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Blasius Flow with Suction and Blowing

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Abstract

In this chapter, we have to study the variation of velocity in fluid flow over flat plate with suction and injection. The boundary layer equations are transferred from partial differential equations to thermal layer thickness equations and extended to Lagrange's equation with force representation. The thickness of the thermal boundary layers and the corresponding skinfriction coefficients are computed for the non dimensional velocity values of H from -0.60 to + 0.60. The values are given in Annexure I.

INTRODUCTION

The problem under consideration is that of boundary layer flow along as flat plate with suction or blowing. In physics and fluid mechanics, a Balasius boundary layer (named after Paul Richard Heinrich Blasius). describe the steady two dimensional laminar boundary layer that forms on a semi- infinite plate.

Let the leading edge of the plate be at x = 0, the plate being parallel to the x -axis and infinitely long down the stream. We consider steady flow with free steam velocity, U_∞ which is parallel to the x axis and assumed longitudial velocity $\frac{u}{u_\infty} = \frac{2y}{d} - \frac{2y^3}{d^3} + \frac{y^4}{d^4},\ 0 \le y \le d\,(5)$ where d is the boundary layer thickness.

From the exact equations of the Navier - stokes equations by GPDP method we formulate the thickness equation. Force representation of the variational principle of Least dissipation of energy is applied to find out the skin friction coefficients.

Problems related to convention boundary layer flow are important in Engineering and industrial activities. Such flows are applied to manage thermal effects in many industrial out puts for example in electronic devices, computer power supply and also in engine cooling system such as a heat sink in a car radiator . A few practical examples are industrial processes such as extursion of metals and plastics, cooling and drying of paper and textiles.

Blasius solution for flow past a flate plate was investigated by Abussia [1], and the existence of a solution was established. Asithambi [2] presented a finite difference method for the solution of the "Falkner - Skan" equation, and recently ; Wang [3] obtained an approximate solution for the classical Blasius equation using Adomian decomposition method.

Blasius flow was also numerically analysized by Cortel [4], and very recenlty a shooting procedure has been used by Zhang and Chen [5] in order to obtain interesting results of the Falkner - Skan quation.

The object of the present paper is to find out the mechanical characteristions of such boundary layer flows by GPDP.

BOUNDARY LAYER FLOW

Blasius solution for laminar flow over a flat plate

Assume : Governing equations .

1. Conservation of mass : $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$ (1)

2. Momentum Balance (*x* direction), $u\frac{\partial u}{\partial x} + u\frac{\partial u}{\partial y} = \gamma\frac{\partial^2 u}{\partial^2 y}$ (2)

3. Longitudial velocity $u = u_{\infty}\left(\frac{2y}{d} - \frac{2y^3}{d^3} + \frac{y^4}{d^4}\right)$; $0 \le y \le d$ (5)

4. Boundary conditions : y = 0, u = 0 (no slip), $\frac{\partial^2 u}{\partial y^2} = 0$ (6)

$$y = d; u = U_{\infty}; \frac{\partial u}{\partial y} = 0 \text{ (Smooth fit) ; } \frac{\partial^2 u}{\partial y^2} = 0 \quad (7)$$

5. Dimensionless velocity : $H = \frac{U_{\circ}}{U_{\infty}}\sqrt{\frac{U_{\infty}x}{\gamma}}$

6. When y = 0, u=0, v = v_0 (x)

7. Boundary layer thickness : $d = d * \sqrt{\frac{\upsilon x}{u_{\infty}}}$

8. Skin friction $\tau^* = \sqrt{\frac{\gamma x}{v_{\infty}^3}}\left(\frac{\partial u*}{\partial y}\right)_{y=o}$

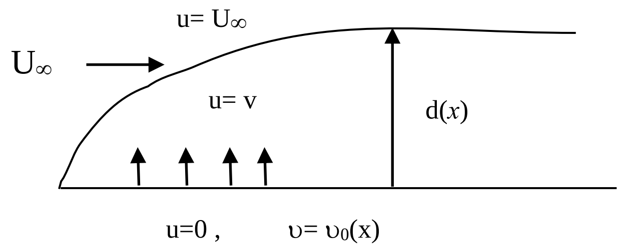
9. Kinetic viscocity of the fluid : γ

BLASIUS FLOW WITH SUCTION AND BLOWING

Mathematical Modelling:

The constitutive equations are $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$ (1) and $u\frac{\partial u}{\partial x} + \upsilon\frac{\partial u}{\partial y} = \gamma\frac{\partial^2 u}{\partial^2 y}$ (2) Where u and υ are the velocity components along x and y directions respectively and γ is the Kinetic Viscosity.

The boundary conditions of the system are y = 0, u = 0, v = v_0 (3) and y $\to \infty$, u $\to u_{\infty}$ (4)



Method of Solution :

We select a fourth degree trial function for longitudial velocity as

$$\frac{u}{u_\infty} = \frac{2y}{d} - \frac{2y^3}{d^3} + \frac{y^4}{d^4} \quad 0 \le y \le d \tag{5}$$

Which satisfies the compatibility conditions

$$y = 0,\ u = 0 \text{ (no slip)},\ \frac{\partial^2 u}{\partial y^2} = 0 \tag{6}$$

$$y = d; u = U_{\infty;}\ \frac{\partial u}{\partial y} = 0 \text{ (Smooth fit) ; } \frac{\partial^2 u}{\partial y^2} = 0 \tag{7}$$

Here d is the extent of the hypothetical hydrodynamical boundary layer thickness.

$$u = U_{\infty;}\left(\frac{2y}{d} - \frac{2y^3}{d^3} + \frac{y^4}{d^4}\right),\ 0 \le y \le d$$

Differentiating w.r.x $\frac{\partial u}{\partial x} = U_{\infty;}\left(\frac{-2y}{d^2}\, d^1 + \frac{6y^3}{d^4}\, d^1 - \frac{4y^4}{d^5} d^1\right)$

From (1) $\frac{\partial \upsilon}{\partial y} = \frac{-\partial u}{\partial x} = -U_{\infty;}\left(\frac{-2y}{d^2} d^1 + \frac{6y^3}{d^4} d^1 - \frac{4y^4}{d^5} d^1\right)$

$$\frac{\partial \upsilon}{\partial y} = U_{\infty;}\left(\frac{2y}{d^2}\, d^1 - 6\,\frac{y^3}{d^4}\, d^1 + 4\,\frac{y^4}{d^5}\, d^1\right)$$

Integrating w.r.x.y $\upsilon = U_\infty\left(\frac{y^2}{d^2}\, d^1 - \frac{3}{2}\,\frac{y^4}{d^4}\, d^1 + \frac{4}{5}\,\frac{y^5}{d^5}\, d^1\right) + A$

From (3) y=0, u =0, $\upsilon = \upsilon_0\,(x)$

$$\therefore\ \upsilon_o(x) = U_\infty\left(\frac{0}{d^2}\, d^1 - \frac{3}{2}\cdot\frac{0.d^1}{d^4} + \frac{4}{5}\cdot 0\,\frac{\cdot d^1}{d^5}\right) + A$$

$$\therefore\ A = \upsilon_o(x)$$

$$\therefore\quad \upsilon = U_\infty\left(\frac{y^2}{d^2}\, d^1 - {}^3\!/_2\ \frac{y^4}{d^4}\, d^1 + \frac{4}{5}\ \frac{y^5}{d^5}\, d^1\right) + \upsilon_\circ(x)$$

Form (5) $\frac{\partial u}{\partial y} = U_\infty\left(\frac{2}{d} - 6\frac{y^2}{d^3} + 4\frac{y^3}{d^4}\right)$

Form (2) $\gamma \frac{\partial^2 u*}{\partial y^2} = u \frac{\partial u}{\partial x} + \upsilon \frac{\partial u}{\partial y}$, using dual field method $\left(P_{12} = -\mu \frac{\partial u*}{\partial y} \right)$

$$\gamma \frac{\partial^2 u*}{\partial y^2} = U_\infty \left[\frac{2y}{d} - \frac{2y^3}{d^3} + \frac{y^4}{d^4} \right] \left[U_\infty \left(\frac{-2y}{d^2} d^1 + \frac{6y^3}{d^4} d^1 - 4 \frac{y^4}{d^5} d^1 \right) \right]$$

$$+ \left[U_\infty \left(\frac{y^2}{d^2} d^1 - \frac{3}{2} \frac{y^4}{d^4} d^1 + \frac{4}{5} \frac{y^5}{d^5} d^1 \right) + \upsilon_0 \right] \left[U_\infty \left(\frac{2}{d} - \frac{6y^2}{d^3} + 4 \frac{y^3}{d^4} \right) \right]$$

$$\gamma \frac{\partial^2 u*}{\partial y^2} = U_\infty^2 d^1 \left[-4 \frac{y^2}{d^3} + 12 \frac{y^4}{d^5} - 8 \frac{y^5}{d^6} + 4 \frac{y^4}{d^5} - 12 \frac{y^6}{d^7} + 8 \frac{y^7}{d^8} - \frac{2y^5}{d^6} + \frac{6y^7}{d^8} - \frac{4y^8}{d^9} \right]$$

$$+ U_\infty^2 d^1 \left[\frac{2y^2}{d^3} - \frac{6y^4}{d^5} + \frac{4y^5}{d^6} - \frac{3y^4}{d^5} + \frac{9y^6}{d^7} - \frac{6y^7}{d^8} + \frac{8}{5} \frac{y^5}{d^6} - \frac{24}{5} \frac{y^7}{d^8} + \frac{16}{5} \frac{y^8}{d^9} \right]$$

$$+ U_\infty \upsilon_0 \left(\frac{2}{d} - \frac{6y^2}{d^3} + \frac{4y^3}{d^4} \right)$$

$$\gamma \frac{\partial^2 u*}{\partial y^2} = U_\infty^2 d^1 \left| -2 \frac{y^2}{d^3} + 7 \frac{y^4}{d^5} - \frac{22}{5} \frac{y^5}{y^6} - 3 \frac{y^6}{d^7} + \frac{16}{5} \frac{y^7}{d^8} \frac{-4}{5} \frac{y^8}{d^9} \right|$$

$$+ U_\infty \upsilon_0 \left(\frac{2}{d} - \frac{6y^2}{d^3} + 4 \frac{y^3}{d^4} \right)$$

Integrating w.r.t.y

$$\gamma \frac{\partial u^*}{\partial y} = U_\infty^2 d^1 \left[\frac{-2}{3} \frac{y^3}{d^3} + \frac{7}{5} \frac{y^5}{d^5} - \frac{11}{15} \frac{y^6}{d^6} - \frac{3}{7} \frac{y^7}{d^7} + \frac{2}{5} \frac{y^8}{d^8} - \frac{4}{45} \frac{y^9}{d^9} \right]$$

$$+ U_\infty \upsilon_0 \left[\frac{2y}{d} - \frac{2y^3}{d^3} + \frac{y^4}{d^4} \right] + A$$

$$\gamma u^* = U_\infty^2 d^1 \left[-\frac{1}{6} \frac{y^4}{d^3} + \frac{7}{30} \frac{y^6}{d^5} - \frac{11}{105} \frac{y^7}{d^6} - \frac{3}{56} \frac{y^8}{d^7} + \frac{2}{45} \frac{y^9}{d^8} - \frac{2}{225} \frac{y^{10}}{d^9} \right]$$

$$+ U_\infty \upsilon_0 \left[\frac{y^2}{d} - \frac{1}{2} \frac{y^4}{d^3} + \frac{1}{5} \frac{y^5}{d^4} \right] + Ay + B$$

From (7), When $y = d, \frac{du*}{dy} = 0$

$u* = 0,\ When\ y = 0\ \therefore B = 0$

$u* = U_\infty\ When\ y = d$

$$\gamma U_\infty = U_\infty^2 dd^1\left[-\frac{1}{6}+\frac{7}{30}-\frac{11}{105}-\frac{3}{56}+\frac{2}{45}-\frac{2}{225}\right]+U_\infty \upsilon_0 d\left[1-\frac{1}{2}+\frac{1}{5}\right]+Ad$$

$$\gamma U_\infty = U_\infty^2 dd^1\left[\frac{-600+840-377.14286-192.85714+160-32}{3600}\right]+.7U_\infty\upsilon_0 d+Ad$$

$$=U_\infty^2 dd^1\left[\frac{-101}{1800}\right]+.7U_\infty \upsilon_0 d+Ad$$

$$\therefore A=\frac{\gamma U_\infty}{d}+\frac{101}{1800}\ U_\infty^2 d^1-.7\upsilon_o U_\infty$$

$$\gamma U^* = U_\infty^2 d^1\left[-\frac{1}{6}\frac{y^4}{d^3}+\frac{7}{30}\frac{y^6}{d^5}-\frac{11}{105}\frac{y^7}{d^6}-\frac{3}{56}\frac{y^8}{d^7}+\frac{2}{45}\frac{y^9}{d^8}-\frac{2}{225}\frac{y^{10}}{d^9}\right]$$

$$+U_\infty\upsilon_0\left[\frac{y^2}{d}-\frac{1}{2}\frac{y^4}{d^3}+\frac{1}{5}\frac{y^5}{d^4}\right]+y\left(\gamma\frac{U_\infty}{d}+\frac{101}{1800}U_\infty^2 d^1-.7U_0U_\infty\right)$$

$$\gamma U^* = U_\infty^2 d^1\left[\frac{101}{1800}y-\frac{1}{6}\frac{y^4}{d^3}+\frac{7}{30}\frac{y^6}{d^5}-\frac{11}{105}\frac{y^7}{d^6}-\frac{3}{50}\frac{y^8}{d^7}+\frac{2}{45}\frac{y^9}{d^8}-\frac{2}{225}\frac{y^{10}}{d^9}\right]$$

$$+\upsilon_0 U_\infty\left[\frac{y^2}{d}-\frac{1}{2}\frac{y^4}{d^3}+\frac{1}{5}\frac{y^5}{d^4}-\cdot 7y\right]+\frac{y\gamma U_\infty}{d}$$

$$\frac{\partial u^*}{\partial y}=\frac{U_\infty}{d}+\frac{U_\infty^2 d}{\gamma}\left[\frac{101}{1800}-\frac{2}{3}\frac{y^3}{d^3}+\frac{7}{5}\frac{y^5}{d^5}-\frac{11}{15}\frac{y^6}{d^6}-\frac{3}{7}\frac{y^7}{d^7}+\frac{2}{5}\frac{y^8}{d^8}-\frac{4}{45}\frac{y^9}{d^9}\right]$$

$$+\frac{U_\infty U_0}{\gamma}\left[\frac{2y}{d}-\frac{2y^3}{d^3}+\frac{y^4}{d^4}-\cdot 7\right]$$

Formulate force representation

$$\delta\int\left[\left(\frac{\partial u^*}{\partial y}\right)_\circ\frac{\partial u}{\partial y}-\frac{1}{2}\left(\frac{\partial u}{\partial y}\right)^2\right]dv=0$$

$$\delta\int\left[\left\{\frac{U_\infty}{d}+\frac{U_\infty^2 d^1}{\gamma}\left(\cdot 05611116-\frac{2}{3}\frac{y^3}{d^3}+\frac{7}{5}\frac{y^5}{d^5}-\frac{11}{15}\frac{y^6}{d^6}-\frac{3}{7}\frac{y^7}{d^7}+\frac{2}{5}\frac{y^8}{d^8}-\frac{4}{45}\frac{y^9}{d^9}\right)\right.\right.$$

$$\left.\left.+\frac{U_\infty\upsilon_0}{\gamma}\left(\frac{2y}{d}-\frac{2y^3}{d^3}+\frac{y^4}{d^4}-\cdot 7\right)\right\}\left\{U_\infty\left(\frac{2}{d}-6\frac{y^2}{d^3}+4\frac{y^3}{d^4}\right)\right\}-\frac{1}{2}U_\infty^2\left(\frac{2}{d}-\frac{6y^2}{d^3}+\frac{4y^3}{d^4}\right)^2\right]dxdy=0$$

$$\int_{x=0}^{L}\int_{y=0}^{d}\left[\left\{\frac{U_\infty}{d}+\frac{U_\infty^2 d^1}{\gamma}\left(\cdot 0561116-\frac{2}{3}\frac{y^3}{d^3}+\frac{7}{5}\frac{y^5}{d^5}-\frac{11}{15}\frac{y^6}{d^6}-\frac{3}{7}\frac{y^7}{d^7}+\frac{2}{5}\frac{y^8}{d^8}-\frac{4}{45}\frac{y^9}{d^9}\right)\right.\right.$$

$$+\frac{U_\infty \upsilon_0}{\gamma}\bigg(\frac{2y}{d}-\frac{2y^3}{d^3}+\frac{y^4}{d^4}-\cdot 7\bigg)\bigg\}\bigg\}\bigg\{U_\infty\bigg(-\frac{2}{d^2}+18\frac{y^2}{d^4}-16\frac{y^3}{d^5}\bigg)\bigg\}$$

$$-\frac{1}{2}U_\infty^2\bigg(-\frac{8}{d^3}-216\frac{y^4}{d^7}-128\frac{y^6}{d^9}+96\frac{y^2}{d^5}+336\frac{y^5}{d^8}-80\frac{y^3}{d^6}\bigg)\bigg]\delta d\,dx\,dy=0$$

$$\int_0^L\int_0^d\bigg[\bigg\{\frac{U_\infty}{d}+\frac{U_\infty^2 d^1}{\gamma}(\cdot 05611116)\bigg\}\bigg\{U_\infty\bigg(-\frac{2}{d^2}+18\frac{y^2}{d^4}-16\frac{y^3}{d^5}\bigg)\bigg\}+\frac{U_\infty^3 d^1}{\gamma}\bigg\{-\frac{2}{3}\frac{y^3}{d^3}\bigg(\frac{-2}{d^2}+\frac{18y^2}{d^4}-\frac{16y^3}{d^5}\bigg)$$

$$+\frac{7}{5}\frac{y^5}{d^5}\bigg(-\frac{2}{d^2}+\frac{18y^2}{d^4}-\frac{16y^3}{d^5}\bigg)-\frac{11}{15}\frac{y^6}{d^6}\bigg(-\frac{2}{d^2}+\frac{18y^2}{d^4}-\frac{16y^3}{d^5}\bigg)-\frac{3}{7}\frac{y^7}{d^7}\bigg(-\frac{2}{d^2}+\frac{18y^2}{d^4}-\frac{16y^3}{d^5}\bigg)+\frac{2}{5}\frac{y^8}{d^8}\bigg(-\frac{2}{d^2}+\frac{18y^2}{d^4}-\frac{16y^3}{d^5}\bigg)$$

$$-\frac{4}{45}\frac{y^9}{d^9}\bigg(-\frac{2}{d^2}+\frac{18y^2}{d^4}-\frac{16y^3}{d^5}\bigg)+\frac{U_\infty^2\upsilon_0}{\gamma}\bigg\{\frac{2y}{d}\bigg(-\frac{2}{d^2}+\frac{18y^2}{d^4}-\frac{16y^3}{d^5}\bigg)-\frac{2y^3}{d^3}\bigg(-\frac{2}{d^2}+\frac{18y^2}{d^4}-\frac{16y^3}{d^5}\bigg)+\frac{y^4}{d^4}\bigg(-\frac{2}{d^2}+\frac{18y^2}{d^4}-\frac{16y^3}{d^5}\bigg)$$

$$-\cdot 7\bigg(--\frac{2}{d^2}+\frac{18y^2}{d^4}-\frac{16y^3}{d^5}\bigg)\bigg\}\bigg\}-\frac{1}{2}U_\infty^2\bigg(-\frac{8}{d^3}-216\frac{y^4}{d^7}-128\frac{y^6}{d^9}+96\frac{y^2}{d^5}+336\frac{y^5}{d^8}-80\frac{y^3}{d^6}\bigg)\bigg]\delta d\,dx\,dy=0$$

$$\int_0^L\int_0^d\bigg[\bigg\{\frac{U_\infty^2}{d}+\frac{U_\infty^3 d^1}{\upsilon}(.5611116)\bigg\}\bigg\{\frac{-2}{d^2}+18\frac{y^2}{d^4}-16\frac{y^3}{d^5}\bigg\}+\frac{U_\infty^3 d^1}{\gamma}\bigg\{\frac{4}{3}\frac{y^3}{d^5}-12\frac{y^5}{d^7}+\frac{32}{5}\frac{y^6}{d^8}-\frac{14}{5}\frac{y^5}{d^7}+\frac{126}{5}\frac{y^7}{d^9}$$

$$\frac{-112}{5}\frac{y^8}{d^{10}}+\frac{22}{5}\frac{y^6}{d^8}-\frac{198}{15}\frac{y^8}{d^{10}}+\frac{176}{15}\frac{y^9}{d^{11}}+\frac{6}{7}\frac{y^7}{d^9}-\frac{54}{7}\frac{y^9}{d^{11}}+\frac{48}{7}\frac{y^{10}}{y^{12}}-\frac{4}{5}\frac{y^8}{d^{10}}+\frac{36}{5}\frac{y^{10}}{d^{12}}-\frac{32}{5}\frac{y^{11}}{d^{13}}+\frac{8}{45}\frac{y^9}{d^{11}}$$

$$\frac{-72}{45}\frac{y^{11}}{d^{13}}+\frac{64}{45}\frac{y^{12}}{d^{14}}\bigg\}+\frac{U_\infty^2\upsilon_0}{\gamma}\bigg\{\frac{-4y}{d^3}+36\frac{y^3}{d^5}-32\frac{y^4}{d^6}+4\frac{y^3}{d^5}-36\frac{y^5}{d^7}+32\frac{y^6}{d^8}-2\frac{y^4}{d^6}+18\frac{y^6}{d^8}-16\frac{y^7}{d^9}$$

$$-\cdot 7\bigg(-\frac{2}{d^2}+18\frac{y^2}{d^4}-16\frac{y^3}{d^5}\bigg)\bigg\}-\frac{1}{2}U_\infty^2\bigg(-\frac{8}{d^3}-216\frac{y^4}{d^7}-128\frac{y^6}{d^9}+96\frac{y^2}{d^5}+336\frac{y^5}{d^8}-80\frac{y^3}{d^6}\bigg)\bigg]\partial d\,dxdy=0$$

Integrating w.r.t*y*

$$\int_0^L\bigg[\bigg\{\frac{U_\infty^2}{d}+\frac{U_\infty^3 d^1}{\gamma}(.0561116)\bigg\}\bigg\{\frac{-2y}{d^2}+\frac{6y^3}{d^4}-\frac{4y^4}{d^5}\bigg\}+\frac{U_\infty^3 d^1}{\gamma}\bigg\{\frac{1}{3}\frac{y^4}{d^5}-\frac{2y^6}{d^7}+\frac{32}{21}\frac{y^7}{d^8}-\frac{14}{30}\frac{y^6}{d^7}+\frac{126}{40}\frac{y^8}{d^9}$$

$$-\frac{112}{45}\frac{y^9}{y^{10}}+\frac{22}{105}\frac{y^7}{d^8}-\frac{198}{135}\frac{y^9}{d^{10}}+\frac{176}{150}\frac{y^{10}}{y^{11}}+\frac{6}{56}\frac{y^8}{d^9}-\frac{54}{70}\frac{y^{10}}{d^{11}}+\frac{48}{77}\frac{y^{11}}{d^{12}}-\frac{4}{45}\frac{y^9}{d^{10}}+\frac{36}{55}\frac{y^{11}}{d^{12}}-\frac{32}{60}\frac{y^{12}}{d^{13}}$$

$$+\frac{8}{450}\frac{y^{10}}{d^{11}}-\frac{72}{540}\frac{y^{12}}{d^{13}}+\frac{64}{585}\frac{y^{13}}{d^{14}}\Bigg\}+\frac{U_\infty^2 U_0}{\gamma}\Bigg\{-\frac{2y^2}{d^3}+9\frac{y^4}{d^5}-\frac{32}{5}\frac{y^5}{d^6}+\frac{y^4}{d^5}-\frac{6y^6}{d^7}+\frac{32}{7}\frac{y^7}{d^8}-\frac{2}{5}\frac{y5}{d^6}$$

$$+\frac{18}{7}\frac{y^7}{d^8}-\frac{2y^8}{d^9}-\cdot 7\left(\frac{-2y}{d^2}+\frac{6y^3}{d^4}-\frac{4y^4}{d^5}\right)\Bigg\}-\frac{1}{2}U_\infty^2\left(\frac{-8y}{d3}-\frac{216}{5}\frac{y^5}{d^7}-\frac{128}{7}\frac{y^7}{d^9}+\frac{96}{3}\frac{y^3}{d^5}+\frac{336}{6}\frac{y^6}{d^8}-\frac{80}{4}\frac{y^4}{d^4}\right)_0^d \delta ddx = 0$$

$$\int_0^L\Bigg[\frac{U_\infty^3 d^1}{\gamma}\frac{1}{d}\Bigg\{\frac{1}{3}-2+\frac{32}{21}-\frac{14}{30}+\frac{126}{40}-\frac{112}{45}+\frac{22}{105}-\frac{198}{135}+\frac{176}{150}+\frac{6}{56}-\frac{54}{70}+\frac{48}{77}-\frac{4}{45}+\frac{36}{55}-\frac{32}{60}$$

$$+\frac{8}{450}-\frac{72}{540}+\frac{64}{585}\Bigg\}+\frac{U_\infty^2\upsilon_0}{\gamma d}\Bigg\{-2+9-\frac{32}{5}+1-6+\frac{32}{7}-\frac{2}{5}+\frac{18}{7}-2\Bigg\}$$

$$-\frac{1}{2}\frac{U_\infty^2}{d^2}\Bigg\{-8-\frac{126}{5}-\frac{128}{7}+32+56-20\Bigg\}\Bigg]\delta ddx = 0$$

$$\int_0^L\Bigg[\frac{U_\infty^3 d^1}{\gamma d}(-0.046961928)+\frac{U_\infty^2\upsilon_0}{\gamma d}(0.342857142)-\frac{1}{2}\frac{U_\infty^2}{d^2}(-1.48571429)\Bigg]\delta ddx = 0$$

$$d = d^*\sqrt{\frac{\gamma x}{U_\infty}} \qquad H = \frac{\upsilon_0}{U_\infty}\sqrt{\frac{U_\infty x}{\gamma}} \qquad \frac{HU_\infty}{\upsilon_0 x} = \sqrt{\frac{U_\infty}{\gamma x}}$$

$$d^1 = d^*\frac{1}{2}\sqrt{\frac{\gamma}{U_\infty x}} \qquad \frac{H^2U_\infty^2\gamma_x}{U_\infty} = \upsilon_0^2 x^2$$

$$\int_0^L\left|\frac{U_\infty^3 d^* \frac{1}{2}\sqrt{\frac{\gamma}{U_\infty x}}\, d^1}{\gamma\, d^*\sqrt{\frac{\gamma x}{U\infty}}}\left(-0.046961928)+\frac{U_\infty^2\upsilon_0}{\gamma d^*}\sqrt{\frac{U_\infty}{\gamma x}}(0.342857142)-\frac{1}{2}\frac{U_\infty^3}{d^{x2}\gamma x}(-1.48571429)\right]\right.\delta ddx = 0$$

$$\int_0^L\Bigg[\frac{U_\infty^3}{2\gamma x}(-0.046961928)+\frac{U_\infty^2\upsilon_0}{\gamma d^*}\frac{HU_\infty}{\upsilon_0 x}(0.342857142)-\frac{1}{2}\frac{U_\infty^3}{d^{*2}\gamma x}(-1.48571429)\Bigg]\delta ddx = 0$$

$$\int_0^L\Bigg[\frac{U_\infty^3}{\gamma x}\{\frac{1}{2}(-0.046961928)+\frac{H}{d^*}(0.342857142)+\frac{1}{d^{*2}}(\frac{1}{2}(-1.48571429)\Bigg]\delta ddx = 0$$

The Euler's Lagrange's equation is

$$-0.023480964+\frac{H}{d^*}(0.342857142)+\frac{1}{dx^2}(0.742857145) = 0$$

$$d^{*2}(0.023480964) - Hd^*(0.342857142) - (0.742857145) = 0$$

$$\text{When } H = O., d^{*} = 5.624639482$$

$$\text{Skin friction } \tau_{W}^{*} = \sqrt{\frac{\gamma x}{U_{\infty}^{3}}}\left(\frac{\partial \upsilon^{*}}{\partial y}\right)_{y=0}$$

$$= \frac{1}{d^{*}} + \frac{101}{3600} d^{*} - (0.7)H$$

$$= \frac{1}{d^{*}} + \frac{d^{*}}{2}(0.05611111) - (0.7)H$$

CONCLUSION

Boundary layer separation occurs when H = 0.60. When H =0, skin friction coefficient τ^{*} is calculated as 0.3355914 which is nearly equal to exact accepted skin friction coefficient of 0.332. Separation of boundary layer occurs due to excess injection when H > 0 and suction when H<0. The present studies have been taken by in view of its applications in environmental, chemical, engineering and industrial applications. Flow parameters such as thermal boundary layer thicknesses and skinfriction coefficients are computed for every nondimension velocity for different flow of conditions (suction and injection). The results have also been compared to that of earlier works and found that the present model agrees well with the observation of earlier works.

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ANNEXURE I

The Euler's Lagrange's equation d*[2] (0.023480964) - Hd* (0.3428571427 - (0.742857145) = 0

Skin friction $\tau_w^* = \sqrt{\frac{\gamma x}{U_\infty^3}} \left(\frac{\partial u^*}{\partial y}\right)_{y=0} = \frac{1}{d^*} + \frac{101}{3600} d^* - (0.7)H$

H	d*	τ_w^*	H	d*	τ_w^*	H	d*	τ_w^*
-.60	2.748711	0.8609233	-.19	4.406021	0.4835755	.22	7.455638	0.1892987
-.59	2.777094	0.8510014	-.18	4.518818	0.475299	.23	7.549112	0.1832604
-.58	2.865951	0.8411078	-.17	4.561983	0.4670748	.24	7.643419	0.1772718
-.57	2.835291	0.831243	-.16	4.576537	0.4589031	.25	7.738551	0.1713325
-.56	2.865124	0.8214076	-.15	4.635145	0.4507846	.26	7.834499	0.1654418
-.55	2.895458	0.811602	-.14	4.694649	0.4427195	.27	7.931253	0.1595992
-.54	2.926303	0.801827	-.13	4.755055	0.4347082	.28	8.028803	0.1538041
-.53	2.957668	0.792083	-.12	4.816371	0.4267512	.29	8.127140	0.1480559
-.52	2.989564	0.7823707	-.11	4.8786	0.4188487	.30	8.226254	0.1423541
-.51	3.021999	0.7726906	-.10	4.941748	0.411001	.31	8.326134	0.1366981
-.50	3.054983	0.7630432	-.09	5.005821	0.402085	.32	8.426770	0.1310872
-.49	3.088527	0.7534292	-.08	5.070823	0.3954714	.33	8.528154	0.1255208
-.48	3.12264	0.7438491	-.07	5.136757	0.3877899	.34	8.630274	0.1199984
-.47	3.157333	0.7343030	-.06	5.203627	0.3801643	.35	8.733118	0.1145192
-.46	3.192614	0.7247933	-.05	5.271436	0.3725947	.36	8.836679	0.1090827
-.45	3.228496	0.7153189	-.04	5.340185	0.3650813	.37	8.940944	0.1036882
-.44	3.264988	0.7058808	-.03	5.40988	0.3576242	.38	9.045903	0.09833518
-.43	3.3021	0.6964797	-.02	5.480519	0.3502235	.39	9.151546	0.09302294
-.42	3.339844	0.6871163	-.01	5.552106	0.3428793	.40	9.257861	0.08775082
-.41	3.378229	0.6777911	0	5.624639482	0.3355961	.41	9.364839	0.08251825
-.40	3.417266	0.6685048	.01	5.698121	0.3283604	.42	9.472469	0.7732457
-.39	3.456965	0.659258	.02	5.97255	0.3211858	.43	9.580741	0.07216919
-.38	3.497337	0.6500515	.03	5.847925	0.3140675	.44	9.689643	0.06705138
-.37	3.538393	0.6408857	.04	5.924245	0.3070059	.45	9.799166	0.06197065
-.36	3.580143	0.6317613	.05	6.00151	0.3000004	.46	9.909299	0.05692631
-.35	3.622597	0.622679	.06	6.079717	0.2930512	.47	10.02003	0.05191776
-.34	3.665767	0.6136393	.07	6.158861	0.2861579	.48	10.13135	0.04694438
-.33	3.709662	0.604643	.08	6.238942	0.2793206	.49	10.24326	0.04200557
-.32	3.754294	0.5956905	.09	6.319956	0.2725388	.50	10.35573	0.03710073
-.31	3.799672	0.5867825	.10	6.401898	0.2658125	.51	10.46876	0.03222925
-.30	3.845860	0.5779198	.11	6.484764	0.2591413	.52	10.58234	0.02739054
-.29	3.892707	0.5691027	.12	6.56855	0.2525249	.53	10.69646	0.02258408
-.28	3.940385	0.560332	.13	6.65325	0.245931	.54	10.81111	0.01780918
-.27	3.988849	0.5516083	.14	6.738858	0.2394555	.55	10.92628	0.01306537
-.26	4.038111	0.5429321	.15	6.825369	0.2330018	.56	11.04196	0.008352041
-.25	4.088177	0.5343039	.16	6.912775	0.2266015	.57	11.5814	0.003668606
-.24	4.13906	0.5257245	.17	7.001072	0.2202542	.58	11.27481	0.0009854436
-.23	4.190767	0.5171941	.18	7.090251	0.2139597	.59	11.39197	-0.005610645
-.22	4.243309	0.5087136	.19	7.180305	0.2077173	.60	11.5096	-0.01020756
-.21	4.296693	0.5002833	.20	7.271226	0.2015266			
-.20	4.350927	0.4919038	.21	7.363006	0.1953873			