use fuzzy set theory in production management. First, imprecision and vagueness are inherent to the decision maker's mental mode of the problem under study. Thus, the decision maker's experience and judgment may be used to complement established theories to foster a better understanding of the problem. Second, in the production management environment, the information required to formulate a model's objective, decision variables, constraints and parameters may be vague or not precisely measurable. Third, imprecision and vagueness as a result of personal bias and subjective opinion may further dampen the quality and quantity of available information. Hence, fuzzy logic can be used to bridge modeling gaps in descriptive and prescriptive decision models in production management.

A fuzzy set $\tilde{A}$ is a collection of elements in a universe of information where the boundary of the set contained in the universe is ambiguous, vague and otherwise fuzzy and is defined by $\tilde{A} = \{(x, \mu_A(x)) : x \in A, \mu_A(x) \in [0,1]\}$. In the pair $(x, \mu_A(x))$, the first

element *x* belong to the classical set A, the second element $\mu_A(x)$ belong to the interval [0,1], called membership function.

A fuzzy set $\tilde{A}$ defined on the universal set of real numbers R, is said to be a fuzzy number if its membership function has the following characteristics:

(i) $\mu_{\tilde{A}} : R \to [0,1]$ is continuous.

(ii) $\mu_{\tilde{A}} = 0$ for all $x \in (-\infty, a_1) \cup (a_3, \infty)$

(iii) $\mu_{\tilde{A}}$ is strictly increasing on [a_1, a_2] and strictly decreasing on [a_2, a_3].

(iv) $\mu_{\tilde{A}} = 1$ for x = a_2.

In the present research paper, we have used triangular fuzzy numbers to represent the vagueness in processing time of jobs. A triangular fuzzy number is represented with three points as follows: $\tilde{A} = (a_1, a_2, a_3)$, where a_1 and a_3 denote the lower and upper limits of support of a fuzzy set $\tilde{A}$.The membership value of the x denoted by $\mu(x), x \in R^+$, can be calculated according to the following formula.

$$\mu(x) = \begin{cases} 0; x \le a_1 \\ \dfrac{x - a_1}{a_2 - a_1}; a_1 < x < a_2 \\ \dfrac{a_3 - x}{a_3 - a_2}; a_2 < x < a_3 \\ o; x \ge a_3 \end{cases}$$

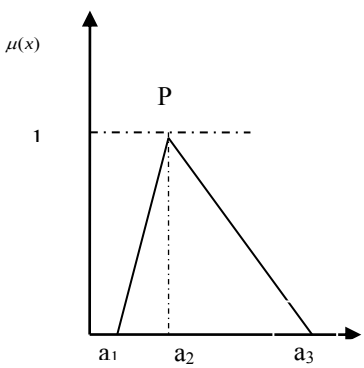

Figure 1: Triangular fuzzy number

To find the optimal sequence, the processing times of the jobs are calculated by using Yager's [6] average high ranking formula (AHR) = $h(A) = \dfrac{3a_2 + a_3 - a_1}{3}$.

If $A_1 = (m_{A_1}, \alpha_{A_1}, \beta_{A_1})$ and $A_2 = (m_{A_2}, \alpha_{A_2}, \beta_{A_2})$ be the two triangular fuzzy numbers, then

(i) $A_1 + A_2 = (m_{A_1}, \alpha_{A_1}, \beta_{A_1}) + (m_{A_2}, \alpha_{A_2}, \beta_{A_2})$

$= (m_{A_1} + m_{A_2}, \alpha_{A_1} + \alpha_{A_2}, \beta_{A_1} + \beta_{A_1})$

(ii) $A_1 - A_2 = (m_{A_1}, \alpha_{A_1}, \beta_{A_1}) - (m_{A_2}, \alpha_{A_2}, \beta_{A_2})$

$= (m_{A_1} - m_{A_2}, \alpha_{A_1} - \alpha_{A_2}, \beta_{A_1} - \beta_{A_2})$ if the following condition is satisfied $DP\left(\tilde{A}_1\right) \geq DP\left(\tilde{A}_2\right)$,

where $DP(\tilde{A}_1) = \dfrac{\beta_{A_1} - m_{A_1}}{2}$ and $DP(\tilde{A}_2) = \dfrac{\beta_{A_2} - m_{A_2}}{2}$. H, say ere DP denotes difference point of a Triangular fuzzy number.

Otherwise; $A_1 - A_2 = (m_{A_1}, \alpha_{A_1}, \beta_{A_1}) - (m_{A_2}, \alpha_{A_2}, \beta_{A_2})$

$= (m_{A_1} - \beta_{A_2}, \alpha_{A_1} - \alpha_{A_2}, \beta_{A_1} - m_{A_2})$

(iii) $kA_1 = k(m_{A_1}, \alpha_{A_1}, \beta_{A_1}) = (km_{A_1}, k\alpha_{A_1}, k\beta_{A_1})$; if $k>0$.

(iv) $kA_1 = k(m_{A_1}, \alpha_{A_1}, \beta_{A_1}) = (k\beta_{A_1}, k\alpha_{A_1}, km_{A_1})$; if $k<0$.

IV. PROBLEM FORMULATION

The following notations will be used all the way through the present paper

A_{ij} : Processing time of the ith job on j^{th} machine

S_{ij} : Setup time of the ith job on j^{th} machine

t_i : Transportation time of the ith job form M_1 to M_2

r_i : Returning time of transporting agent from M_2 to M_1

$h_i(A_j)$: AHR of processing time of ith job on j^{th} machine

$S_i(A_j)$: AHR of setup time of ith job on j^{th} machine.

I_{M1} : Idle time for machine M_1

I_{M2} : Idle time for machine M_2

I_{M1} : Idle time for transporting agent.

Let some job i (i=1, 2, 3… n) are to be processed on two machines M_1 and M_2 in the order M_1M_2 such that no passing is allowed. Let A_{ij} be the processing time and S_{ij} be the setup time of ith job on j^{th} (i=1, 2, 3…., n; j=1,2) machine in fuzzy environment. Let t_i be the transportation time of ith job from machine M_1 to machine M_2 by a transporter. The transporting agent who transports a job from machine M_1 to the machine M_2 returns back to M_1 for carrying next job. Let r_i be the time taken by transporting agent to returns back to machine M_1 for having next job. Let $\beta = (k, m)$ be

the job block. The mathematical model of the problem in matrix form is as shown in Table 1. Our objective is to find the optimal schedule of the jobs processing so as to minimize the total production run time for completing all the jobs.

TABLE 1:

THE PROBLEM IN MATRIX FORM

Jobs	Machine M_1		t_i	r_i	Machine M_2	
i	A_{i1}	S_{i1}			A_{i1}	S_{i2}
1	A_{11}	S_{11}	t_1	r_1	A_{12}	S_{12}
2	A_{21}	S_{21}	t_2	r_2	A_{22}	S_{22}
3	A_{31}	S_{31}	t_3	r_3	A_{32}	S_{32}
4	A_{41}	s_{41}	t_4	r_4	A_{42}	s_{42}
-	-	-	-	-	-	-
n	A_{n1}	S_{n1}	t_n	r_n	A_{n2}	S_{n2}

V. THEOREMS

The following theorems has been established to find the optimal sequence of jobs processing.

Theorem 1: In processing a schedule S = $\{J_1, J_2, J_3, \ldots\ldots, J_k, J_{k+1}, \ldots\ldots\ldots, J_n\}$ of n jobs on two machines M_1 and M_2 in the order M_1M_2 with no passing allowed. The job block (J_k, J_{k+1}) having processing times $\{A_{k,1}, A_{k,2}, A_{(k+1),1}, A_{(k+1),2}\}$ is equal to the single job β. The processing time of job block β on machine M_1 and M_2 denoted respectively by $A_{\beta,1}$ and $A_{\beta,2}$ are given by

$$A_{\beta,1} = A_{k,1} + A_{k+1,1} - \min\{A_{k,2}, A_{k+1,1}\}$$

$$A_{\beta,2} = A_{k,2} + A_{k+1,2} - \min\{A_{k,2}, A_{k+1,1}\}$$

Proof: Let $C_{k,l}$ denote the completion time of kth job (k =1, 2, 3,---, n) on l^{th} machine (l = 1, 2) for the sequence S of jobs.

Therefore, by definition

$$C_{k,2} = \max\left(C_{k,1}, C_{k-1,2}\right) + A_{k,2} \qquad = \max\left(C_{k,1} + A_{k,2}, C_{k-1,2} + A_{k,2}\right)$$

$$C_{k+1,2} = \max\{C_{k+1,1}, C_{k,2}\} + A_{k+1,2}$$

$$= \max\{C_{k+1,1}, C_{k,1} + A_{k,2}, C_{k-1,2} + A_{k,2}\} + A_{k+1,2}$$

$= \max\{C_{k+1,1} + A_{k+1,2}, C_{k,1} + A_{k,2} + A_{k+1,2}, C_{k-1,2} + A_{k,2} + A_{k+1,2}\}$ Since, $C_{k+1,1} = C_{k,1} + A_{k+1,1}$

$$C_{k+1,2} = \max\begin{Bmatrix} C_{k,1} + A_{k+1,1} + A_{k+1,2}, C_{k,1} + A_{k,2} + A_{k+1,2} \\ , C_{k-1,2} + A_{k,2} + A_{k+1,2} \end{Bmatrix}$$

Also, $C_{k+2,2} = \max\{C_{k+2,1}, C_{k+1,2}\} + A_{k+2,2} = \max\begin{Bmatrix} C_{k+2,1}, C_{k,1} + A_{k+1,1} + A_{k+1,2}, \\ C_{k,1} + A_{k,2} + A_{k+1,2}, C_{k-1,2} + A_{k,2} + A_{k+1,2} \end{Bmatrix} + A_{k+2,2}$

Since, $C_{k+2,1} = C_{k+1} + A_{k+1,1} + A_{k+2,1}$

Therefore, we have

$C_{k+2,2} = \max\begin{Bmatrix} C_{k,1} + A_{k+1,1} + A_{k+2,1}, C_{k,1} + A_{k+1,1} + A_{k+1,2}, \\ C_{k,1} + A_{k,2} + A_{k+1,2}, C_{k-1,2} + A_{k,2} + A_{k+1,2} \end{Bmatrix} + A_{k+2,2}$ Since,

$$\max\begin{Bmatrix} C_{k,1} + A_{k+1,1} + A_{k+1,2} \\ , C_{k,1} + A_{k,1} + A_{k+1,2} \end{Bmatrix} = C_{k,1} + \max\{A_{k+1,1}, A_{k,1}\} + A_{k+1,2}$$

Therefore, we have

$$C_{k+2,2} = \max\begin{Bmatrix} C_{k,1} + A_{k+1,1} + A_{k+2,1}, C_{k,1} + \max\{A_{k+1,1}, A_{k,2}\} + A_{k+1,2} \\ , C_{k-1,2} + A_{k,2} + A_{k+1,2} \end{Bmatrix} + A_{k+2,2} \qquad \textbf{--- (1)}$$

Also, $C_{k+2,1} = C_{k-1,1} + A_{k,1} + A_{k+1,1} + A_{k+2,1} = C_{k,1} + A_{k+1,1} + A_{k+2,1}$

--- (2)

Now, let us define a sequence S' of jobs as

$$S' = \{J_1, J_2, J_3, —, J_{k-1}, \beta, J_{k+2}, —, J_n\}$$

Where $A_{\beta,1} = A_{k,1} + A_{K+1,1} - c$ and $A_{\beta,2} = A_{k,2} + A_{K+1,2} - c$; c is a constant. **--- (3)**

Let $C'_{k,l}$ denote the completion time of kth job (k =1, 2, 3,---, n) on l^{th} machine (l = 1, 2) for the sequence S' of jobs.

Therefore, by definition

$$C'_{\beta,2} = \max\left(C'_{\beta,1}, C'_{\beta-1,2}\right) + A_{\beta,2} = \max\left(C'_{\beta,1} + A_{\beta,2}, C'_{\beta-1,2} + A_{\beta,2}\right) \qquad \textbf{--- (4)}$$

$$C'_{k+1,2} = \max\{C'_{k+1,1}, C'_{\beta,2}\} + A_{k+2,2} = \max\{C'_{k+2,1}, C'_{\beta,1} + A_{\beta,2}, C'_{\beta-1,2} + A_{\beta,2}\} + A_{k+2,2} \qquad \textbf{--- (5)}$$

Since, $C'_{k+2,1} = C'_{k-1,1} + A_{\beta,1} + A_{k+2,1}$

$= C_{k-1,1} + (A_{k,1} + A_{k+1,1} - c) + A_{k+2,1} \left[\because C'_{k-1,1} = C_{k-1,1}\right]$

$$= C_{k,1} + A_{k+1,1} - c + A_{k+2,1} \left[\because C_{k,1} = C_{k-1,1} + A_{k,1} \right] \quad \textbf{--- (6)}$$

Also, $C'_{\beta,1} = C'_{k-1,1} + A_{\beta,1} = C_{k-1,1} + A_{k,1} + A_{k+1,1} - c \;\; = C_{k,1} + A_{k+1,1} - c$ **--- (7)**

On combining the results (3), (4), (5), (6) and (7), we have

$$C'_{k+2,2} = \max \begin{Bmatrix} C_{k,1} + A_{k+1,1} - c + A_{k+2,1}, C_{k,1} + A_{k+1,1} - c + A_{k,2} + \\ A_{k+1,2} - c, C_{k,1} + A_{k+1,1} - c + C_{k,2} + C_{k+1,2} - c \end{Bmatrix} + A_{k+2,2} \quad \textbf{--- (8)}$$

Let $c = \min\left\{A_{k+1,1}, A_{k,2}\right\}$, then **---(9)**

$$A_{k+1,1} - c + A_{k,2} = A_{k+1,1} - \min\left\{A_{k+1}, A_{k,2}\right\} + A_{k,2} = \max\left\{A_{k+1,1}, A_{k,2}\right\} \quad \textbf{-- (10)}$$

Also, $C'_{k-1,2} = C_{k-1,2}$ **--- (11)**

On combining results (8), (9), (10) and (11), we have

$$C'_{k+2,2} = \max \begin{Bmatrix} C_{k,1} + A_{k+1,1} + A_{k+2,1} - c, C_{k,1} + \max\left\{A_{k+1,1}, A_{k,2}\right\} \\ + A_{k+1,2} - c, C_{k-2,2} + A_{k,2} + A_{k+1,2} - c \end{Bmatrix} + A_{k+2,2}$$

$$= \max \begin{Bmatrix} C_{k,1} + A_{k+1,1} + A_{k+2,1}, C_{k,1} + \max\left\{A_{k+1}, A_{k,2}\right\} \\ + A_{k+1,2}, C_{k-1,2} + A_{k,2} + A_{k+1,2} \end{Bmatrix} + A_{k+2,2} - c \quad \textbf{--- (12)}$$

From (1) and (12), we have

$$C'_{k+2,2} = C_{k+2,2} - c \quad \textbf{--- (13)}$$

From (2) and (6), we conclude that

$$C'_{k+2,1} = C_{k+2,1} - c \quad \textbf{--- (14)}$$

From results (13) and (14), we observe that the replacement of job-block (J_k,J_{k+1}) in S by job β decreases the completion times of the later job J_{k+1} on both the machines by a constant c in S'. .i.e. if T and T' be the completion times of sequence S and S', then we have $T = T' - c$, .i.e. the completion times on both the machines are changed by a value which is independent of the particular sequence S. Hence, the substitution does not change the relative merit of different sequences. Hence, job block β is equivalent job for job block (J_k,J_{k+1}).

Theorem 2: The optimal schedule for n jobs with processing time A_{ij} ; $i = 1,2,3....,n$; $j = 1,2$ and set up time S_{ij};i =1,2,3….,n ; j= 1,2 for two machines M_1 and M_2 in the fuzzy environment including the transportation time t_i and with r_i as time taken by transporting agent to get back to M_1 for next job is obtained by sequencing the jobs (i-1), i and ($i+1$) such that

$$Min\left(h_i(A_1) + R_{i-1} + t_i + S_{i-1}(A_1), h_{i+1}(A_2) + t_{i+1} + R_i + S_i(A_2)\right) <$$
$$Min\left(h_{i+1}(A_1) + R_i + t_{i+1} + S_i(A_1), h_i(A_2) + t_i + R_{i-1} + S_{i-1}(A_2)\right)$$

Where $R_{i-1} = t_{i-1} + r_{i-1} - A_{i1}$; if $t_{i-1} + r_{i-1} - A_{i1} > 0$ and $R_{i-1} = 0$;otherwise.

Proof: Let S = $\{1,2,3,.....,i-1,i,i+1,.....,n\}$ and S$^{'}$= $\{1,2,3,.....,i-1,i+1,i,.....,n\}$ be two sequences of the jobs. Let $\left(A_{pj}, A'_{pj}\right), j=1,2$ and $\left(CA_{pj}, CA'_{pj}\right), j=1,2$ be the processing time and completion time of the pth job on two machines M_1 and M_2 for the sequences S and S$^{'}$. Let $h_p\left(A_j\right)$, $h_p\left(A'_j\right)$ and $h_pC\left(A_j\right), h_pC\left(A'_j\right)$ are the average high ranking of the processing time and completion time of pth job on machine $M_1$and $M_2$for the sequences S and S$^{'}$ respectively. Let $\left(t_p, t'_p\right)$be the transportation times of pth from machine $M_1$to machine M_2 for the two sequences. Let r_p is the returning time of the transportation agent form machine M_2 to machine M_1after delivering the pth job at machine M_2.

The completion time of the pth job on machine M_2 is

$$h_pC(A_2) = max\left\{(h_pC(A_1)+t_p+R_{p-1}),(h_{p-1}C(A_2)\right\}+S_{p-1}(A_2)+h_p(A_2) \qquad \text{----- (1)}$$

We will choose the sequence S if $h_pC(A_2) < h_pC(A'_2)$ ---- (2)

i.e.if $max\left\{(h_pC(A_1)+t_p+R_{p-1}),(h_{p-1}C(A_2)\right\}+S_{p-1}(A_2)+h_p(A_2) <$
$max\left\{(h_pC(A'_1)+t'_p+R'_{p-1}),(h_{p-1}C(A'_2)\right\}+S_{p-1}(A'_2)+h_p(A'_2)$

Also, $h_nC(A_1)+t_n+R_{n-1} = \sum_{i=1}^{n} h_i(A_1)+t_n+R_{n-1} = h_nC(A'_1)+t'_n+R'_{n-1}$

and $h_n(A_2) = h_n(A'_2)$. ---- (3)

.i.e. result (2) will hold only if $h_{n-1}C(A_2) < h_{n-1}C(A'_2)$

Continuing in this way, we conclude that the inequality (2) will hold if

$h_pC(A_2) < h_pC(A'_2)$; $p = i+1, i+2, i+3, ...n; i = 1,2,3,.....,n-1.$

Now $h_{i+1}C(A_2) = max\left\{(h_{i+1}C(A_1)+t_{i+1}+R_i),(h_iC(A_2)\right\}+S_i(A_2)+h_{i+1}(A_2)$

$= max\left\{(h_{i+1}C(A_1)+t_{i+1}+R_i), max\left\{(h_iC(A_1)+t_i+R_{i-1}),(h_{i-1}C(A_2)\right\}+S_{i-1}(A_2)+h_i(A_2))\right\}+S_i(A_2)+h_{i+1}(A_2)$

$= max\left\{(h_{i+1}C(A_1)+t_{i+1}+R_i), max\left\{(h_iC(A_1)+t_i+R_{i-1}+S_{i-1}(A_2)+h_i(A_2)),(h_{i-1}C(A_2)+S_{i-1}(A_2)+h_i(A_2))\right\}\right\}+S_i(A_2)+h_{i+1}(A_2)$

$$= max\left\{\begin{array}{l}(h_{i+1}C(A_1)+t_{i+1}+R_i+S_i(A_2)+h_{i+1}(A_2)),(h_iC(A_1)+t_i+R_{i-1}+S_{i-1}(A_2)+h_i(A_2)+S_i(A_2)+h_{i+1}(A_2)),\\(h_{i-1}C(A_2)+S_{i-1}(A_2)+h_i(A_2)+S_i(A_2)+h_{i+1}(A_2))\end{array}\right\}$$

$$= max\left\{\begin{array}{l}(h_{i-1}C(A_1)+h_i(A_1)+S_{i-1}(A_1)+h_{i+1}(A_1)+S_i(A_1)+t_{i+1}+R_i+S_i(A_2)+h_{i+1}(A_2)),(h_{i-1}C(A_1)+h_i(A_1)+S_{i-1}(A_1))\\+t_i+R_{i-1}+S_{i-1}(A_2)+h_i(A_2)+S_i(A_2)+h_{i+1}(A_2)),(h_{i-1}C(A_2)+S_{i-1}(A_2)+h_i(A_2)+S_i(A_2)+h_{i+1}(A_2))\end{array}\right\}$$

---- (4)

Similarly, We can have

$h_{i+1}C(A'_2) =$

$$max\left\{\begin{array}{l}(h_{i-1}C(A_1^{'})+h_i(A_1^{'})+S_{i-1}(A_1^{'})+h_{i+1}(A_1^{'})+S_i(A_1^{'})+t_{i+1}^{'}+R_i^{'}+S_i(A_2^{'})+h_{i+1}(A_2^{'})),(h_{i-1}C(A_1^{'})+h_i(A_1^{'})+S_{i-1}(A_1^{'})+t_i^{'}+R_{i-1}^{'}+\\ S_{i-1}(A_2^{'})+h_i(A_2^{'})+S_i(A_2^{'})+h_{i+1}(A_2^{'})),(h_{i-1}C(A_2^{'})+S_{i-1}(A_2^{'})+h_i(A_2^{'})+S_i(A_2^{'})+h_{i+1}(A_2^{'}))\end{array}\right\}$$

---- (5)

Further, for the sequences S and S$^{'}$, we have

$h_{i-1}C(A_1)=h_{i-1}C(A_1^{'});h_{i-1}C(A_2)=h_{i-1}C(A_2^{'}).$

$h_i(A_j)=h_{i+1}(A_j^{'}),\ j=1,2;\ t_i=t_{i+1}^{'};\ h_{i+1}(A_j)=h_i(A_j^{'}),\ j=1,2;$

$t_{i+1}=t_i^{'};\ R_{i-1}=R_i^{'},R_i=R_{i-1}^{'}$ and

$S_{i-1}(A_2)=S_i(A_2^{'}),S_i(A_2)=S_{i-1}(A_2^{'})\ \ S_{i-1}(A_1)=S_i(A_1^{'}),S_i(A_1)=S_{i-1}(A_1^{'})$ ---- (6)

On writing the result (2) for $p = i+1$, we have $h_{i+1}C(A_2)<h_{i+1}C(A_2^{'})$ ---- (7)

On using results (4), (5) & (6) ; (7) can be written as

$$max\left\{\begin{array}{l}(h_{i-1}C(A_1)+h_i(A_1)+S_{i-1}(A_1)+h_{i+1}(A_1)+S_i(A_1)+t_{i+1}+R_i+\\ S_i(A_2)+h_{i+1}(A_2)),(h_{i-1}C(A_1)+h_i(A_1)+S_{i-1}(A_1)+t_i+R_{i-1}+\\ S_{i-1}(A_2)+h_i(A_2)+S_i(A_2)+h_{i+1}(A_2)),(h_{i-1}C(A_2)+S_{i-1}(A_2)+\\ h_i(A_2)+S_i(A_2)+h_{i+1}(A_2))\end{array}\right\}<$$

$$max\left\{\begin{array}{l}(h_{i-1}C(A_1)+h_i(A_1)+S_i(A_1)+h_{i+1}(A_1)+S_{i-1}(A_1)+t_i+R_{i-1}\\ +S_{i-1}(A_2)+h_i(A_2)),(h_{i-1}C(A_1)+h_{i+1}(A_1)+S_i(A_1)+t_{i+1}+R_i\\ +S_{i-1}(A_2)+h_i(A_2)+S_i(A_2)+h_{i+1}(A_2)),(h_{i-1}C(A_2)+S_{i-1}(A_2)\\ +h_i(A_2)+S_i(A_2)+h_{i+1}(A_2))\end{array}\right\}$$

On subtracting $h_{i-1}C(A_2)+S_{i-1}(A_2)+h_i(A_2)+S_i(A_2)+h_{i+1}(A_2)$ from each side we have,

$$max\left\{\begin{array}{l}(h_{i-1}C(A_1)+h_i(A_1)+S_{i-1}(A_1)+h_{i+1}(A_1)+S_i(A_1)+t_{i+1}+\\ R_i+S_i(A_2)+h_{i+1}(A_2)),(h_{i-1}C(A_1)+h_i(A_1)+S_{i-1}(A_1)\\ +t_i+R_{i-1}+S_{i-1}(A_2)+h_i(A_2)+S_i(A_2)+h_{i+1}(A_2))\end{array}\right\}$$

$$<\ max\left\{\begin{array}{l}(h_{i-1}C(A_1)+h_i(A_1)+S_i(A_1)+h_{i+1}(A_1)+S_{i-1}(A_1)+t_i+\\ R_{i-1}+S_{i-1}(A_2)+h_i(A_2)),(h_{i-1}C(A_1)+h_{i+1}(A_1)+S_i(A_1)\\ +t_{i+1}+R_i+S_{i-1}(A_2)+h_i(A_2)+S_i(A_2)+h_{i+1}(A_2))\end{array}\right\}$$

On further subtracting

$(h_{i-1}C(A_1)+h_{i+1}(A_1)+h_i(A_1)+t_i+R_{i-1}+S_{i-1}(A_1)+S_{i-1}(A_2)+h_i(A_2)\ +t_{i+1}+R_i+S_i(A_1)\ +S_i(A_2)+h_{i+1}(A_2))$
from each side, we have

$$max\{(-h_i(A_2)-t_i-R_{i-1}-S_{i-1}(A_2)),(-h_{i+1}(A_1)-t_{i+1}-R_i-S_i(A_1)\}<$$

$$max\{(-h_{i+1}(A_2)-t_{i+1}-R_i-S_i(A_2)),(-h_i(A_1)-t_i-R_{i-1}-S_{i-1}(A_1)\}$$

$$=Min\left(h_i(A_1)+R_{i-1}+t_i+S_{i-1}(A_1),h_{i+1}(A_2)+t_{i+1}+R_i+S_i(A_2)\right)<Min\left(h_{i+1}(A_1)+R_i+t_{i+1}+S_i(A_1),h_i(A_2)+t_i+R_{i-1}+S_{i-1}(A_2)\right)$$

Hence, the theorem is verified for optimality is free.

VI. ALGORITHM

The following algorithm is proposed to obtain the optimal sequence of jobs processing with minimum total elapsed time:

Step 1: Find the average high ranking (AHR) $h_i(A_j)$ and $S_i(A_j)$; *i=1, 2, 3…., n; j=1,2* of the processing time and setup time for all the jobs on two machines M_1 and M_2.

Step 2: Calculate R_{i-1}, where $R_{i-1} = t_{i-1} + r_{i-1} - A_{i1}$ if $t_{i-1} + r_{i-1} - A_{i1} > 0$ and $R_{i-1} = 0$ otherwise.

Step 3: Introduce the two fictitious machines G_i and H_i with processing time

$G_i = R_{i-1} + t_i + h_i(A_1) - S_i(A_2)$ and $H_i = R_{i-1} + t_i + h_i(A_2) - S_i(A_1)$.

Step 4: Take equivalent job $\beta(k,m)$ and calculate the processing time $A_{\beta,1}$ and $A_{\beta,2}$ on the guide lines of Maggu and Das [11] as follows

$A_{\beta,1} = A_{k,1} + A_{m,1} - \min(A_{m,1}, A_{k,2})$; $A_{\beta,2} = A_{k,2} + A_{m,2} - \min(A_{m,1}, A_{k,2})$

Step 5: Using modified Johnson's technique as established in the previous section, find the optimal sequence of jobs processing for two fictious machines G_i and H_i .

Step 6: Prepare the In-Out table for the optimal sequence obtained in step 5 by considering the various parameters. Find the total production run time and idle time for the machines and idle time for which the transporting agent.

VII. NUMERICAL ILLUSTARTION

Consider 5 jobs and 2 machines flowshop scheduling problem in which the processing time, setup time each are under fuzzy environment with significant transportation time and there is a transporting agent taking a job from M_1 to M_2 and returns back to M_1 for next job as given in table 2. Further the jobs 2 and 4 processed as a group job $\beta = (2,4)$. Find the optimal sequence of jobs processing with minimum production run time and the idle time of machines M_1, M_2 and of transporting agent.

TABLE 2

THE MACHINES WITH PROCESSING AND SETUP TIME

Jobs	Machine M_1		t_i	r_i	Machine M_2	
i	A_{i1}	S_{i1}			A_{i1}	S_{i2}
1	(8,9,10)	(1,2,3)	3	2	(7,8,9)	(3,4,5)
2	(10,11,12)	(2,3,4)	6	2	(10,12,14)	(2,4,6)
3	(6,7,8)	(1,2,3)	4	2	(6,7,8)	(2,3,4)
4	(8,10,12)	(2,4,6)	8	2	(9,10,11)	(3,4,5)

5	(7,8,9)	(2,3,4)	5	2	(6,7,8)	(1,2,3)

Solution: As per step 1 & 2, the AHR of the processing time, setup time of the job with transportation time t_i is as follows:

TABLE 3

MACHINES WITH A.H.R

Job	Machine M_1		t_i	r_i	R_{i-1}	Machine M_2	
I	$h_i(A_1)$	$S_i(A_1)$				$h_i(A_2)$	$S_i(A_2)$
1	29/3	8/3	3	2	---	26/3	14/3
2	35/3	11/3	6	2	0	38/3	16/3
3	23/3	8/3	4	2	1/3	23/3	11/3
4	32/3	14/3	8	2	0	32/3	14/3
5	26/3	11/3	5	2	4/3	23/3	8/3

As per step 3, the processing time for the two fictions machines G_i and H_i are

TABLE 4

THE PROCESSING TIME FORTWO FICTIOUS MACHINE

Job(i)	1	β	3	5
G_i	8	37/3	25/3	16
H_i	9	15	28/3	7

Now, by applying modified Johnson's technique, the optimal sequence is S = 1 – 3 – β – 5 = 1 – 3 –2– 4 – 5.

The flow table for the sequence S is as shown in table 5

TABLE 5

FLOW TABLE FOR THE OPTIMAL SEQUENCE S

J	A_{i1}	S_{i1}	R_{M1}	t_i	r_i	R_{M2}	A_{i2}	S_{i2}
1	(8,9,10)	(2,3,4)	(0,0,0)	3	2	(11,12,13)	(18,20,22)	(3,4,5)

3	(15,18,21)	(1,2,3)	(13,14,15)	4	2	(19,22,25)	(27,31,35)	(2,3,4)
2	(26,21,36)	(1,2,3)	(21,24,27)	6	2	(32,27,42)	(42,39,56)	(2,4,6)
4	(36,34,52)	(2,4,6)	(29,32,35)	8	2	(37,40,43)	(53,53,73)	(3,4,5)
5	(46,45,67)	(2,3,4)	(39,42,45)	5	2	(44,47,50)	(62,64,86)	(1,2,3)

The total processing time (Production time) of all the jobs in the system is (62, 64, 86) hours. Idle time for machine M_1 is(17, 18, 19) hours, Idle time for machine M_2 is (17, 15, 23) hours and the transporting agent is idle for (22, 15, 48) hours.

VIII. CONCLUSIONS

In the past, the processing time for each job was usually assumed to be exactly known, but in many real world applications, processing times may vary dynamically due to human factors or operating faults. Fuzzy programming techniques have been developed for flow shop scheduling problems with uncertain processing times. In this paper the concept of job block is introduced in addition to fuzzy processing time and fuzzy setup time. Further we have considered a single constraint of transporting agent which returns back to 1st machine after delivering the job on 2nd machine. The proposed algorithm yields an optimal schedule of job processing with minimum total elapsed time. The present work can further be extended by introducing more than two machines, taking Trapezoidal fuzzy numbers and by considering weighted job.

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