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Deszcz pseudo symmetric type of α-Sasakian manifolds

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Abstract

Recently the present author introduced the notion of *generalized quasi-conformal curvature tensor which bridges Conformal curvature tensor, Concircular curvature tensor, Projective curvature tensor and Conharmonic curvature tensor*. This article attempts to charectrize α-Sasakian manifolds with $\omega(X, Y)\cdot\mathcal{W} = L\{(X\wedge_g Y)\cdot\mathcal{W}\}$. *On the basis of this curvature conditions and by taking into account, the permutation of different curvature tensors we obtained and tabled the nature of the Ricci tensor for the respective pseudo symmetry type* α-Sasakian manifolds.

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1. Introduction

In tune with Yano and Sawaki [10], recently the authors in [6] has introduced and studied *generalized quasi-conformal curvature tensor in the context of* $N(k, \mu)$-manifold. The *generalized quasi-conformal curvature tensor* is defined as

$$\begin{aligned}\mathcal{W}(X, Y)Z &= \frac{2n-1}{2n+1}\Big[(1+2na-b)-\{1+2n(a+b)\}c\Big]C(X, Y)Z \\ &\quad+\Big[1-b+2na\Big]E(X, Y)Z + 2n(b-a)P(X, Y)Z \\ &\quad+\frac{2n-1}{2n+1}(c-1)\{1+2n(a+b)\}\hat{C}(X, Y)Z \qquad (1.1)\end{aligned}$$

for all X, Y & $Z \in \chi(M)$, the set of all vector field of the manifold M, where a, b & c are real constants. The beauty of *generalized quasi-conformal curvature tensor* lies in the fact that it has the flavour of Riemann curvature tensor R for $a = b = c = 0$, Conformal curvature tensor C [11] for $a = b = -\frac{1}{2n-1}$ & $c = 1$, Conharmonic curvature tensor $\hat{C}$ [16] for $a = b = -\frac{1}{2n-1}$& $c = 0$, Concircular curvature tensor E ([9],p.84) for $a = b = 0$ & $c = 1$, Projective curvature tensor P([9],p.84) for $a = -\frac{1}{2n}$, $b = 0$ & $c = 0$ and m-projective curvature tensor H [5], for $a = b = -\frac{1}{4n}$ & $c = 0$. The equation (1.1) can also be written as

$$\begin{aligned} \mathcal{W}(X,Y)Z &= R(X,Y)Z + a\Big[S(Y,Z)X - S(X,Z)Y\Big] \\ &\quad +b\Big[g(Y,Z)QX - g(X,Z)QY\Big] \\ &\quad -\frac{cr}{2n+1}\left(\frac{1}{2n} + a + b\right)\Big[g(Y,Z)X - g(X,Z)Y\Big]. \end{aligned} \tag{1.2}$$

An Sasakian manifold (M^{2n+1}, g), $n \geq 1$, is said to be pseudo-symmetry type according to Deszcz ([13]) (respectively Ricci pseudo symmetry type [14]) if

$$\omega(X,Y)\cdot\mathcal{W} = L\{(X\wedge_g Y)\cdot\mathcal{W}\} \text{ (resp. } \omega(X,Y)\cdot\mathcal{W} = L\{(X\wedge_S Y)\cdot\mathcal{W}\}), \tag{1.3}$$

where ω and $\mathcal{W}$ stand for *generalized quasi-conformal curvature tensor* with the associated scalar triples $(\bar{a}, \bar{b}, \bar{c})$ and (a, b, c) respectively, the dot means that $\omega(X,Y)$ acts as a derivation on $\mathcal{W}$, L is a smooth function and the endomorphism field $X \wedge_A Y$ is defined by

$$(X\wedge_A Y)Z = A(Y,Z)X - A(X,Z)Y, \tag{1.4}$$

for all vector fields X, Y, Z on M and it similarly acts as a derivation on $\mathcal{W}$. In particular, manifold satisfying the condition $\mathcal{R}(X,Y)\cdot\mathcal{R} = L\{(X\wedge_g Y)\cdot\mathcal{R}\}$ (obtained from (1.3) by setting $\bar{a} = \bar{b} = \bar{c}$=0 $= a = b = c$ in (1.3)) is said to be pseudo symmetric. If L is constant, M is called a pseudo symmetric manifold of constant type and if particularly $L = 0$ then M is called a semi-symmetry type manifold for details (see [17], [18] and the references therein). Semi-symmetric spaces are a generalization of locally symmetric spaces ($\nabla R = 0$, [2]) while pseudo symmetric spaces are a natural generalization of semi-symmetric spaces. Note that the Schwarzschild spacetime, the Kottler spacetime, the Reissner-Nordström spacetime, as well as some Friedmann-Lemaître-Robertson-Walker spacetimes are the "oldest" examples of non semi-symmetric pseudo symmetric warped product manifolds ([15]).

Our work is structured as follows. Section 2 is a very brief account of α-Sasakian manifolds. In section 3, we paid special attention to investigate pseudo-symmetry type α-Sasakian manifold. Based *on this condition, we obtained the nature of the Ricci tensor for the respective pseudo-symmetry type Sasakian manifold.*

2. α-Sasakian manifolds

A contact manifold is a $(2n+1)$-dimensional C^∞-manifold M equipped with a global form η, called a contact form of M such that $\eta \wedge (d\eta)^n \neq 0$ everywhere on M. In particular, $\eta \wedge (d\eta)^n \neq 0$ is a volume element of M so that a contact manifold is orientable. A contact manifold associated with the Riemannian metric g is called contact metric manifold if it satisfy the following relation

$$d\eta(X, Y) = g(X, \phi Y),\ \eta(X) = g(X, \xi),\ \phi^2 = -I + \eta \circ \xi, \tag{2.5}$$

Where ϕ is a $(1, 1)$-tensor field and is a unique vector field such that $d\eta(\xi, X) = 0$ and $\eta(\xi) = 1$. We denote the symbols ∇, R and Q by Levi-Civita connection, curvature tensor and Ricci operator of g respectively. We define a $(1, 1)$ type tensor field h by $h = \frac{1}{2}\pounds_\xi \phi$ and we know that h and $h\phi$ are trace free and $h\phi = -\phi h$.

An almost contact manifold $M(\phi, \xi, \eta, g)$ is trans-Sasakian manifold if there exist two function and on M such that

$$(\nabla_X \phi)(Y) = \alpha\{g(X, Y)\xi - \eta(Y)X\} + \beta\{g(\phi X, Y)\xi - \eta(Y)\phi X\}, \tag{2.6}$$

for any vector X, Y on M. If $\beta = 0$ then M is α-Sasakian manifold. Sasakian manifolds is a case of α-Sasakian manifold with $\alpha = 1$. If $\alpha = 0$ then M is called β-Kenmotsu manifold. Kenmotsu manifolds are case of β-Kenmotsu with $\beta = 1$. If both α and β vanish, then M is a cosymplectic manifold. In a α-Sasakian manifold $M^{2n+1}(\phi, \xi, \eta, g)$ the following relations hold [1]:

$$\nabla_X \xi = -\alpha\phi X, \tag{2.7}$$

$$(\nabla_X \phi)(Y) = \alpha\{g(X, Y)\xi - \eta(Y)X\} \tag{2.8}$$

$$g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y) \tag{2.9}$$

for all vector fields X,Y on M^{2n+1}. where α be a smooth functions on M and we say that the α-Sasakian structure is of type $(\alpha, 0)$. From (2.8), it follows that

$$(\nabla_X \eta)(Y) = -\alpha g(\phi X, Y). \tag{2.10}$$

$$R(X, Y)\xi = \alpha^2[\eta(Y)X - \eta(X)Y], \tag{2.11}$$

$$\eta(R(X, Y)Z) = \alpha^2[g(Y, Z)\eta(X) - g(X, Z)\eta(Y)], \tag{2.12}$$

$$R(\xi, X)Y = \alpha^2[g(X, Y)\xi - \eta(Y)X], \tag{2.13}$$

$$S(X, \xi) = 2n\alpha^2\eta(X), \tag{2.14}$$

for any vector fields X, Y on M.

Let A, B be two symmetric $(0, 2)$-tensors on M. Their Kulkarni-Nomizu[8] product $A \wedge B$ is defined on $\chi(M)$ by:

$$\begin{aligned}(A \wedge B)(X, Y, Z, U) \ =\ & A(X, U)B(Y, Z) + A(Y, Z)B(X, U) \\ & -A(X, Z)B(Y, U) - A(Y, U)B(X, Z).\end{aligned} \tag{2.15}$$

In particular, when $A = B = g$, we have the Kulkarni-Nomizu squared $g \wedge g$:

$$(g \wedge g)(X, Y, Z, W) = 2[g(X, W)g(Y, Z) - g(X, Z)g(Y, W)]. \tag{2.16}$$

We notice that

$$(g \wedge g)(X, Y, Z, W) = 2g((X \wedge_g Y)(Z), W). \tag{2.17}$$

This leads to the (0, 4)-tensor $G = (1/2)(g \wedge g)$; it is defined as follows:

$$G(X, Y, Z, W) = g(X, W)g(Y, Z) - g(X, Z)g(Y, W). \tag{2.18}$$

3. Pseudo symmetry type α-Sasakian manifold

Definition 3.1. An α-Sasakian manifold (M^{2n+1}, g), $n \geq 1$ is said to be pseudo symmetry type, if the generalized quasi-conformal tensor ω(or $\mathcal{W}$) admits[7]

$$(\omega(X, Y) \cdot \mathcal{W})(Z, U)V = L_\omega((X \wedge Y) \cdot \mathcal{W})(Z, U)V \tag{3.1}$$

$$\begin{aligned} & \omega(X, Y)\mathcal{W}(Z, U)V - \mathcal{W}(\omega(X, Y)Z, U)V \\ & -\mathcal{W}(Z, \omega(X, Y)U)V - \mathcal{W}(Z, U)\omega(X, Y)V \\ = \; & L\Big[\bar{\mathcal{W}}(Z, U, V, Y)X + \bar{\mathcal{W}}(Z, U, V, X)Y + g(Y, Z)\mathcal{W}(X, U)V \\ & -g(X, Z)\mathcal{W}(Y, U)V + g(Y, U)\mathcal{W}(Z, X)V - g(X, U)\mathcal{W}(Z, Y)V \\ & +g(Y, V)\mathcal{W}(Z, U)X - g(X, V)\mathcal{W}(Z, U)Y\Big]. \end{aligned} \tag{3.2}$$

where ω and $\mathcal{W}$ stands for generalized quasi-conformal curvature tensor with associate scalars $(\bar{a}, \bar{b}, \bar{c})$ and (a, b, c) respectively.

If $\omega \cdot \mathcal{W} = 0$ then M is called generalized quasi-conformally semi-symmetric. A pseudo-symmetric space is said to be proper if it is not semi-symmetric. For details we refer to ([3], [12]).

Let us consider a $(2n+1)$-dimensional pseudo-symmetry type α-Sasakian manifold. Then from the equation (3.2), we get

$$\begin{aligned} & g(\omega(\xi, X)\mathcal{W}(Y, Z)U, \xi) - g(\mathcal{W}(\omega(\xi, X)Y, Z)U, \xi) \\ & -g(\mathcal{W}(Y, \omega(\xi, X)Z)U, \xi) - g(\mathcal{W}(Y, Z)\omega(\xi, X)U, \xi) \\ = \; & L\Big[\bar{\mathcal{W}}(Y, Z, U, /X) - \eta(\mathcal{W}(Y, Z)U)\eta(X) - g(X, Y)\eta(\mathcal{W}(\xi, Z)U) \\ & +\eta(Y)\eta(\mathcal{W}(X, Z)U) - g(X, Z)\eta(\mathcal{W}(Y, \xi)U) + \eta(Z)\eta(\mathcal{W}(Y, X)U) \\ & +\eta(U)\eta(\mathcal{W}(Y, Z)X)\Big]. \end{aligned} \tag{3.3}$$

Putting $X = Y = e_i$ in (3.3) where $\{e_1, e_2, e_3, \ldots, e_{2n}, e_{2n+1} = \xi\}$ is an orthonormal basis of the tangent space at each point of the manifold M and taking the summation over i, $1 \le i \le 2n+1$, we get

$$\begin{aligned}
&\sum_{i=1}^{2n+1}\Big[g(\omega(\xi,e_i)\mathcal{W}(e_i,Z)U,\xi) - g(\mathcal{W}(\omega(\xi,e_i)e_i,Z)U,\xi)\\
&-g(\mathcal{W}(e_i,\omega(\xi,e_i)Z)U,\xi) - g(\mathcal{W}(e_i,Z)\omega(\xi,e_i)U,\xi)\Big]\\
=\;& L\sum_{i=1}^{2n+1}\Big[\bar{\mathcal{W}}(e_i,Z,U,e_i) - \eta(\mathcal{W}(e_i,Z)U)\eta(e_i) - g(e_i,e_i)\eta(\mathcal{W}(\xi,Z)U)\\
&+\eta(e_i)\eta(\mathcal{W}(e_i,Z)U) - g(e_i,Z)\eta(\mathcal{W}(e_i,\xi)U) + \eta(Z)\eta(\mathcal{W}(e_i,e_i)U)\\
&+\eta(U)\eta(\mathcal{W}(e,Z)e_i)\Big].
\end{aligned} \tag{3.4}$$

From the equation (1.2), we can easily bring out the followings

$$\begin{aligned}
&\eta(\mathcal{W}(\xi,U)Z)\\
=\;&\left[\frac{cr}{2n+1}\left(\frac{1}{2n}+a+b\right) - 2n\alpha^2 a - 2n\alpha^2 b - \alpha^2\right]\eta(Z)\eta(U)\\
&+\left[\alpha^2 + 2nb\alpha^2 - \frac{cr}{2n+1}\left(\frac{1}{2n}+a+b\right)\right]g(Z,U) + aS(Z,U).
\end{aligned} \tag{3.5}$$

$$\begin{aligned}
&\sum_{i=1}^{2n+1}\bar{\mathcal{W}}(e_i,Z,U,e_i)\\
=\;&(1-b+2na)S(Z,U) + \left\{br - \frac{2ncr}{2n+1}\left(\frac{1}{2n}+a+b\right)\right\}g(Z,U),
\end{aligned} \tag{3.6}$$

$$\begin{aligned}
&\sum_{i=1}^{2n+1}\eta(\mathcal{W}(e_i,Z)e_i)\\
=\;&\left[-2n\alpha^2(1-a+2nb) - \left\{ar - \frac{2ncr}{2n+1}\left(\frac{1}{2n}+a+b\right)\right\}\right]\eta(Z).
\end{aligned} \tag{3.7}$$

$$\begin{aligned}
&\sum_{i=1}^{2n+1}S(\mathcal{W}(e_i,Z)U,e_i)\\
=\;&\left\{ar + \frac{cr}{2n+1}\left(\frac{1}{2n}+a+b\right)\right\}S(Z,U) - (a+b-1)S^2(Z,U)\\
&+\left\{b\parallel Q\parallel^2 - \frac{cr^2}{2n+1}\left(\frac{1}{2n}+a+b\right)\right\}g(Z,U).
\end{aligned} \tag{3.8}$$

$$\sum_{i=1}^{2n+1} \eta(e_i)\eta(\mathcal{W}(Qe_i, Z)U)$$
$$= 2n\alpha^2\left[\alpha^2 + 2n\alpha^2 b - \frac{cr}{2n+1}\left(\frac{1}{2n} + a + b\right)\right]g(Z,U) + 2n\alpha^2 aS(Z,U)$$
$$-2n\alpha^2\left[\alpha^2 + 2n\alpha^2(a+b) - \frac{cr}{2n+1}\left(\frac{1}{2n} + a + b\right)\right]\eta(Z)\eta(U) \tag{3.9}$$

$$\sum_{i=1}^{2n+1} S(e_i, Z)\eta(\mathcal{W}(e_i, \xi)U)$$
$$= 2n\alpha^2\left[\alpha^2 + 2n\alpha^2(a+b) - \frac{cr}{2n+1}\left(\frac{1}{2n} + a + b\right)\right]\eta(Z)\eta(U)$$
$$-\left\{\alpha^2 + 2n\alpha^2 b - \frac{cr}{2n+1}\left(\frac{1}{2n} + a + b\right)\right\}S(Z,U) - aS^2(Z,U) \tag{3.10}$$

Now, $$\sum_{i=1}^{2n+1} g(\omega(\xi, e_i)\mathcal{W}(e_i, Z)U, \xi)$$
$$= \left[\alpha^2 + 2n\alpha^2\bar{b} - \frac{\bar{c}r}{2n+1}\left(\frac{1}{2n} + \bar{a} + \bar{b}\right)\right]\{\bar{\mathcal{W}}(e_i, Z, U, e_i)$$
$$-\eta(\mathcal{W}(e_i, Z)U)\eta(e_i)\}$$
$$+\bar{a}\Big[S(\mathcal{W}(e_i, Z)U, e_i) - 2n\alpha^2\eta(\mathcal{W}(e_i, Z)U)\eta(e_i)\Big]$$
$$= \left[\alpha^2 + 2n\alpha^2\bar{b} - \frac{\bar{c}r}{2n+1}\left(\frac{1}{2n} + \bar{a} + \bar{b}\right)\right]\bar{\mathcal{W}}(e_i, Z, U, e_i)$$
$$+\bar{a}S(\mathcal{W}(e_i, Z)U, e_i)$$
$$+\left[\frac{\bar{c}r}{2n+1}\left(\frac{1}{2n} + \bar{a} + \bar{b}\right) - 2n\alpha^2\bar{b} - \alpha^2 - 2n\alpha^2\bar{a}\right]\eta(\mathcal{W}(\xi, U)Z). \tag{3.11}$$

In view of (3.6) & (3.8), (3.11) becomes

$$\sum_{i=1}^{2n+1} g(\omega(\xi, e_i)\mathcal{W}(e_i, Z)U, \xi)$$
$$= \Big[\left\{\alpha^2 + 2n\alpha^2\bar{b} - \frac{\bar{c}r}{2n+1}\left(\frac{1}{2n} + \bar{a} + \bar{b}\right)\right\}(1 + 2na - b)$$
$$+\bar{a}\left\{ar + \frac{cr}{2n+1}\left(\frac{1}{2n} + a + b\right)\right\}\Big]S(Z,U) + \bar{a}(1 - a - b)S^2(Z,U)$$

$$+\left\{\frac{\bar{c}r}{2n+1}\left(\frac{1}{2n}+\bar{a}+\bar{b}\right)-2n\alpha^2(\bar{a}+\bar{b})-\alpha^2\right\}\eta(\mathcal{W}(\xi,U)Z)$$
$$+\Big[\left\{\alpha^2+2n\alpha^2\bar{b}-\frac{\bar{c}r}{2n+1}\left(\frac{1}{2n}+\bar{a}+\bar{b}\right)\right\}\left\{br-\frac{2ncr}{2n+1}\left(\frac{1}{2n}+a+b\right)\right\}$$
$$+\bar{a}\left\{b\parallel Q\parallel^2-\frac{cr^2}{2n+1}\left(\frac{1}{2n}+a+b\right)\right\}\Big]g(Z,U). \tag{3.12}$$

$$\begin{aligned}
&\sum_{i=1}^{2n+1} g(\mathcal{W}(\omega(\xi,e_i)e_i,Z)U,\xi)\\
=\;&\left[(2n+1)\left\{\alpha^2+2n\alpha^2\bar{b}-\frac{\bar{c}r}{2n+1}\left(\frac{1}{2n}+\bar{a}+\bar{b}\right)\right\}+\bar{a}r\right]\eta(\mathcal{W}(\xi,U)Z)\\
&+\left\{\frac{\bar{c}r}{2n+1}\left(\frac{1}{2n}+\bar{a}+\bar{b}\right)-2n\alpha^2\bar{a}-\alpha^2\right\}g(\mathcal{W}(e_i,Z)U,\xi)\eta(e_i)\\
&-\bar{b}g(\mathcal{W}(Qe_i,Z)U,\xi)\eta(e_i)\\
=\;&\left[2n\left\{\alpha^2+2n\alpha^2\bar{b}-\frac{\bar{c}r}{2n+1}\left(\frac{1}{2n}+\bar{a}+\bar{b}\right)\right\}+\bar{a}r+2n\alpha^2(\bar{b}-\bar{a})\right]\eta(\mathcal{W}(\xi,U)Z)\\
&-2n\alpha^2\bar{b}\left[\alpha^2+2n\alpha^2b-\frac{cr}{2n+1}\left(\frac{1}{2n}+a+b\right)\right]g(Z,U)-2n\alpha^2a\bar{b}S(Z,U)\\
&+2n\alpha^2\bar{b}\left[\alpha^2+2n\alpha^2(a+b)-\frac{cr}{2n+1}\left(\frac{1}{2n}+a+b\right)\right]\eta(Z)\eta(U)
\end{aligned}\tag{3.13}$$

$$\begin{aligned}
&\sum_{i=1}^{2n+1} g(\mathcal{W}(e_i,\omega(\xi,e_i)Z)U,\xi)\\
=\;&\left\{\alpha^2+2n\alpha^2\bar{b}-\frac{\bar{c}r}{2n+1}\left(\frac{1}{2n}+\bar{a}+\bar{b}\right)\right\}g(e_i,Z)\eta(\mathcal{W}(e_i,\xi)U)\\
&\left\{\frac{\bar{c}r}{2n+1}\left(\frac{1}{2n}+\bar{a}+\bar{b}\right)-2n\alpha^2\bar{a}-\alpha^2\right\}\eta(\mathcal{W}(e_i,e_i)U)\eta(Z)\\
&+\bar{a}S(e_i,Z)\eta(\mathcal{W}(e_i,\xi)U)-\bar{b}\eta(\mathcal{W}(e_i,Qe_i)U)\eta(Z)\\
=\;&\left\{\frac{\bar{c}r}{2n+1}\left(\frac{1}{2n}+\bar{a}+\bar{b}\right)-2n\alpha^2\bar{b}-\alpha^2\right\}\eta(\mathcal{W}(\xi,U)Z)\\
&+2n\alpha^2\bar{a}\left[\alpha^2+2n\alpha^2(a+b)-\frac{cr}{2n+1}\left(\frac{1}{2n}+a+b\right)\right]\eta(Z)\eta(U)\\
&-\bar{a}\left\{\alpha^2+2n\alpha^2b-\frac{cr}{2n+1}\left(\frac{1}{2n}+a+b\right)\right\}S(Z,U)-\bar{a}aS^2(Z,U).
\end{aligned}\tag{3.14}$$

Finally, $\sum_{i=1}^{2n+1} g(\mathcal{W}(e_i, Z)\omega(\xi, e_i)U, \xi)$

$$
\begin{aligned}
&= \left\{\alpha^2 + 2n\alpha^2\bar{b} - \frac{\bar{c}r}{2n+1}\left(\frac{1}{2n} + \bar{a} + \bar{b}\right)\right\} g(e_i, U)\eta(\mathcal{W}(e_i, Z)\xi) \\
&\quad + \left\{\frac{\bar{c}r}{2n+1}\left(\frac{1}{2n} + \bar{a} + \bar{b}\right) - 2n\alpha^2\bar{a} - \alpha^2\right\} \eta(\mathcal{W}(e_i, Z)e_i)\eta(U) \\
&\quad + \bar{a}S(e_i, U)\eta(\mathcal{W}(e_i, Z)\xi) \\
&= \left\{\frac{\bar{c}r}{2n+1}\left(\frac{1}{2n} + \bar{a} + \bar{b}\right) - 2n\alpha^2\bar{a} - \alpha^2\right\} \eta(\mathcal{W}(e_i, Z)e_i)\eta(U) \qquad (3.15)
\end{aligned}
$$

In view of (3.7), (3.15) reduces to

$$
\begin{aligned}
&\sum_{i=1}^{2n+1} g(\mathcal{W}(e_i, Z)\omega(\xi, e_i)U, \xi) \\
&= -\left\{\frac{\bar{c}r}{2n+1}\left(\frac{1}{2n} + \bar{a} + \bar{b}\right) - 2n\alpha^2\bar{a} - \alpha^2\right\} \Big[2n\alpha^2(1 - a + 2nb) \\
&\quad + \left\{ar - \frac{2ncr}{2n+1}\left(\frac{1}{2n} + a + b\right)\right\}\Big]\eta(U)\eta(Z). \qquad (3.16)
\end{aligned}
$$

By virtue of (3.12), (3.13), (3.14) & (3.16), we obtain from (3.4) that

$$
\begin{aligned}
&\Big[\left\{\alpha^2 + 2n\alpha^2\bar{b} - \frac{\bar{c}r}{2n+1}\left(\frac{1}{2n} + \bar{a} + \bar{b}\right) - L\right\}(1 - b) + \alpha^2\bar{a}(1 + 2nb)\Big]S(Z,U) \\
&+\Big[\left\{2n\alpha^2(1 - a + 2nb) + ar - \frac{2ncr}{2n+1}\left(\frac{1}{2n} + a + b\right)\right\} \times \\
&\left\{\frac{\bar{c}r}{2n+1}\left(\frac{1}{2n} + \bar{a} + \bar{b}\right) - 2n\alpha^2\bar{a} - \alpha^2 + L\right\} \\
&+ \left\{\alpha^2 + 2n\alpha^2(a + b) - \frac{cr}{2n+1}\left(\frac{1}{2n} + a + b\right)\right\} \times \\
&\left\{2n\left\{\alpha^2 + 2n\alpha^2\bar{b} - \frac{\bar{c}r}{2n+1}\left(\frac{1}{2n} + \bar{a} + \bar{b}\right) - L\right\} + \bar{a}r - 2n\alpha^2\bar{a}\right\}\Big]\eta(U)\eta(Z) \\
&+\Big[\left\{\alpha^2 + 2n\alpha^2\bar{b} - \frac{\bar{c}r}{2n+1}\left(\frac{1}{2n} + \bar{a} + \bar{b}\right) - L\right\}\left\{br - 2n\alpha^2 - 4n^2\alpha^2 b\right\} \\
&+\bar{a}\left\{b \parallel Q \parallel^2 - \alpha^2 r - 2n\alpha^2 rb\right\} + \bar{a}(1 - b)S^2(Z,U) = 0. \qquad (3.17)
\end{aligned}
$$

Theorem 3.2. Let (M^{2n+1}, g), $n > 1$ be an α-Sasakian manifold. Then for respective pseudo symmetry type conditions, the Ricci tensor of the manifold M takes the respective forms as follows:

Curvature condition	Expression for Ricci tensor
$R(X,Y)\cdot R = L\{(X\wedge Y)\cdot R\}$ (by $\bar{a}=\bar{b}=\bar{c}=0$ & $a=b=c=0$)	For $L\neq\alpha^2$, $S=2n\alpha^2 g$.
$R(X,Y)\cdot C = L\{(X\wedge Y)\cdot C\}$ (by $\bar{a}=\bar{b}=\bar{c}=0$, $a=b=-\frac{1}{2n-1}$ & $c=1$)	For $L\neq\alpha^2$, $S=\left(\frac{r}{2n}-\alpha^2\right)g - \left\{\frac{r}{2n}-(2n+1)\alpha^2\right\}\eta\otimes\eta$.
$R(X,Y)\cdot \hat{C} = L\{(X\wedge Y)\cdot \hat{C}\}$ (by $\bar{a}=\bar{b}=\bar{c}=0$, $a=b=-\frac{1}{2n-1}$ & $c=0$)	For $L\neq\alpha^2$, $S=\left(\frac{r}{2n}-\alpha^2\right)g - \left\{\frac{r}{2n}-(2n+1)\alpha^2\right\}\eta\otimes\eta$.
$R(X,Y)\cdot E = L\{(X\wedge Y)\cdot E\}$ (by $\bar{a}=\bar{b}=\bar{c}=0$, $a=b=0$ & $c=1$)	For $L\neq\alpha^2$, $S=2n\alpha^2 g$.
$R(X,Y)\cdot P =$ $L\{(X\wedge Y)\cdot P\}$ (by $\bar{a}=\bar{b}=\bar{c}=0$, $a=-\frac{1}{2n}$ & $b=c=0$)	For $L\neq\alpha^2$, $S=2n\alpha^2 g$ $-(\frac{r}{2n}-2n-1)\eta\otimes\eta$.
$R(X,Y)\cdot H =$ $L\{(X\wedge Y)\cdot H\}$ (by $\bar{a}=\bar{b}=\bar{c}=0$, $a=b=-\frac{1}{4n}$ & $c=0$)	For $L\neq\alpha^2$, $S=\left(\frac{r+4n^2\alpha^2}{4n+1}\right)g-\left\{\frac{r-2n\alpha^2(2n+1)}{4n+1}\right\}\eta\otimes\eta$.
$E(X,Y)\cdot R =$ $L\{(X\wedge Y)\cdot R\}$ (by $\bar{a}=\bar{b}=0, \bar{c}=1$, & $a=b=c=0$)	For $L\neq\alpha^2-\frac{r}{2n(2n+1)}$, $S=2n\alpha^2 g$.

Curvature condition	Expression for Ricci tensor
$E(X,Y)\cdot C = L\{(X\wedge Y)\cdot C\}$ (by $\bar{a}=0=\bar{b}, \bar{c}=1$ $a=b=-\frac{1}{2n-1}, c=1$)	For $L \neq \alpha^2 - \frac{r}{2n(2n+1)}$, $S = \left(\frac{r}{2n} - \alpha^2\right) g - \left\{\frac{r}{2n} - (2n+1)\alpha^2\right\} \eta\otimes\eta.$
$E(X,Y)\cdot \hat{C} = L\{(X\wedge Y)\cdot \hat{C}\}$ (by $\bar{a}=0=\bar{b}, \bar{c}=1$ $a=b=-\frac{1}{2n-1}$ & $c=0$)	For $L \neq \alpha^2 - \frac{r}{2n(2n+1)}$, $S = \left(\frac{r}{2n} - \alpha^2\right) g - \left\{\frac{r}{2n} - (2n+1)\alpha^2\right\} \eta\otimes\eta.$
$E(X,Y)\cdot E = L\{(X\wedge Y)\cdot E\}$ (by $\bar{a}=0=\bar{b}, \bar{c}=1$ $a=b=0$ & $c=1$)	For $L \neq \alpha^2 - \frac{r}{2n(2n+1)}$, $S = 2n\alpha^2 g.$
$E(X,Y)\cdot P = L\{(X\wedge Y)\cdot P\}$ (by $\bar{a}=0=\bar{b}, \bar{c}=1$ $a=-\frac{1}{2n}, b=0$ & $c=1$)	For $L \neq \alpha^2 - \frac{r}{2n(2n+1)}$, $S = 2n\alpha^2 g - \left\{\frac{r}{2n} - 2\alpha^2\right\} \eta\otimes\eta.$
$E(X,Y)\cdot H = L\{(X\wedge Y)\cdot H\}$ (by $\bar{a}=0=\bar{b}, \bar{c}=1$ $a=b=-\frac{1}{4n}$ & $c=1$)	For $L \neq \alpha^2 - \frac{r}{2n(2n+1)}$, $S = \left(\frac{r+4n^2\alpha^2}{4n+1}\right) g - \left\{\frac{r-2n(2n+1)\alpha^2}{4n+1}\right\} \eta\otimes\eta.$
$\hat{C}(X,Y)\cdot R = L\{(X\wedge Y)\cdot R\}$ (by $\bar{a}=\bar{b}=-\frac{1}{2n-1}, \bar{c}=0$ & $a=b=c=0$)	$\left\{\frac{r}{2n} - 2\alpha^2 - (2n-1)L\right\} S = -\alpha^2 \left\{2n\alpha^2 + 2n(2n-1)L\right\} g + \alpha^2\left\{r - 2n\alpha^2\right\} \eta\otimes\eta + S^2$

Curvature condition	Expression for Ricci tensor
$\hat{C}(X,Y)\cdot\hat{C}=L\{(X\wedge Y)\cdot\hat{C}\}$ (by $\bar{a}=\bar{b}=-\frac{1}{2n-1}$, $\bar{c}=0$, $a=b=-\frac{1}{2n-1}$ & $c=0$)	$\{r-(2n-1)\alpha^2-2n(2n-1)L\}S = [\{\frac{r}{2n}-2n\alpha^2-(2n-1)L\}(r-2n\alpha^2) +\alpha^2 r-\parallel Q\parallel^2\}]g+2nS^2$ $[\{\frac{r}{2n}-\alpha^2-(2n-1)L\}\{2n\alpha^2(2n+1)-r\} -\alpha^2(2n+1)\left\{r-2n\alpha^2\right\}]\eta\otimes\eta$
$\hat{C}(X,Y)\cdot C=L\{(X\wedge Y)\cdot C\}$ (by $\bar{a}=\bar{b}=-\frac{1}{2n-1}$, $\bar{c}=0$, $a=b=-\frac{1}{2n-1}$ & $c=1$)	$\{r-(2n-1)\alpha^2-2n(2n-1)L\}S = [\{\frac{r}{2n}-2n\alpha^2-(2n-1)L\}(r-2n\alpha^2) +\alpha^2 r-\parallel Q\parallel^2\}]g+2nS^2$ $[\{\frac{r}{2n}-\alpha^2-(2n-1)L\}\{2n\alpha^2(2n+1)-r\} +\{\frac{r}{2n}-\alpha^2(2n+1)\}\left\{r-2n\alpha^2\right\}]\eta\otimes\eta$
$\hat{C}(X,Y)\cdot E=L\{(X\wedge Y)\cdot E\}$ (by $\bar{a}=\bar{b}=-\frac{1}{2n-1}$, $\bar{c}=0$ & $a=b=0$, $c=1$)	$\{2+(2n-1)L\}\,S=-S^2 +[r+2n+2n(2n-1)L]g +(2n-r)\left\{\frac{r}{2n(2n+1)}-1\right\}\eta\otimes\eta$
$\hat{C}(X,Y)\cdot P=L\{(X\wedge Y)\cdot P\}$ (by $\bar{a}=\bar{b}=-\frac{1}{2n-1}$, $\bar{c}=0$ $a=-\frac{1}{2n}$, $b=0$ & $c=0$)	$\left\{\frac{r}{2n}-2\alpha^2-(2n-1)L\right\}S= -\alpha^2\{2n\alpha^2+2n\,(2n-1)\,L\}g +\left\{\frac{r}{2n}-\alpha^2-(2n-1)\,L\right\}\times \left\{(2n+1)\alpha^2-\frac{r}{2n}\right\}\eta\otimes\eta+S^2$
$\hat{C}(X,Y)\cdot H=L\{(X\wedge Y)\cdot H\}$ (by $\bar{a}=\bar{b}=-\frac{1}{2n-1}$, $\bar{c}=0$ $a=b=-\frac{1}{4n}$ & $c=0$)	$\frac{\{1+(2n-1)L\}(4n+1)+2n}{4n}S =[\left(\frac{r}{4n}+n\right)\{1+(2n-1)L\} +\frac{r}{2}+\frac{1}{4n}\parallel Q\parallel^2]g-\left(1+\frac{1}{4n}\right)S^2 +\left(\frac{r}{4n}-\frac{2n+1}{2}\right)\{1+(2n-1)L\}\eta\otimes\eta$
$P(X,Y)\cdot R=L\{(X\wedge Y)\cdot R\}$ (by $\bar{a}=-\frac{1}{2n}$, $\bar{b}=0,\bar{c}=0$ & $a=b=c=0$)	$\left(\frac{2n-1}{2n}\alpha^2-L\right)S=\alpha^2\left(-\frac{r}{2n}+2n\alpha^2-2nL\right)g +L\left(\frac{r}{2n}-2n\alpha^2-\alpha^2\right)\eta\otimes\eta+\frac{1}{2n}S^2$
$P(X,Y)\cdot\hat{C}=L\{(X\wedge Y)\cdot\hat{C}\}$ (by $\bar{a}=-\frac{1}{2n}$, $\bar{b}=0,\bar{c}=0$ $a=b=-\frac{1}{2n-1}$ & $c=0$)	$\left(\frac{4n^2+1}{2n}\alpha^2-2nL\right)S=S^2 +\left\{\begin{array}{c}(\alpha^2-L)(r-2n\alpha^2)\\ +\frac{1}{2n}(\alpha^2 r-\parallel Q\parallel^2)\end{array}\right\}g +\left\{\begin{array}{c}rL-2n\alpha^2(\alpha^2-L)(2n+1)\\ +\left(\alpha^2-\frac{r}{2n}\right)(2n+1)\alpha^2\end{array}\right\}\eta\otimes\eta$

Curvature condition	Expression for Ricci tensor
$P(X,Y)\cdot C = L\{(X\wedge Y)\cdot C\}$ (Obtain by $\bar{a}=-\frac{1}{2n}, \bar{b}=0, \bar{c}=0$ $a=b=-\frac{1}{2n-1}$ & $c=1$)	$\left(\frac{4n^2+1}{2n}\alpha^2-2nL\right)S=S^2$ $+\left\{\begin{array}{c}(\alpha^2-L)(r-2n\alpha^2)\\ +\frac{1}{2n}(\alpha^2 r- \parallel Q\parallel^2)\end{array}\right\}g$ $+\left[\begin{array}{c}\left(\alpha^2-\frac{r}{2n}\right)(2n+1)\alpha^2-\\ (\alpha^2-L)\left\{r+2n\alpha^2(2n+1)\right\}\end{array}\right]\eta\otimes\eta$
$P(X,Y)\cdot E = L\{(X\wedge Y)\cdot E\}$ (by $\bar{a}=-\frac{1}{2n}, \bar{b}=0, \bar{c}=0$ $a=b=0$ & $c=1$)	$\left\{\frac{2n-1}{2n}\alpha^2-L\right\}S$ $=\alpha^2\left(-\frac{r}{2n}+2n\alpha^2-2nL\right)g$ $+\frac{1}{2n}\left(2n\alpha^2-\frac{r}{2n+1}\right)\times$ $\left\{\frac{r}{2n}-(2n+1)\alpha^2\right\}\eta\otimes\eta$ $+\frac{1}{2n}S^2$
$P(X,Y)\cdot P = L\{(X\wedge Y)\cdot P\}$ (by $\bar{a}=-\frac{1}{2n}, \bar{b}=0, \bar{c}=0$ $a=-\frac{1}{2n}, b=0$ & $c=0$)	$\left\{\frac{2n-1}{2n}\alpha^2-L\right\}S=\frac{1}{2n}S^2$ $+\alpha^2\left(-\frac{r}{2n}+2n\alpha^2-2nL\right)g$ $+L\left\{\frac{r}{2n}-(2n+1)\alpha^2\right\}\eta\otimes\eta$
$P(X,Y)\cdot H = L\{(X\wedge Y)\cdot H\}$ (by$\bar{a}=-\frac{1}{2n}, \bar{b}=0, \bar{c}=0$ $a=b=-\frac{1}{4n}$ & $c=0$)	$\left\{\alpha^2-\frac{4n+1}{4n}L\right\}S=\frac{1}{2n}\left(1+\frac{1}{4n}\right)S^2$ $+\left\{n(\alpha^2-L)\alpha^2-\frac{1}{8n^2}\parallel Q\parallel^2-\frac{r}{4n}L\right\}g+$ $+L\left\{\frac{r}{4n}-\frac{(2n+1)}{2}\alpha^2\right\}\eta\otimes\eta$
$H(X,Y)\cdot R = L\{(X\wedge Y)\cdot R\}$ by$\bar{a}=\bar{b}=-\frac{1}{4n}, \bar{c}=0$ & $a=b=c=0$)	$\left\{\frac{2n-1}{4n}\alpha^2-L\right\}S=$ $\alpha^2\left(-\frac{r}{4n}+n\alpha^2-2nL\right)g$ $+\alpha^2\left(\frac{r}{4n}-\frac{1}{2}\right)\eta\otimes\eta+\frac{1}{4n}S^2$

Curvature condition	Expression for Ricci tensor
$H(X,Y)\cdot\hat{C} = L\{(X\wedge Y)\cdot\hat{C}\}$ by $\bar{a}=\bar{b}=-\frac{1}{4n}$, $\bar{c}=0$ $a=b=-\frac{1}{2n-1}$ & $c=0)$	$\left(\frac{4n^2+1}{4n}\alpha^2-2nL\right)S=\frac{1}{2n}S^2 +\left\{\begin{array}{c}\left(\frac{\alpha^2}{2}-L\right)(r-2n\alpha^2)\\ +\frac{1}{4n}(\alpha^2 r-\parallel Q\parallel^2)\end{array}\right\}g +\left[\begin{array}{c} r\left(L-\frac{\alpha^2}{2}\right)+\alpha^2(2n+1)\times\\ \left\{\frac{2n+1}{2}\alpha^2-2nL-\frac{r}{4n}\right\}\end{array}\right]\eta\otimes\eta$
$H(X,Y)\cdot E = L\{(X\wedge Y)\cdot E\}$ by $\bar{a}=\bar{b}=-\frac{1}{4n}$, , $\bar{c}=0$ $a=b=0$ & $c=1)$	$\left\{\frac{2n-1}{4n}\alpha^2-L\right\}S=\alpha^2\left(-\frac{r}{4n}+n\alpha^2-2nL\right)g +\{\frac{(n+1)\alpha^2 r}{2n(2n+1)}-\frac{r^2}{8n^2(2n+1)}+\frac{\alpha^4}{2}\}\eta\otimes\eta+\frac{1}{4n}S^2$
$H(X,Y)\cdot P = L\{(X\wedge Y)\cdot P\}$ by $\bar{a}=\bar{b}=-\frac{1}{4n}$, $\bar{c}=0$, $a=-\frac{1}{2n}$, $b=c=0)$	$\left\{\frac{2n-1}{4n}\alpha^2-L\right\}S=\alpha^2\left(-\frac{r}{4n}+n\alpha^2-2nL\right)g +\{\frac{r}{2n}-(2n+1)\alpha^2\}(L-\frac{\alpha^2}{2})\eta\otimes\eta +\frac{1}{4n}S^2$
$H(X,Y)\cdot H = L\{(X\wedge Y)\cdot H\}$ by $\bar{a}=\bar{b}=-\frac{1}{4n}$, , $\bar{c}=0$ $a=b=-\frac{1}{4n}$ & $c=0)$	$\left\{\frac{1}{2}\alpha^2-\frac{4n+1}{4n}L\right\}S=\{-\frac{\alpha^2 r}{8n}-\frac{1}{16n^2}\parallel Q\parallel^2 +(\frac{\alpha^2}{2}-L)(\frac{r}{4n}+n\alpha^2)\}g+\frac{1}{4n}\left(1+\frac{1}{4n}\right)S^2 +\{\frac{r}{4n}-\frac{2n+1}{2}\alpha^2\}(L-\frac{\alpha^2}{2})\eta\otimes\eta$
$C(X,Y)\cdot R = L\{(X\wedge Y)\cdot R\}$ (by $\bar{a}=\bar{b}=-\frac{1}{2n-1}$, $\bar{c}=1$ $a=b=c=0)$	$\left\{\frac{1}{2}\alpha^2-\frac{4n+1}{4n}L\right\}S=\{-\frac{\alpha^2 r}{8n}-\frac{1}{16n^2}\parallel Q\parallel^2 +(\frac{\alpha^2}{2}-L)(\frac{r}{4n}+n\alpha^2)\}g+\frac{1}{4n}\left(1+\frac{1}{4n}\right)S^2 +\{\frac{r}{4n}-\frac{2n+1}{2}\alpha^2\}(L-\frac{\alpha^2}{2})\eta\otimes\eta$
$C(X,Y)\cdot R = L\{(X\wedge Y)\cdot R\}$ (by $\bar{a}=\bar{b}=-\frac{1}{2n-1}$, $\bar{c}=1$ $a=b=c=0)$	$\left\{\frac{r}{2n}-2\alpha^2-(2n-1)L\right\}S =-\alpha^2\left\{2n\alpha^2+2n(2n-1)L\right\}g +\alpha^2\left\{r-2n\alpha^2\right\}\eta\otimes\eta+S^2$

Curvature condition	Expression for Ricci tensor
$C(X,Y)\cdot\hat{C}=L\{(X\wedge Y)\cdot\hat{C}\}$ by $\bar{a}=\bar{b}=-\frac{1}{2n-1}0, \bar{c}=1$ $a=b=-\frac{1}{2n-1}, c=0)$	$\{r-(2n-1)\alpha^2-2n(2n-1)L\}S$ $=[\{\frac{r}{2n}-2n\alpha^2-(2n-1)L\}(r-2n\alpha^2)$ $+\alpha^2 r-\parallel Q\parallel^2\}]g+2nS^2$ $[\{\frac{r}{2n}-\alpha^2-(2n-1)L\}\{2n\alpha^2(2n+1)-r\}$ $-\alpha^2(2n+1)\left\{r-2n\alpha^2\right\}]\eta\otimes\eta$
$C(X,Y)\cdot P=L\{(X\wedge Y)\cdot P\}$ (by $\bar{a}=\bar{b}=-\frac{1}{2n-1}0, \bar{c}=1$ $a=-\frac{1}{2n}, b=c=0)$	$\left\{\frac{r}{2n}-2\alpha^2-(2n-1)L\right\}S=$ -$\alpha^2\{2n\alpha^2+2n\,(2n-1)\,L\}g$ $+\left\{\frac{r}{2n}-\alpha^2-(2n-1)\,L\right\}\times$ $\left\{(2n+1)\alpha^2-\frac{r}{2n}\right\}\eta\otimes\eta+S^2$
$C(X,Y)\cdot E=L\{(X\wedge Y)\cdot E\}$ (by $\bar{a}=\bar{b}=-\frac{1}{2n-1}0, \bar{c}=1$ $a=b=0, c=1)$	$\left\{\frac{r}{2n}-2\alpha^2-(2n-1)L\right\}S=$ $-\alpha^2\left\{2n\alpha^2+2n(2n-1)L\right\}g$ $+\alpha^2\left\{r-2n\alpha^2\right\}\eta\otimes\eta+S^2$
$C(X,Y)\cdot C=L\{(X\wedge Y)\cdot H\}$ (by $\bar{a}=\bar{b}=-\frac{1}{2n-1}0, \bar{c}=1$ $a=b=-\frac{1}{4n}, c=0)$	$\{r-(2n-1)\alpha^2-2n(2n-1)L\}S$ $=[\{\frac{r}{2n}-2n\alpha^2-(2n-1)L\}(r-2n\alpha^2)$ $+\alpha^2 r-\parallel Q\parallel^2\}]g+2nS^2$ $[\{\frac{r}{2n}-\alpha^2-(2n-1)L\}\{2n\alpha^2(2n+1)-r\}$ $+\{\frac{r}{2n}-\alpha^2(2n+1)\}\left\{r-2n\alpha^2\right\}]\eta\otimes\eta$

Remark 3.3. For $L=0$, the above theorem gives us the nature of the Ricci tensor of the manifold M for respective semi-symmetry type conditions.

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References

[1] Blair, D. E., *Contact manifolds in Riemannian geometry,* Lecture Notes in Mathematics, **509**, Springer-Verlag, Berlin, 1976.

[2] Cartan, E., *Sur une classe remarquable d'espaces de Riemannian*. Bull. Soc. Math. France, 54 (1926) 214–264.

[3] Boeckx, E., Kowalski, O., Vanhecke, L., *Riemannian Manifolds of Conullity Two*, World Scientific Publishing Co., Inc., River Edge, NJ, 1996.

[4] *Defever, F., Deszcz, R., On warped product manifolds satisfying a certain curvature condition*. Atti. Acad. Peloritana Pericolanti Cl. Sci. Fis. Mat. Natur. 69, 213–236 (1991).

[5] Pokhariyal, G.K., Mishra, R. S., *Curvatur tensors$'$and their relativistics significance I.*, Yokohama Math. J., vol. 18, pp. 105–108, 1970.

[6] Baishya, K. K., Chowdhury, P. R., *On generalized quasi-conformal $N(k,\mu)$-manifolds*, Commun. Korean Math. Soc., 31(2016), No. 1, 163–176.

[7] Baishya, K. K., Chowdhury, P. R., *Deszcz pseudo symmetric type LP-Sasakian manifolds*, Analele Universităţii de Vest Timişoara, LIV, 1, (2016), 35–53.

[8] Nomizu, K., *On the decomposition of generalized curvature tensor fields*, Differential Geometry in honor of K. Yano, Kinokuniya, Tokyo, 1972, 335–345.

[9] Yano, K., Bochner, S., *Curvature and Betti numbers*, Annals of Math. Studies 32, Princeton University Press, 1953.

[10] Yano, K., Sawaki, S., *Riemannian manifolds admitting a conformal transformation group*, J. Diff. Geom., **2**(1968), 161–184.

[11] Eisenhart, L. P., *Riemannian Geometry*, Princeton University Press, 1949.

[12] Belkhelfa, M., Deszcz, R., Verstraelen, L., *Symmetry properties of Sasakian space forms*, Soochow J. Math., 31 (2005), 611–616.

[13] Deszcz, R., *On pseudosymmetric spaces*, Bull. Soc. Math. Belg. Ser. A, 44 (1992), 1–34.

[14] Deszcz, R., *On four-dimensional warped product manifolds satisfying certain pseudosymmetry curvature conditions.*, Colloq. Math. 62, 103–120 (1991).

[15] Deszcz, R., Haesen, S. , Verstraelen, L., *On natural symmetries*, "Topics in Differential Geometry", Ch.6, Editors A. Mihai, I. Mihai and R. Miron, Editura Academiei Române, 2008.

[16] Ishii, Y., *On conharmonic transformations*, Tensor (N.S.), 7 (1957), 73–80.

[17] Szabő, Z. I., *Classification and construction of complete hypersurfaces satisfying $R(X,Y)\cdot R=0$.*, Acta. Sci. Math., 47(1984), 321–348.

[18] Szabő, Z. I., *Structure theorems on Riemannian spaces satisfying $R(X,Y)\cdot R=0$*, J. Differential Geom, 17: I, The local version (1982), 531–582.