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Strong $B_{\mathcal{I}s}$-set and another decomposition of continuity

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Abstract

In this paper we introduce and investigate the notions of $Q_{\mathcal{I}s}$-sets, (t-$\mathcal{I}_s$, $Q_{\mathcal{I}s}$)-set and strong $B_{\mathcal{I}s}$-set in ideal topological spaces. Also we obtained a new decomposition of continuity via idealization.

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1. Introduction

Ideal in topological space have been considered since 1930 by Kuratowski[10] and Vaidyanathaswamy[17]. After several decades, in 1990, Jankovic and Hamlett[6] investigated the topological ideals which is the generalization of general topology. Where as in 2010, Khan and Noiri[8] introduced and studied the concept of semi-local functions. Recently, we [15] introduced α-$\mathcal{I}_s$-open, semi-$\mathcal{I}_s$-open, pre-$\mathcal{I}_s$-open, t-$\mathcal{I}_s$-set, $B_{\mathcal{I}s}$-set and obtained decomposition of continuity.

In this paper we introduce and investigate the notions of $Q_{\mathcal{I}s}$-sets, (t-$\mathcal{I}_s$, $Q_{\mathcal{I}s}$)-set, strong $B_{\mathcal{I}s}$-set, (t-$\mathcal{I}_s$, $Q_{\mathcal{I}s}$)-continuous, strongly $B_{\mathcal{I}s}$-continuous in ideal topological spaces. Also we obtained a new decomposition of continuity via idealization. Then we show that a function $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ is continuous if and only if f is α-$\mathcal{I}_s$-continuous and strongly $B_{\mathcal{I}s}$-continuous.

2. Preliminaries

Let (X, τ) be a topological space with no separation properties assumed. For a subset A of a topological space (X, τ), $cl(A)$ and $int(A)$ denote the closure and interior of A in (X, τ) respectively.

An ideal $\mathcal{I}$ on a topological space (X, τ) is a nonempty collection of subsets of X which satisfies:

(1) $A \in \mathcal{I}$ and $B \subseteq A$ implies $B \in \mathcal{I}$

(2) $A \in \mathcal{I}$ and $B \in \mathcal{I}$ implies $A \cup B \in \mathcal{I}$.

If $(X, \tau)$is a topological space and $\mathcal{I}$ is an ideal on X, then $(X, \tau, \mathcal{I})$ is called an ideal topological space or an ideal space.

Let $P(X)$ be the power set of X. Then the operator $()^* : P(X) \to P(X)$ called a local function [10] of A with respect to τ and $\mathcal{I}$, is defined as follows: for $A \subseteq X$, $A^*(\mathcal{I}, \tau) = \{x \in X : U \cap A \notin \mathcal{I}$ for every open set U containing $x\}$. We simply write A^* instead of $A^*(\mathcal{I}, \tau)$ in case there is no confusion. For every ideal topological space $(X, \tau, \mathcal{I})$ there exists topology τ^* finer than τ, generated by $\beta(\mathcal{I}, \tau) = \{U \setminus J : U \in \tau$ and $J \in \mathcal{I}\}$ but in general $\beta(\mathcal{I}, \tau)$ is not always a topology. Additionally $cl^*(A) = A \cup A^*$ defines Kuratowski closure operator for a topology τ^* finer than τ. Throughout this paper X denotes the ideal topological space $(X, \tau, \mathcal{I})$ and also $int^*(A)$ denotes the interior of A with respect to τ^*.

Definition 2.1. Let (X, τ) be a topological space. A subset A of X is said to be semi-open [11] if there exists an open set U in X such that $U \subseteq A \subseteq cl(U)$. The complement of a semi-open set is said to be semi-closed. The collection of all semi-open (resp. semi-closed) sets in X is denoted by SO(X) (resp. SC(X)). The semi-closure of A in (X, τ) is denoted by the intersection of all semi-closed sets containing A and is denoted by $scl(A)$.

Definition 2.2. For $A \subseteq X$, $A_*(\mathcal{I}, \tau) = \{ x \in X / U \cap A \notin \mathcal{I}$for every $U \in SO(X)\}$ is called the semi-local function [8] of A with respect to $\mathcal{I}$ and τ, where $SO(X, x) = \{U \in SO(X) : x \in U\}$. We simply write A_* instead of $A_*(\mathcal{I}, \tau)$ in this case there is no ambiguity.

It is given in [2] that $\tau^{*s}(\mathcal{I})$ is a topology on X, generated by the sub basis $\{ U - E :$ $U \in$ SO(X) and E$\in$ I $\}$ or equivalently $\tau^{*s}\mathcal{I} = \{U \subseteq X : cl^{*s}(X - U) = X - U\}$. The closure operator cl^{*s} for a topology $\tau^{*s}(\mathcal{I})$ is defined as follows: for A $\subseteq X$, $cl^{*s}(A) = A \cup A_*$ and int^{*s} denotes the interior of the set A in $(X, \tau^{*s}, \mathcal{I})$. It is known that $\tau \subseteq \tau^*(\mathcal{I}) \subseteq \tau^{*s}(\mathcal{I})$. A subset A of $(X, \tau, \mathcal{I})$ is called semi-$*$-perfect [9] if $A = A_*$. $A \subseteq (X, \tau, \mathcal{I})$ is called $*$-semi dense in-itself [9](resp. semi-$*$-closed [9]) if $A \subset A_*$ (resp. $A_* \subseteq A$).

Definition 2.3. A subset A of a topological space X is said to be

1. α-open [14] if $A \subseteq int(cl(int(A)))$,

2. semi-open [11] if $A \subseteq cl(int(A))$,
3. pre-open [13] if $A \subseteq int(cl(A))$,
4. β-open if [1] $A \subseteq cl(int(cl(A)))$,
5. a t-set [16] if $int(A) = int(cl(A))$,
6. a Q-set [12] if $cl(int(A)) = int(cl(A))$,
7. a B-set [16] if $A = U \cap V$, where U is an open and V is an t-set,
8. a strong-B-set [4] if $A = U \cap V$, where U is an open and V is both a t-set and a Q-set.

Definition 2.4. A subset A of an ideal topological space $(X, \tau, \mathcal{I})$ is said to be

1. α-$\mathcal{I}$-open [5] if $A \subseteq int(cl^*(int(A)))$,
2. pre-$\mathcal{I}$-open [3] if $A \subseteq int(cl^*(A))$,
3. semi-$\mathcal{I}$-open [5] if $A \subseteq cl^*(int(A))$,
4. a t-$\mathcal{I}$-set[5] if $int(A) = int(cl^*(A))$,
5. a $Q_{\mathcal{I}}$-set [7] if $cl(int(A)) = int(cl^*(A))$,
6. a $B_{\mathcal{I}}$-set [5] if $A = U \cap V$, where U is an open and V is a t-$\mathcal{I}$-set,
7. a strong-$B_{\mathcal{I}}$-set [7] if $A = U \cap V$, where U is an open and V is both a t-$\mathcal{I}$-set and a $Q_{\mathcal{I}}$-set.

Definition 2.5. A subset A of an ideal topological space $(X, \tau, \mathcal{I})$ is said to be

1. α-$\mathcal{I}_s$-open [15]if $A \subseteq int(cl^{*s}(int(A)))$,
2. pre-$\mathcal{I}_s$-open [15] if $A \subseteq int(cl^{*s}(A))$,
3. semi-$\mathcal{I}_s$-open [15] if $A \subseteq cl^{*s}(int(A))$,
4. a t-$\mathcal{I}_s$-set [15] if $int(A) = int(cl^{*s}(A))$,
5. a $B_{\mathcal{I}s}$-set [15] if $A = U \cap V$, where U is an open and V is a t-$\mathcal{I}_s$-set,

Lemma 2.6. [8] Let $(X, \tau, \mathcal{I})$ be an ideal topological space and A, B be subsets of X. Then for the semi-local function the following properties hold:

1. If $A \subseteq B$ then $A_* \subseteq B_*$,
2. If $U \in \tau$ then $U \cap A_* \subseteq (U \cap A)_*$,

3. $A_* = scl(A_*) \subseteq scl(A)$ and A_* is semi-closed in X,
4. $(A_*)_* \subseteq A_*$, .
5. $(A \cup B)_* = A_* \cup B_*$,
6. If $\mathcal{I} = \{\phi\}$, then $A_* = scl(A)$.

3. $Q_{\mathcal{I}s}$-sets and strong $B_{\mathcal{I}s}$-sets

Definition 3.1. A subset A of an ideal topological space $(X, \tau, \mathcal{I})$ is called

1. a $Q_{\mathcal{I}s}$-set if $cl(int(A)) = int(cl^{*s}(A))$,
2. a strong $B_{\mathcal{I}s}$-set if $A = U \cap V$, where U is an open and V is both A t-$\mathcal{I}_s$-set and a $Q_{\mathcal{I}s}$-set.

We denote by $Q_{\mathcal{I}s}(X, \tau)$(resp. $sB_{\mathcal{I}s}(X, \tau)$, t-$\mathcal{I}_s(X, \tau)$) the family of all $Q_{\mathcal{I}s}$-sets(resp. strong $B_{\mathcal{I}s}$-sets, t-$\mathcal{I}_s$-sets) of $(X, \tau, \mathcal{I})$, when there is no chance for confusion with the ideal.

Remark 3.2. Note that t-$\mathcal{I}_s$-sets and $Q_{\mathcal{I}s}$-sets are independent concepts as seen from the following examples.

Example 3.3. Let X ={a, b, c, d}, $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{a, d\}, \{a, b, d\}\}$ and $\mathcal{I} = \{\phi, \{b\}, \{c\}, \{b, c\}\}$

(1) Set $A = \{a, b, c\}$. Then A is a $Q_{\mathcal{I}s}$-set which is not a t-$\mathcal{I}_s$-set. For $A = \{a, b, c\} \subseteq X$, since $A_* = X$ and $cl^{*s}(A) = A \cup A_* = X$, we have $int(cl^{*s}(A)) = X$. Furthermore, since $int(A) = \{a, b\}$, we have $cl(int(A)) = X$. Consequently, $int(cl^{*s}(A)) = X = cl(int(A))$ and hence A is a $Q_{\mathcal{I}s}$-set. On the other hand, since $int(cl^{*s}(A)) = X \neq \{a, b\} = int(A)$, A is not a t-$\mathcal{I}_s$-set.

(2) Set $A = \{a, b, d\}$. Then A is a t-$\mathcal{I}_s$-set which is not a Q-$\mathcal{I}_s$-set. For $A = \{a, b, d\} \subseteq X$, since $A_* = \{a, d\}$ and $cl^{*s}(A) = A \cup A_* = \{a, b, d\}$, we have $int(cl^{*s}(A)) = \{a, b, d\} = int(A)$ and hence A is a t-$\mathcal{I}_s$-set. On the other hand $cl(int(A)) = cl(\{a, b, d\}) = X \neq int(cl^{*s}(A))$, A is not a $Q_{\mathcal{I}s}$-set.

Definition 3.4. If A is both a t-$\mathcal{I}_s$-set(resp. t-$\mathcal{I}$-set, t-set) and a $Q_{\mathcal{I}s}$-set(resp. $Q_{\mathcal{I}}$-set, Q-set), it will be called a (t-$\mathcal{I}_s$, $Q_{\mathcal{I}s}$)-set (resp. (t-$\mathcal{I}$, $Q_{\mathcal{I}}$)-set [7], (t, Q)-set [4]).

Proposition 3.5. For a subset A of an ideal topological space $(X, \tau, \mathcal{I})$ the following properties hold.

1. Every (t-$\mathcal{I}_s$, $Q_{\mathcal{I}s}$)-set is a strong $B_{\mathcal{I}s}$-set.
2. Every strong $B_{\mathcal{I}s}$-set is a $B_{\mathcal{I}s}$-set.

3. Every open set is a strong $B_{\mathcal{I}s}$-set.

Proof.

(1) Since $X \cap t\text{-}\mathcal{I}_s(x, \tau) \cap Q_{\mathcal{I}s}(x, \tau) = t\text{-}\mathcal{I}_s(x, \tau) \cap Q_{\mathcal{I}s}(x, \tau)$, the proof is obvious.

(2) Proof is trivial.

(3) Since $X \in t\text{-}\mathcal{I}_s(x, \tau) \cap Q_{\mathcal{I}s}(x, \tau)$, the proof is obvious.

Proposition 3.6. For a subset A of an ideal topological space $(X, \tau, \mathcal{I})$ the following properties hold.

1. Every (t, Q)-set is a $(t\text{-}\mathcal{I}_s, Q_{\mathcal{I}s})$-set.
2. Every strong B-set is a strong $B_{\mathcal{I}s}$-set.
3. Every B-set is a $B_{\mathcal{I}s}$-set.

Proof.

(1) Let A be a (t, Q)-set. Then $int(cl^{*s}(A)) = int(A \cup A_*) \subseteq int(A \cup scl(A)) \subseteq int(scl(A)) \subseteq int(cl(A)) = int(A)$. Moreover, we have $cl(int(A)) = int(cl(A)) = int(A) = int(cl^{*s}(A))$ and hence A is a $(t\text{-}\mathcal{I}_s, Q_{\mathcal{I}s})$-set.

(2) Let A be a strong B-set. Then $A = U \cap V$, where $U \in \tau$ and V is a (t, Q)-set. By (a), V is a $(t\text{-}\mathcal{I}_s, Q_{\mathcal{I}s})$-set and A is a strong $B_{\mathcal{I}s}$-set.

(3) The proof is obvious since every t-set is a $t\text{-}\mathcal{I}_s$-set by [15, Proposition 4.3] ■

Proposition 3.7. For a subset A of an ideal topological space $(X, \tau, \mathcal{I})$ the following properties hold.

1. Every $(t\text{-}\mathcal{I}, Q\text{-}\mathcal{I})$-set is a $(t\text{-}\mathcal{I}_s, Q_{\mathcal{I}s})$-set.
2. Every strong $B_{\mathcal{I}}$-set is a strong $B_{\mathcal{I}s}$-set.
3. Every $B_{\mathcal{I}}$-set is a $B_{\mathcal{I}s}$-set.

Proof.

(1) Let A be a $(t\text{-}\mathcal{I}, Q\text{-}\mathcal{I})$-set. Then $int(cl^{*s}(A)) = int(A \cup A_*) \subseteq int(A \cup A^*)) = int(cl^*(A)) = int(A)$. Moreover, we have $cl(int(A)) = int(cl^*(A)) = int(A) = int(cl^{*s}(A))$ and hence A is a $(t\text{-}\mathcal{I}_s, Q_{\mathcal{I}s})$-set.

(2) Let A be a strong $B_{\mathcal{I}}$-set. Then $A = U \cap V$, where $U \in \tau$ and V is a $(t\text{-}\mathcal{I}, Q\text{-}\mathcal{I})$-set. By (a), V is a $(t\text{-}\mathcal{I}_s, Q_{\mathcal{I}s})$-set and A is a strong $B_{\mathcal{I}s}$-set.

(3) Let A be a t-$\mathcal{I}$-set. Then we have $int(cl^{*s}(A)) = int(A_* \cup A) \subseteq int(A^* \cup A) = int(cl^*(A)) = int(A)$ and hence $int(cl^{*s}(A)) = int(A)$. Therefore A is a t-$\mathcal{I}_s$-set and hence the proof. ■

Remark 3.8. By Proposition 3.5, 3.6 and 3.7, we have the following diagram, where none of the implications is reversible as shown IN examples below:

$$\begin{array}{ccccc} (t,\, Q)\text{-set} & \longrightarrow & \text{strong } B\text{-set} & \longrightarrow & B\text{-set} \\ \downarrow & & \downarrow & & \downarrow \\ (t\text{-}\mathcal{I},\, Q\text{-}\mathcal{I})\text{-set} & \longrightarrow & \text{strong } B_{\mathcal{I}}\text{-set} & \longrightarrow & B_{\mathcal{I}}\text{-set} \\ \downarrow & & \downarrow & & \downarrow \\ (t\text{-}\mathcal{I}_s,\, Q_{\mathcal{I}s})\text{-set} & \longrightarrow & \text{strong } B_{\mathcal{I}s}\text{-set} & \longrightarrow & B_{\mathcal{I}s}\text{-set} \end{array}$$

Example 3.9. Let X={a, b, c, d}, $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$ and $\mathcal{I} = \{\phi, \{b\}, \{c\}, \{b, c\}\}$. Set $A = \{a, b, d\}$. Then A is a $B_{\mathcal{I}s}$-set which is not a $B_{\mathcal{I}}$-set. For $A = \{a, b, d\} \subseteq X$, since $A_* = \{a, d\}$ and $cl^{*s}(A) = A \cup A_* = \{a, b, d\}$, we have $int(cl^{*s}(A)) = \{a, b\} = int(A)$. Consequently A is a $B_{\mathcal{I}s}$-set. On the other hand, since $A^* = \{a, c, d\}$ and $cl^{*s}(A) = A \cup A_* = X \neq \{a, b\} = int(A)$. This shows that A is not t-$\mathcal{I}$-set. Which implies A is not a $B_{\mathcal{I}}$-set.

Example 3.10. By Example 3.9, A is a $B_{\mathcal{I}s}$-set which is not strong $B_{\mathcal{I}s}$-set. Since $cl(int(A)) = X \neq \{a, b\} = int(cl^{*s}(A))$.

Example 3.11. In Example 3.3, Set $A = \{a, b, d\}$. Then A is a strong $B_{\mathcal{I}s}$-set which is not (t-$\mathcal{I}$, Q-$\mathcal{I}$)-set. For $A = \{a, b, d\}$, since A is open, we have A is a strong $B_{\mathcal{I}s}$-set. On the other hand, since $A_* = \{a, c, d\}$ and $cl^{*s}(A) = A \cup A_* = X$, we have $int(cl^{*s}(A)) = X \neq int(A) = \{a, b, d\}$. This shows that A is not a t-$\mathcal{I}$-set. Hence A is not (t-$\mathcal{I}$, Q-$\mathcal{I}$)-set.

Example 3.12. Let X ={a, b, c, d}, $\tau = \{\phi, X, \{c\}, \{b, d\}, \{a, b, d\}, \{b, c, d\}\}$ and $\mathcal{I} = \{\phi, \{c\}, \{d\}, \{c, d\}\}$. Set $A = \{c, d\}$. Then A is a (t-$\mathcal{I}_s$, $Q_{\mathcal{I}s}$)-set which is not a B-set. For $A = \{c, d\} \subseteq X$, since $A_* = \phi$ and $cl^{*s}(A) = A \cup A_* = A$, we have $int(cl^{*s}(A)) = \{c\} = int(A)$. This shows that A is a t-$\mathcal{I}_s$-set. Furthermore, since $cl(int(A)) = \{c\}$, we have $int(cl^{*s}(A)) = \{c\} = cl(int(A))$ and hence A is a $Q_{\mathcal{I}s}$-set. Consequently, A is a (t-$\mathcal{I}_s$, $Q_{\mathcal{I}s}$)-set. On the other hand,there are two probabilities for A to be B-set, that is (i) $A = X \cap A$ and (ii) $A = \{b, c, d\} \cap \{a, c, d\}$. However neither A nor {a, c, d} is a t-set. Therefore $A = \{c, d\}$ is not a B-set.

Remark 3.13. α-$\mathcal{I}_s$-open and strong $B_{\mathcal{I}s}$-set are independent concepts as shown by the following examples.

Example 3.14. Let X ={a, b, c, d}, $\tau = \{\phi, X, \{d\}, \{a, c\}, \{a, c, d\}\}$ and $\mathcal{I} = \{\phi, \{c\}, \{d\}, \{c, d\}\}$. Set $A = \{b, c\}$. Then A is a strong $B_{\mathcal{I}s}$-set which is not a α-$\mathcal{I}_s$-open.

For $A = \{b, c\} \subseteq X$, since $A_* = \{b\}$ and $cl^{*s}(A) = A \cup A_* = \{b, c\}$, we have $int(cl^{*s}(A)) = \{\phi\} = int(A)$. This shows that A is a t-$\mathcal{I}_s$-set. Furthermore, since $cl(int(A)) = \{\phi\}$, we have $int(cl^{*s}(A)) = \{\phi\} = cl(int(A))$ and hence A is a $Q_{\mathcal{I}s}$-set. Consequently, A is a strong $B_{\mathcal{I}s}$-set. On the other hand, since $int(cl^{*s}(A)) = \{\phi\} = int(A)$, we have $int(cl^{*s}(int(A)) = int(cl^{*s}(\phi) = \phi \not\supseteq \{b, c\} = A$. This shows that A is not an α-$\mathcal{I}_s$-open.

Example 3.15. Let X ={a, b, c, d}, $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$ and $\mathcal{I} = \{\phi, \{c\}\}$. Set $A = \{a, b, c\}$. Then A is a α-$\mathcal{I}_s$-open which is not strong $B_{\mathcal{I}s}$-set. For $A = \{a, b, c\} \subseteq X$, since $int(A) = \{a, b\}$, $cl^{*s}(int(A)) = int(A) \cup (int(A))_* = X$, $int(cl^{*s}(int(A))) = int(X) = X \supseteq A$ and hence A is an α-$\mathcal{I}_s$-open set. On the other hand, since $cl^{*s}(A) \supseteq cl^{*s}(int(A)) = X$, $int(cl^{*s}(A)) = X \neq int(A) = \{a, b\}$ and hence A is not t-$\mathcal{I}_s$-set which is not open, A is not strongly $B_{\mathcal{I}s}$-set.

Proposition 3.16. For a subset A of an ideal topological space $(X, \tau, \mathcal{I})$ the following properties are equivalent:

1. A is an open set,
2. A is an α-$\mathcal{I}_s$-open set and a strong $B_{\mathcal{I}s}$-set,
3. A is a pre-$\mathcal{I}_s$-open(or semi-$\mathcal{I}_s$-open) set and a strong $B_{\mathcal{I}s}$-set.

Proof. (1) $\Longrightarrow$ (2) and (2) $\Longrightarrow$ (3) are obvious.

(3) $\Longrightarrow$ (1) Let A be a pre-$\mathcal{I}_s$-open and a strong $B_{\mathcal{I}s}$-set. Then $A = U \cap V$, where $V \in t$-$\mathcal{I}_s(x, \tau) \cap Q_{\mathcal{I}s}(X, \tau)$ and $U \in \tau$. $A \subseteq int(cl^{*s}(A)) = int(cl^{*s}(U \cap V)) \subseteq int(cl^{*s}(U) \cap cl^{*s}(V)) = int(cl^{*s}(U)) \cap int(cl^{*s}(V)) = int(cl^{*s}(U)) \cap int(V)$. Hence $A \subseteq U$ and $int(A) \subseteq A \subseteq U \cap int(V) = int(A)$. Thus we obtain $A \in \tau$. ■

4. Decomposition of Continuity

Definition 4.1. [15] A function $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ is said to be α-$\mathcal{I}_s$-continuous (resp. semi -$\mathcal{I}_s$-continuous, pre -$\mathcal{I}_s$-continuous and $B_{\mathcal{I}s}$-continuous) if for every $V \in \sigma$, $f^{-1}(V)$ is an α-$\mathcal{I}_s$-set (resp. semi-$\mathcal{I}_s$-set, pre -$\mathcal{I}_s$-set and $B_{\mathcal{I}s}$-set) of $(X, \tau, \mathcal{I})$.

Definition 4.2. A function $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ is said to be (t-$\mathcal{I}_s$, Q-$\mathcal{I}_s$)-continuous (resp. strongly $B_{\mathcal{I}s}$-continuous) if for every $V \in \sigma$, $f^{-1}(V)$ is an (t-$\mathcal{I}_s$, Q-$\mathcal{I}_s$)-set. (resp. strong $B_{\mathcal{I}s}$-set) of $(X, \tau, \mathcal{I})$.

Proposition 4.3. For a function $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$, the following properties hold:

1. If f is (t-$\mathcal{I}_s$, Q-$\mathcal{I}_s$)-continuous then f is strongly $B_{\mathcal{I}s}$-continuous,
2. If f is strongly $B_{\mathcal{I}s}$-continuous then f is $B_{\mathcal{I}s}$-continuous,
3. If f is continuous then f is strongly $B_{\mathcal{I}s}$-continuous.

Proof. The proof is obvious from Proposition 3.5. ■

Remark 4.4. Converse of the proposition 4.3 need not be true as the following examples shows.

Example 4.5. Let $(X, \tau, \mathcal{I})$ be the same ideal topological space as in Example 3.3. Let $Y = \{a, b\}$ and $\sigma = \{Y, \phi, \{a\}\}$.

(1) Let $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ be a function defined as follows $f(a) = b$, $f(c) = f(d) = c$ and $f(b) = a$. Then f is strongly $B_{\mathcal{I}s}$-continuous, but it is not $(t\text{-}\mathcal{I}_s, Q\text{-}\mathcal{I}_s)$-continuous by Example 3.3.

(2) Let $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ be a function defined as follows $f(a) = b$, $f(c) = d$, $f(d) = c$ and $f(b) = a$. Then f is $B_{\mathcal{I}s}$-continuous, but it is not strongly $B_{\mathcal{I}s}$-continuous by Example 3.3.

Example 4.6. Let $(X, \tau, \mathcal{I})$ be the same ideal topological space as in Example 3.14. Let $Y = \{a, b\}$ and $\sigma = \{Y, \phi, \{a, b\}\}$.Let $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ be a function defined as follows $f(a) = c$, $f(c) = f(d) = d$ and $f(b) = b$. Then f is strongly $B_{\mathcal{I}s}$-continuous, but it is not $\alpha\text{-}\mathcal{I}_s$continuous and hence not continuous by Example 3.14.

Theorem 4.7. Let $(X, \tau, \mathcal{I})$ be an ideal topological space for a function $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ the following conditions are equivalent:

1. f is continuous.
2. f is $\alpha\text{-}\mathcal{I}_s$-continuous and strongly $B_{\mathcal{I}s}$-continuous.
3. f is pre-$\mathcal{I}_s$-continuous(or semi-$\mathcal{I}_s$-continuous) and strongly $B_{\mathcal{I}s}$-continuous.

Proof. Proof is trivial from Proposition 3.16. ■

Corollary 4.8. Let $(X, \tau, \mathcal{I})$ be an ideal topological space and $\mathcal{I} = \{\phi\}$ and A is open. For a function $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ the following conditions are equivalent:

1. f is continuous.
2. f is α-continuous and strongly B-continuous.
3. f is pre-continuous(or semi-continuous) and strongly B-continuous.

Proof. Since $\mathcal{I} = \{\phi\}$, we have $A_* = scl(A)$ and $cl^{*s}(A) = A_* \cup A = scl(A)$ for any open subset A of X. Therefore we obtain (a)$\alpha\text{-}\mathcal{I}_s$-open (resp. pre -$\mathcal{I}_s$-open) if and only if it is α-open (resp. Pre-open) (b) A is a strong $B_{\mathcal{I}s}$-set if and only if it is a B-set. The proof follows from Theorem 4.7 immediately. ■

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