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Intuitionistic Fuzzy γ* Generalized Continuous Mappings

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Abstract

In this paper we have introduced intuitionistic fuzzy γ* generalized continuous mappings and investigated some of their properties. Also we have provided some characterization of intuitionistic fuzzy γ* generalized continuous mappings.

Keywords: Intuitionistic fuzzy topology, intuitionistic fuzzy $\gamma^*T_{1/2}$ space, intuitionistic fuzzy γ* generalized continuous mappings, intuitionistic fuzzy γ* generalized irresolute mappings.

I. INTRODUCTION

Atanassov [1] introduced intuitionistic fuzzy sets using the notion of fuzzy sets. Coker [2] introduced intuitionistic fuzzy topological spaces. In this paper we have introduced intuitionistic fuzzy γ* generalized continuous mappings, intuitionistic fuzzy γ* generalized irresolute mappings and investigated some of their properties. Also we have provided some characterization of intuitionistic fuzzy γ* generalized continuous mappings.

2. PRELIMINARIES

Definition 2.1: [1] An ***intuitionistic fuzzy set*** (IFS for short) A is an object having the form

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$$

where the functions μ_A: $X \to [0,1]$ and $\nu_A : X \to [0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$. Denote by IFS(X), the set of all intuitionistic fuzzy sets in X.

An intuitionistic fuzzy set A in X is simply denoted by $A = \langle x, \mu_A, \nu_A\rangle$ instead of denoting $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle: x \in X\}$.

Definition 2.2: [1] Let A and B be two IFSs of the form

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle: x \in X\} \text{ and}$$

$$B = \{\langle x, \mu_B(x), \nu_B(x)\rangle : x \in X\}.$$

Then,

(a) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$,

(b) $A = B$ if and only if $A \subseteq B$ and $A \supseteq B$,

(c) $A^c = \{\langle x, \nu_A(x), \mu_A(x)\rangle : x \in X\}$,

(d) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x)\rangle: x \in X\}$,

(e) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x)\rangle: x \in X\}$.

The intuitionistic fuzzy sets $0_\sim = \langle x, 0, 1\rangle$ and $1_\sim = \langle x, 1, 0\rangle$ are respectively the empty set and the whole set of X.

Definition 2.3: [2] An ***intuitionistic fuzzy topology*** (IFT in short) on X is a family τ of IFSs in X satisfying the following axioms:

(i) $0_\sim, 1_\sim \in \tau$,

(ii) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$,

(iii) $\cup G_i \in \tau$ for any family $\{G_i: i \in J\} \in \tau$.

In this case the pair (X, τ) is called an ***intuitionistic fuzzy topological space*** (IFTS in short) and any IFS in τ is known as an ***intuitionistic fuzzy open set*** (IFOS in short) in

X. The complement A^c of an IFOS A in an IFTS (X, τ) is called an ***intuitionistic fuzzy closed set*** (IFCS in short) in X.

Definition 2.4: [12] Two IFSs A and B are said to be ***q-coincident*** ($A_q B$ in short) if and only if there exits an element x ∈X such that $\mu_A(x) > \nu_B(x)$ or $\nu_A(x) < \mu_B(x)$.

Definition 2.5: [12] Two IFSs A and B are said to be ***not q-coincident*** ($A_{\bar{q}} B$ in short) if and only if $A \subseteq B^c$.

Definition 2.6: [3] An ***intuitionistic fuzzy point*** (IFP for short), written as $p_{(\alpha, \beta)}$, is defined to be an IFS of X given by

$$p_{(\alpha, \beta)}(x) = \begin{cases} (\alpha, \beta) & \text{if } x = p, \\ (0,1) & \text{otherwise.} \end{cases}$$

An IFP $p_{(\alpha, \beta)}$ is said to belong to a set A if $\alpha \leq \mu_A$ and $\beta \geq \nu_A$.

Definition 2.7: [5] An IFS $A = \langle x, \mu_A, \nu_A \rangle$ in an IFTS (X, τ) is said to be an

(i) intuitionistic fuzzy γ closed set (IFγCS in short) if cl(int(A)) ∩ int(cl(A)) ⊆ A

(ii) intuitionistic fuzzy γ open set (IFγOS in short) if A ⊆ int(cl(A)) ∪ cl(int(A))

Definition 2.8: [4] Let A be an IFS in an IFTS (X, τ). Then the γ-interior and γ-closure of A are defined as

$$\gamma int(A) = \cup \{G \,/\, G \text{ is an IF}\gamma\text{OS in X and } G \subseteq A\}$$

$$\gamma cl(A) = \cap \{K \,/\, K \text{ is an IF}\gamma\text{CS in X and } A \subseteq K\}$$

Note that for any IFS A in (X, τ), we have $\gamma cl(A^c) = (\gamma int(A))^c$ and $\gamma int(A)^c = (\gamma cl(A))^c$.

Result 2.9: [6] Let A be an IFS in (X, τ), then

$$\gamma cl(A) \supseteq A \cup cl(int(A)) \cap int(cl(A))$$

$$\gamma int(A) \subseteq A \cap cl(int(A)) \cap int(cl(A))$$

Definition 2.10:[8] An IFS A of an IFTS (X, τ) is said to be an intuitionistic fuzzy γ* generalized closed set (briefly IFγ*GCS) if cl(int(A)) ∩ int(cl(A)) ⊆ U whenever A ⊆ U and U is an IFOS in (X, τ).

The complement A^c of an IFγ*GCS A in an IFTS (X, τ) is called an ***intuitionistic fuzzy γ * generalized open set*** (IFγ*GOS in short) in X.

The family of all IFγ*GOSs of an IFTS (X, τ) is denoted by IFγ*GO(X).

Definition 2.11: [3] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be an *intuitionistic fuzzy continuous* (IF continuous for short) mapping if $f^{-1}(B) \in IFO(X)$ for every $B \in \sigma$

3. INTUITIONISTIC FUZZY γ* GENERALIZED CONTINUOUS MAPPINGS

In this section we have introduced intuitionistic fuzzy γ* generalized continuous mappings and investigated some of their properties. Also we have established the relation between the newly introduced mapping and the already existing mappings.

Definition 3.1: A mapping f: (X, τ) → (Y, σ) is called an ***intuitionistic fuzzy γ* generalized continuous*** (IFγ*G continuous for short) **mapping** if $f^{-1}(V)$ is an IFγ*GCS in (X, τ) for every IFCS V of (Y, σ).

For the sake of simplicity, we shall use the notation $A = \langle x, (\mu_a, \mu_b), (\nu_a, \nu_b)\rangle$ instead of $A = \langle x, (a/\mu_a, b/\mu_b), (a/\nu_a, b/\nu_b)\rangle$ in the following examples.

Similarly we shall use the notation $B = \langle y, (\mu_u, \mu_v), (\nu_u, \nu_v)\rangle$ instead of $B = \langle y, (u/\mu_u, v/\mu_v), (u/\nu_u, v/\nu_v)\rangle$ in the following examples.

Example 3.2: Let X = {a, b} and Y= {u, v}. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively, where $G_1 = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$ and $G_2 = \langle y, (0.6_u, 0.6_v), (0.4_u, 0.4_v)\rangle$. Then (X, τ) is an IFTS. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. The IFS $G_2^c = \langle y, (0.4_u, 0.4_v), (0.6_u, 0.6_v)\rangle$ is an IFCS in Y. Then $f^{-1}(G_2^c) = \langle x, (0.4_a, 0.4_b), (0.6_a, 0.6_b)\rangle$ is an IFγ*GCS in (X, τ) as $f^{-1}(G_2^c) \subseteq G_1$ and $cl(int(f^{-1}(G_2^c))) \cap int(cl(f^{-1}(G_2^c))) = 0_\sim \subseteq G_1$, where G_1 is an IFOS in X. Therefore f is an IFγ*G continuous mapping.

Theorem 3.3: Every IF continuous mapping is an IFγ*G continuous mapping but not conversely in general.

Proof: Let f: (X, τ) → (Y, σ) be an IF continuous mapping. Let V be an IFCS in Y. Then $f^{-1}(V)$ is an IFCS in X. Since every IFCS is an IFγ*GCS [8], $f^{-1}(V)$ is an IFγ*GCS in X. Hence f is an IFγ*G continuous mapping.

Example 3.4: In Example 3.2, f is an IFγ*G continuous mapping but since $f^{-1}(G_2^c) = \langle x, (0.4_a, 0.4_b), (0.6_a, 0.6_b)\rangle$ is not an IFCS in X, as $cl(f^{-1}(G_2^c)) = G_1^c \neq f^{-1}(G_2^c)$, f is not an IF continuous mapping.

Theorem 3.5: Every IFS continuous mapping is an IFγ*G continuous mapping but not conversely in general.

Proof: Let f: (X, τ) → (Y, σ) be an IFS continuous mapping [6]. Let V be an IFCS in Y. Then $f^{-1}(V)$ is an IFSCS in X. Since every IFSCS is an IFγ*GCS [8], $f^{-1}(V)$ is an IFγ*GCS in X. Hence f is an IFγ*G continuous mapping.

Example 3.6: In Example 3.2, f is an IFγ*G continuous mapping. We have $int(cl(f^{-1}(G_2^c))) = int(G_1^c) = G_1 \not\subseteq f^{-1}(G_2^c) = \langle x, (0.4_a, 0.4_b), (0.6_a, 0.6_b)\rangle$. Hence $f^{-1}(G_2^c)$ is not an IFSCS in X. Hence f is not an IFS continuous mapping.

Theorem 3.7: Every IFP continuous mapping is an IFγ*G continuous mapping but not conversely in general.

Proof: Let f: (X, τ) → (Y, σ) be an IFP continuous mapping [6]. Let V be an IFCS in Y. Then $f^{-1}(V)$ is an IFPCS in X. Since every IFPCS is an IFγ*GCS [8], $f^{-1}(V)$ is an IFγ*GCS in X. Hence f is an IFγ*G continuous mapping.

Example 3.8: Let X = {a, b} and $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ be IFTs on X and Y respectively, where $G_1 = \langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)\rangle$ and $G_2 = \langle x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)\rangle$ and $G_3 = \langle y, (0.6_u, 0.6_v), (0.4_u, 0.4_v)\rangle$. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v.

The IFS $G_3^c = \langle y, (0.4_u, 0.4_v), (0.6_u, 0.6_v)\rangle$ is an IFCS in Y. Then $f^{-1}(G_3^c) = \langle x, (0.4_a, 0.4_b), (0.6_a, 0.6_b)\rangle$ is an IFγ*GCS as $cl(int(f^{-1}(G_3^c))) \cap int(cl(f^{-1}(G_3^c))) = G_1^c \cap G_2 = G_2 \subseteq G_2$ where $f^{-1}(G_3^c) \subseteq G_2$ but not an IFPCS as $cl(int(f^{-1}(G_3^c))) = G_1^c \not\subseteq f^{-1}(G_3^c)$. Hence f is not an IFP continuous mapping.

Theorem 3.9: Every IFα continuous mapping is an IFγ*G continuous mapping but not conversely in general.

Proof: Let f: (X, τ) → (Y, σ) be an IFα continuous mapping [6]. Let V be an IFCS in Y. Then $f^{-1}(V)$ is an IFαCS in X. Since every IFαCS is an IFγ*GCS [8], $f^{-1}(V)$ is an IFγ*GCS in X. Hence f is an IFγ*G continuous mapping.

Example 3.10: Let X = {a, b}, Y = {u, v} and G_1 = ⟨x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)⟩, G_2 = ⟨y, (0.5_u, 0.6_v), (0.4_u, 0.4_v)⟩. Then τ = {0~, G_1, 1~} and σ = {0~, G_2, 1~} are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. The IFS G_2^c = ⟨y, (0.4_u, 0.4_v), (0.5_u, 0.6_v)⟩ is an IFCS in Y. Then $f^{-1}(G_2^c)$ = ⟨x, (0.4_a, 0.4_b), (0.5_a, 0.6_b)⟩ is an IFγ*GCS as cl(int($f^{-1}(G_2^c)$)) ∩ int(cl($f^{-1}(G_2^c)$)) = 0~ ∩ G_1 = 0~ ⊆ G_1 where $f^{-1}(G_2^c) \subseteq G_1$ but not an IFαCS in (X, τ) as cl(int(cl($f^{-1}(G_2^c)$))) = $G_1^c \nsubseteq f^{-1}(G_2^c)$. Hence f is not an IFα continuous mapping.

Theorem 3.11: Let f: (X, τ) → (Y, σ) be a mapping and $f^{-1}(A)$ be an IFRCS in X for every IFCS A in Y. Then f is an IFγ*G continuous mapping but not conversely in general.

Proof: Let A be an IFCS in Y and $f^{-1}(A)$ be an IFRCS in X. Since every IFRCS is an IFγ*GCS [8], $f^{-1}(A)$ is an IFγ*GCS in X. Hence f is an IFγ*G continuous mapping.

Example 3.12: Let X = {a, b}, Y = {u, v} and G_1 = ⟨x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)⟩, G_2 = ⟨y, (0.5_u, 0.4_v), (0.5_u, 0.5_v)⟩. Then τ = {0~, G_1, 1~} and σ = {0~, G_2, 1~} are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. The IFS G_2^c = ⟨y, (0.5_u, 0.5_v), (0.5_u, 0.4_v)⟩ is an IFCS in Y. Then $f^{-1}(G_2^c)$ = ⟨x, (0.5_a, 0.5_b), (0.5_a, 0.4_b)⟩ is an IFγ*GCS in X as cl(int($f^{-1}(G_2^c)$)) ∩ int(cl($f^{-1}(G_2^c)$)) = G_1^c ∩ G_1 = G_1 ⊆ 1~ whenever A ⊆ 1~ but not an IFRCS in (X, τ) as cl(int($f^{-1}(G_2^c)$)) = $G_1^c \neq f^{-1}(G_2^c)$.

Theorem 3.13: Every IFγ continuous mapping is an IFγ*G continuous mapping but not conversely in general.

Proof: Let f: (X, τ) → (Y, σ) be an IFγ continuous mapping [5]. Let V be an IFCS in Y. Then $f^{-1}(V)$ is an IFγCS in X. Since every IFγCS is an IFγ*GCS [8], $f^{-1}(V)$ is an IFγ*GCS in X. Hence f is an IFγ*G continuous mapping.

Example 3.14: Let X = {a, b}, Y = {u, v} and G_1 = ⟨x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)⟩, G_2 = ⟨x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)⟩ and G_3 = ⟨y, (0.6_u, 0.4_v), (0.4_u, 0.6_v)⟩. Then τ = {0~, G_1, G_2, 1~} and σ = {0~, G_3, 1~} are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. The IFS G_3^c = ⟨y, (0.4_u, 0.6_v), (0.6_u, 0.4_v)⟩ is an IFCS in Y. Then $f^{-1}(G_3^c)$ = ⟨x, (0.4_a, 0.6_b), (0.6_a, 0.4_b)⟩ is an IFγ*GCS in X as cl(int($f^{-1}(G_3^c)$)) ∩ int(cl($f^{-1}(G_3^c)$)) = G_1^c ∩ G_1 = $G_1^c \subseteq G_1$ and $f^{-1}(G_3^c) \subseteq G_1$ but not an IFγCS in (X, τ) as $cl(int(f^{-1}(G_3^c))) \cap int(cl(f^{-1}(G_3^c))) = G_1^c \cap G_1 = G_1^c \nsubseteq f^{-1}(G_3^c)$. Hence $f^{-1}(G_3^c)$ is not an IFγCS in X. Hence f is not an IFγ continuous mapping.

Remark 3.15: Every IFG continuous mapping is an IFγ*G continuous mapping but not conversely in general.

Proof: Let f: (X, τ) → (Y, σ) be an IFG continuous mapping [11]. Let V be an IFCS in Y. Then $f^{-1}(V)$ is an IFGCS in X. Since every IFGCS is an IFγ*GCS [8], $f^{-1}(V)$ is an IFγ*GCS in X. Hence f is an IFγ*G continuous mapping.

Example 3.16: In Example 3.2, f is an IFγ*G continuous mapping but not an IFG continuous mapping as $cl(f^{-1}(G_2^c)) = G_1^c \nsubseteq G_1$, where $f^{-1}(G_2^c) \subseteq G_1$.

Remark 3.17: Every IFαG continuous mapping is an IFγ*G continuous mapping but not conversely in general.

Proof: Let f: (X, τ) → (Y, σ) be an IFαG continuous mapping [9]. Let V be an IFCS in Y. Then $f^{-1}(V)$ is an IFαGCS in X. Since every IFαGCS is an IFγ*GCS [8], $f^{-1}(V)$ is an IFγ*GCS in X. Hence f is an IFγ*G continuous mapping.

Example 3.18: Let X = {a, b}, Y = {u, v} and $G_1 = \langle x, (0.5_a, 0.3_b), (0.5_a, 0.7_b)\rangle$, $G_2 = \langle x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)\rangle$ and $G_3 = \langle y, (0.7_u, 0.8_v), (0.3_u, 0.2_v)\rangle$. Then $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. The IFS $G_3^c = \langle y, (0.3_u, 0.2_v), (0.7_u, 0.8_v)\rangle$ is an IFCS in Y. Then $f^{-1}(G_3^c) = \langle x, (0.3_a, 0.2_b), (0.7_a, 0.8_b)\rangle$ is an IFγ*GCS in X as $cl(int(f^{-1}(G_3^c))) \cap int(cl(f^{-1}(G_3^c))) = 0_\sim \cap G_1 = 0_\sim \subseteq G_1, G_2$ where $f^{-1}(G_3^c) \subseteq G_1, G_2$ but not an IFαGCS in (X, τ) as $\alpha cl(f^{-1}(G_3^c)) = f^{-1}(G_3^c) \cup cl(int(cl(f^{-1}(G_3^c)))) = f^{-1}(G_3^c) \cup G_1^c = G_1^c \nsubseteq G_1, G_2$ but $f^{-1}(G_3^c) \subseteq G_1, G_2$. Hence f is not an IFαG continuous mapping.

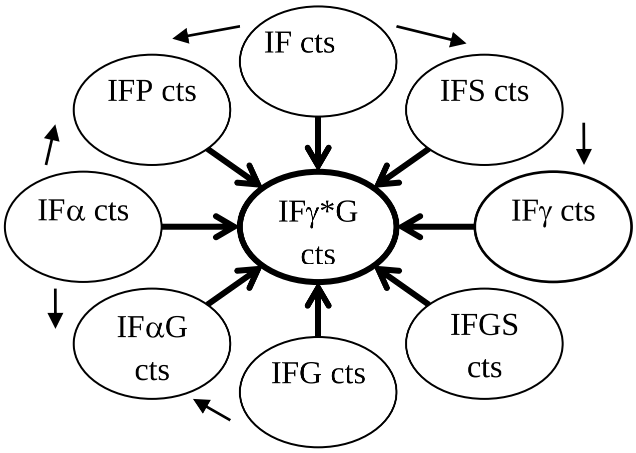
Remark 3.19: Every IFGS continuous mapping is an IFγ*G continuous mapping but not conversely in general.

Proof: Let f: (X, τ) → (Y, σ) be an IFGS continuous mapping [10]. Let V be an IFCS in Y. Then $f^{-1}(V)$ is an IFGSCS in X. Since every IFGSCS is an IFγ*GCS [8], $f^{-1}(V)$ is an IFγ*GCS in X. Hence f is an IFγ*G continuous mapping.

Example 3.20: In Example 3.18, f is an IFγ*G continuous mapping but not an IFGS continuous mapping as $f^{-1}(G_3^c)$ is an IFCS in Y, but not an IFGSCS in X, since

$scl(f^{-1}(G_3^c)) = f^{-1}(G_3^c) \cup int(cl(f^{-1}(G_3^c))) = f^{-1}(G_3^c) \cup G_1 = G_1 \not\subseteq G_2$, but $f^{-1}(G_3^c) \subseteq G_2$.

The relation between various types of intuitionistic fuzzy continuity is given in the following diagram. In this diagram 'cts.' means continous



Theorem 3.21: A mapping f: (X, τ) → (Y, σ) is an IFγ*G continuous mapping if and only if the inverse image of each IFOS in Y is an IFγ*GOS in X.

Proof: (Necessity): Let A be an IFOS in Y. This implies A^c is an IFCS in Y. Since f is an IFγ*G continuous mapping, $f^{-1}(A^c)$ is an IFγ*GCS in X. Since $f^{-1}(A^c) = (f^{-1}(A))^c$, $f^{-1}(A)$ is an IFγ*GOS in X.

(Sufficiency): Let A be an IFCS in Y. This implies A^c is an IFOS in Y. By hypothesis $f^{-1}(A^c)$ is an IFγ*GOS in X. Since $f^{-1}(A^c) = (f^{-1}(A))^c$, $f^{-1}(A)$ is an IFγ*GCS in X. Hence f is an IFγ*G continuous mapping.

Theorem 3.22: If f: (X, τ) → (Y, σ) is an IFγ*G continuous mapping then for each IFP $p_{(\alpha, \beta)}$ of X and each A ∈ σ such that $f(p_{(\alpha, \beta)}) \in A$, there exists an IFγ*GOS B of X such that $p_{(\alpha, \beta)} \in B$ and $f(B) \subseteq A$.

Proof: Let $p_{(\alpha, \beta)}$ be an IFP of X and A ∈ σ such that $f(p_{(\alpha, \beta)}) \in A$. Put $B = f^{-1}(A)$. Then by hypothesis, B is an IFγ*GOS in X such that $p_{(\alpha, \beta)} \in B$ and $f(B) = f(f^{-1}(A)) \subseteq A$.

Theorem 3.23: If f: (X, τ) → (Y, σ) is an IFγ*G continuous mapping then for each IFP $p_{(\alpha, \beta)}$ of X and each A ∈ σ such that $f(p_{(\alpha, \beta)})_q$ A, there exists an IFγ*GOS B of X such that $p_{(\alpha, \beta)\, q}$ B and $f(B) \subseteq A$.

Proof: Let $p_{(\alpha, \beta)}$ be an IFP of X and A ∈ σ such that $f(p_{(\alpha, \beta)})$ $_q$ A. Put B = f^{-1} (A). Then by hypothesis, B is an IFγ*GOS in X such that $p_{(\alpha, \beta)\, q}$ B and f(B) = f (f^{-1} (A)) ⊆ A.

Theorem 3.24: Let f: (X, τ) → (Y, σ) be an IFγ*G continuous mapping, then

(i) f is an IFγ continuous mapping if X is an IFγ*$T_{1/2}$ space

(ii) f is an IF continuous mapping if X is an IFγ*c$T_{1/2}$ space

(iii) f is an IFP continuous mapping if X is an IFγ*p$T_{1/2}$ space

Proof: (i) Let V be an IFCS in Y. Then f^{-1}(V) is an IFγ*GCS in X, by hypothesis. Since X is an IFγ*$T_{1/2}$ space, f^{-1}(V) is an IFγCS in X. Hence f is an IFγ continuous mapping.

(ii) Let V be an IFCS in Y. Then f^{-1}(V) is an IFγ*GCS in X, by hypothesis. Since X is an IFγ*c$T_{1/2}$ space, f^{-1}(V) is an IFCS in X. Hence f is an IF continuous mapping.

(iii) Let V be an IFCS in Y. Then f^{-1}(V) is an IFγ*GCS in X, by hypothesis. Since X is an IFγ*p$T_{1/2}$ space, f^{-1}(V) is an IFPCS in X. Hence f is an IFP continuous mapping.

Theorem 3.25: Let f: (X, τ) → (Y, σ) be an IFγ*G continuous mapping and g: (Y, σ) → (Z, δ) is an IF continuous mapping then g ∘ f: (X, τ) → (Z, δ) is an IFγ*G continuous mapping.

Proof: Let V be an IFCS in Z. Then g^{-1}(V) is an IFCS in Y, by hypothesis. Since f is an IFγ*G continuous mapping, f^{-1}(g^{-1}(V)) is an IFγ*GCS in X. Hence g ∘ f is an IFγ*G continuous mapping.

Remark 3.26: The composition of two IFγ*G continuous mappings need not be an IFγ*G continuous mapping. This can be seen from the following example.

Example 3.27: Let X = {a, b}, Y= {u, v} and Z= {p, q}. Then τ = {$0_\sim$, G_1, G_2, $1_\sim$}, bσ = {$0_\sim$, G_3, $1_\sim$} and δ = {$0_\sim$, G_4, $1_\sim$} are IFTs on X, Y and Z respectively, where G_1 = ⟨x, (0.5_a, 0.7_b), (0.2_a, 0.2_b)⟩ , G_2 = ⟨x, (0.6_a, 0.8_b), (0.2_a, 0.2_b)⟩, G_3 = ⟨y, (0.5_u, 0.4_v), (0.5_u, 0.6_v)⟩ and G_4 = ⟨z, (0.2_p, 0.2_q), (0.5_p, 0.8_q)⟩. Then (X, τ), (Y, σ) and (Z, δ) are IFTSs Now define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v and g: (Y, σ) → (Z, δ) by g(u) = p and g(v) = q. Here f and g are IFγ*G continuous mappings but their composition g ∘ f: (X, τ) → (Z, δ) defined by g(f(a)) =p and g(f(b)) = q is not an IFγ*G continuous mapping since G_4^c = ⟨z, (0.5_p, 0.8_q), (0.2_p, 0.2_q)⟩ is an IFCS in Z but

$f^{-1}(g^{-1}(G_4^c)) = \langle x, (0.5_a, 0.8_b), (0.2_a, 0.2_b)\rangle$ is not an IFγ*GCS in X as $f^{-1}(g^{-1}(G_4^c)) \subseteq G_2$ but $cl(int(f^{-1}(g^{-1}(G_4^c)))) \cap int(cl(f^{-1}(g^{-1}(G_4^c)))) = 1_\sim \nsubseteq G_2$.

Theorem 3.28: The composition of two IFγ*G continuous mappings is an IFγ*G continuous mapping if Y is an IFγ*c$T_{1/2}$ space.

Proof: Let V be an IFCS in Z. Then $g^{-1}(V)$ is an IFγ*GCS in Y, by hypothesis. Since Y is an IFγ*c$T_{1/2}$ space, $g^{-1}(V)$ is an IFCS in Y. Therefore $f^{-1}(g^{-1}(V))$ is an IFγ*GCS in X, by hypothesis. Hence g ∘ f is an IFγ*G continuous mapping.

Theorem 3.29: Let f: (X, τ) → (Y, σ) be a mapping. Then the following conditions are equivalent if X and Y are IFγ*$T_{1/2}$ spaces:

(i) f is an IFγ*G continuous mapping

(ii) $f^{-1}(B)$ is an IFγ*GOS in X for each IFOS B in Y

(iii) for each IFP $p_{(\alpha, \beta)}$ in X and for every IFOS B in Y such that $f(p_{(\alpha, \beta)}) \in B$, there exists an IFγ*GOS A in X such that $p_{(\alpha, \beta)} \in A$ and $f(A) \subseteq B$

Proof: (i) ⇒ (ii) is obvious from the Theorem 3.21.

(ii) ⇒ (iii) Let B be any IFOS in Y and let $p_{(\alpha, \beta)} \in X$. Given $f(p_{(\alpha, \beta)}) \in B$. By hypothesis $f^{-1}(B)$ is an IFγ*GOS in X. Take $A = f^{-1}(B)$. Then $p_{(\alpha, \beta)} \in f^{-1}(B) = A$. This implies $p_{(\alpha, \beta)} \in A$ and $f(A) = f(f^{-1}(B)) \subseteq B$.

(iii) ⇒ (i) Let A be an IFCS in Y. Then its complement, say B is an IFOS in Y. Let $p_{(\alpha, \beta)} \in X$ and $f(p_{(\alpha, \beta)}) \in B$. Then there exists an IFγ*GOS, say C in X such that $p_{(\alpha, \beta)} \in C$ and $f(C) \subseteq B$. Therefore $p_{(\alpha, \beta)} \in C \subseteq f^{-1}(B)$ and hence $f^{-1}(B)$ is an IFγ*GOS in X, by Theorem 3.21. That is $f^{-1}(A^c)$ is an IFγ*GOS in X and hence $f^{-1}(A)$ is an IFγ*GCS in X. Thus f is an IFγ*G continuous mapping.

Theorem 3.30: Let f: (X, τ) → (Y, σ) be a mapping. Then the following conditions are equivalent if X is an IFγ*$T_{1/2}$ space:

(i) f is an IFγ*G continuous mapping

(ii) If B is an IFOS in Y then $f^{-1}(B)$ is an IFγ*GOS in X

(iii) $f^{-1}(int(B)) \subseteq cl(int(f^{-1}(B))) \cup int(cl(f^{-1}(B)))$ for every IFS B in Y

Proof: (i) ⇒ (ii) is obviously true by Theorem 3.21.

(ii) ⇒ (iii) Let B be any IFS in Y. Then int(B) is an IFOS in Y. Then f^{-1}(int(B)) is an IFγ*GOS in X. Since X is an IFγ*$T_{1/2}$ space, f^{-1}(int(B)) is an IFγOS in X. Therefore f^{-1}(int(B)) ⊆ cl(int(f^{-1}(int(B)))) ∪ int(cl(f^{-1}(int(B)))) ⊆ cl(int(f^{-1}(B))) ∪ int(cl(f^{-1}(B))).

(iii) ⇒ (i) Let B be an IFCS in Y. Then its complement, say A is an IFOS in Y, then int(A) = A. Now by hypothesis f^{-1}(int(A)) ⊆ cl(int(f^{-1}(A))) ∪ int(cl(f^{-1}(A))). This implies f^{-1}(A) ⊆ cl(int(f^{-1}(A))) ∪ int(cl(f^{-1}(A))). Hence f^{-1}(A) is an IFγOS in X. Since every IFγOS is an IFγ*GOS, f^{-1} (A) is an IFγ*GOS in X. Thus f^{-1} (B) is an IFγ*GCS in X, since f^{-1} (A) = f^{-1}(B^c). Hence f is an IFγ*G continuous mapping.

Theorem 3.31: Let f: (X, τ) → (Y, σ) be a mapping. Then the following conditions are equivalent if X and Y are IFγ*$T_{1/2}$ spaces:

(i) f is an IFγ*G continuous mapping

(ii) cl(int(f^{-1}(B))) ∩ int(cl(f^{-1}(B))) ⊆ f^{-1}(cl(B)) for each IFCS B in Y

(iii) f^{-1}(int(B)) ⊆ cl(int(f^{-1}(B))) ∪ int(cl(f^{-1}(B))) for each IFOS B of Y

(iv) f(int(cl (A)) ∩ cl(int(A))) ⊆ cl(f(A)) for each IFS A of X.

Proof: (i) ⇒ (ii) Let B be an IFCS in Y. Then f^{-1} (B) is an IFγ*GCS in X. Since X is an IFγ*$T_{1/2}$ space, f^{-1} (B) is an IFγCS in X. Therefore cl(int(f^{-1}(B))) ∩ int(cl(f^{-1}(B))) ⊆ f^{-1}(B) = f^{-1}(cl(B)).

(ii) ⇒ (iii) can be easily proved by taking complement in (ii).

(iii) ⇒ (iv) Let A ∈ X. Then B = f(A) in Y and therefore A ⊆ f^{-1}(f(A)) ⊆ f^{-1} (B). Here int(f(A)) = int(B) is an IFOS in Y. Then (iii) implies that f^{-1}(int(B)) ⊆ cl(int(f^{-1}(int(B)))) ∪ int(cl(f^{-1}(int(B)))) ⊆ cl(int(f^{-1}(B))) ∪ int(cl(f^{-1}(B))). Now (cl(int(A^c)) ∪ int(cl(A^c)))^c ⊆ (cl(int(f^{-1}(B^c))) ∪ int(cl(f^{-1}(B^c))))^c ⊆ (f^{-1}(int(B^c)))^c. Therefore int(cl(A)) ∩ cl(int(A)) ⊆ f^{-1}(cl(B)). Now f(int(cl (A)) ∩ cl(int(A))) ⊆ f(f^{-1}(cl(B))) ⊆ cl(B) = cl(f(A)).

(iv) ⇒ (i) Let B be any IFCS in Y, then f^{-1}(B) is an IFS in X. By hypothesis f(int(cl(f^{-1}(B))) ∩ cl(int(f^{-1}(B)))) ⊆ cl(f(f^{-1}(B))) ⊆ cl(B) = B. Now (int(cl(f^{-1}(B))) ∩ cl(int(f^{-1}(B)))) ⊆ f^{-1}(f(int(cl (f^{-1}(B))) ∩ cl(int(f^{-1}(B)))) ⊆ f^{-1}(B). This implies f^{-1}(B) is an IFγCS and hence it is an IFγ*GCS in X. Thus f is an IFγ*G continuous mapping.

Theorem 3.32: A mapping f: (X, τ) → (Y, σ) is an IFγ*G continuous mapping if $cl(int(cl(f^{-1}(A)))) \subseteq f^{-1}(cl(A))$ for every IFS A in Y.

Proof: Let A be an IFCS in Y. By hypothesis, $cl(int(cl(f^{-1}(A)))) \subseteq f^{-1}(cl(A)) = f^{-1}(A)$. Therefore $f^{-1}(A)$ is an IFαCS and hence it is an IFγ*GCS. Thus f is an IFγ*G continuous mapping.

Theorem 3.33: Let f: (X, τ) → (Y, σ) be a mapping from an IFTS X into an IFTS Y. Then the following conditions are equivalent if X is an IFγ*$T_{1/2}$ space:

(i) f is an IFγ*G continuous mapping

(ii) $cl(int(f^{-1}(A))) \cap int(cl(f^{-1}(A))) \subseteq f^{-1}(cl(A))$ for every IFS A in Y

Proof: (i) ⇒ (ii) Let A be an IFS in Y. Then cl(A) is an IFCS in Y. By hypothesis, $f^{-1}(cl(A))$ is an IFγ*GCS in X. Since X is an IFγ*$T_{1/2}$ space, $f^{-1}(cl(A))$ is an IFγCS in X. Therefore $cl(int(f^{-1}(cl(A)))) \cap int(cl(f^{-1}(cl(A)))) \subseteq f^{-1}(cl(A))$. Now $cl(int(f^{-1}(A))) \cap int(cl(f^{-1}(A))) \subseteq cl(int(f^{-1}(cl(A)))) \cap int(cl(f^{-1}(cl(A)))) \subseteq f^{-1}(cl(A))$.

(ii) ⇒ (i) Let A be an IFCS in Y. By hypothesis $cl(int(f^{-1}(A))) \cap int(cl(f^{-1}(A))) \subseteq f^{-1}(cl(A)) = f^{-1}(A)$. This implies $f^{-1}(A)$ is an IFγCS in X and hence it is an IFγ*GCS. Thus f is an IFγ*G continuous mapping.

4. INTUITIONISTIC FUZZY γ* GENERALIZED IRRESOLUTE MAPPINGS

In this section we have introduced intuitionistic fuzzy γ* generalized irresolute mappings and studied some of their properties.

Definition 4.1: A mapping f: (X, τ) → (Y,σ) is called an *intuitionistic fuzzy γ* generalized irresolute* (IFγ*G irresolute) mapping if $f^{-1}(V)$ is an IFγ*GCS in (X, τ) for every IFγ*GCS V of (Y, σ).

Example 4.2: Let X = {a, b} and Y= {u, v}. Then τ = {$0_\sim$, G_1, $1_\sim$} and σ = {$0_\sim$, G_2, $1_\sim$} are IFTs on X and Y respectively, where $G_1 = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$ and $G_2 = \langle y, (0.4_u, 0.4_v), (0.6_u, 0.6_v)\rangle$ Then (X, τ) is an IFTS. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v.

IFγ*GC(X) ={$0_\sim$, $1_\sim$, $\mu_a \in [0,1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$, $\nu_b \in [0, 1]$ / $0 \le \mu_a + \nu_a \le 1$, $0 \le \mu_b + \nu_b \le 1$}

IFγ*GC(Y) ={$0_\sim$, $1_\sim$, $\mu_a \in [0,1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$, $\nu_b \in [0, 1]$ / $0 \leq \mu_a + \nu_a \leq 1$, $0 \leq \mu_b + \nu_b \leq 1$}.

Then f is an IFγ*G irresolute mapping.

Theorem 4.3: Let f: (X, τ) → (Y, σ) be an IFγ*G irresolute mapping, then f is an IFγ*G continuous mapping but not conversely.

Proof: Let f be an IFγ*G irresolute mapping. Let V be any IFCS in Y. Then V is an IFγ*GCS and by hypothesis $f^{-1}(V)$ is an IFγ*GCS in X. Hence f is an IFγ*G continuous mapping.

Example 4.4: Let X = {a, b}, Y = {u, v} and $G_1 = \langle x, (0.6_a, 0.8_b), (0.2_a, 0.1_b)\rangle$, $G_2 = \langle x, (0.3_a, 0.3_b), (0.2_a, 0.2_b)\rangle$ and $G_3 = \langle y, (0.5_u, 0.6_v), (0.5_u, 0.4_v)\rangle$. Then τ = {$0_\sim$, G_1, G_2, $1_\sim$} and σ = {$0_\sim$, G_3, $1_\sim$} are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v.

Then f is an IFγ*G continuous mapping but not an IFγ*G irresolute mapping, since the IFS $A = \langle y, (0.5_u, 0.3_v), (0.2_u, 0.1_v)\rangle$ is an IFγ*GCS in Y but not in X as $f^{-1}(A) = \langle x, (0.5_a, 0.3_b), (0.2_a, 0.1_b)\rangle \subseteq G_1$ but $int(cl((f^{-1}(A)))) \cap cl(int((f^{-1}(A)))) = 1_\sim \not\subseteq G_1$. Hence f is not an IFγ*G irresolute mapping.

Theorem 4.5: A mapping f: (X, τ) → (Y, σ) is an IFγ*G irresolute mapping if and only if the inverse image of each IFγ*GOS in Y is an IFγ*GOS in X.

Proof: The proof is obvious from the Definition 4.1, since $f^{-1}(A^c) = (f^{-1}(A))^c$.

Theorem 4.6: Let f: (X, τ) → (Y, σ) be an IFγ*G irresolute mapping, then

(i) f is an IFγ irresolute mapping if X is an IFγ*$T_{1/2}$ space

(ii) f is an IFP irresolute mapping if X is an IFγ*p$T_{1/2}$ space

Proof: (i) Let V be an IFγCS in Y. Then V is an IFγ*GCS in Y [8]. Therefore $f^{-1}(V)$ is an IFγ*GCS in X, by hypothesis. Since X is an IFγ*$T_{1/2}$ space, $f^{-1}(V)$ is an IFγCS in X. Hence f is an IFγ irresolute mapping.

(ii) Let V be an IFPCS in Y. Then V is an IFγ*GCS in Y [8]. Therefore $f^{-1}(V)$ is an IFγ*GCS in X, by hypothesis. Since X is an IFγ*P$T_{1/2}$ space, $f^{-1}(V)$ is an IFPCS in X. Hence f is an IFP irresolute mapping.

Theorem 4.7: Composition of two IFγ*G irresolute mappings is an IFγ*G irresolute mapping.

Proof: Let f: (X, τ) → (Y, σ) and g: (Y, σ) → (Z, δ) be IFγ*G irresolute mappings. Let V be an IFγ*GCS in Z. Then $g^{-1}(V)$ is an IFγ*GCS in Y. Since f is an IFγ*G irresolute, $f^{-1}(g^{-1}(V))$ is an IFγ*GCS in X, by hypothesis. Hence g o f is an IFγ*G irresolute mapping.

Theorem 4.8: Let f: (X, τ) → (Y, σ) be an IFγ*G irresolute mapping and g: (Y, σ) → (Z, δ) be an IFγ*G continuous mapping, then g o f: (X, τ) → (Z, δ) is an IFγ*G continuous mapping.

Proof: Let V be an IFCS in Z. Then $g^{-1}(V)$ is an IFγ*GCS in Y. Since f is an IFγ*G irresolute mapping, $f^{-1}(g^{-1}(V))$ is an IFγ*GCS in X. Hence g o f is an IFγ*G continuous mapping.

Theorem 4.9: Let f: (X, τ) → (Y, σ) be an IFγ*G irresolute mapping and g: (Y, σ) → (Z, δ) be an IF continuous mapping, then g o f: (X, τ) → (Z, δ) is an IFγ*G continuous mapping.

Proof: Let V be an IFCS in Z. Then $g^{-1}(V)$ is an IFCS in Y. Since every IFCS is an IFγ*GCS, $g^{-1}(V)$ is an IFγ*GCS in Y. Therefore $f^{-1}(g^{-1}(V))$ is an IFγ*GCS in X, by hypothesis. Hence g o f is an IFγ*G continuous mapping.

Theorem 4.10: Let f: (X, τ) → (Y, σ) be an IFγ*G continuous mapping and g: (Y, σ) → (Z, γ) be an IFγ*G irresolute mapping, then g o f: (X, τ) → (Z, γ) is an IFγ*G irresolute mapping if Y is an IFγ*c$T_{1/2}$ space.

Proof: Let V be an IFγ*GCS in Z. Then $g^{-1}(V)$ is an IFγ*GCS in Y, by hypothesis. Since Y is an IFγ*c$T_{1/2}$ space $g^{-1}(V)$ is an IFCS in Y. Therefore $f^{-1}(g^{-1}(V))$ is an IFγ*GCS in X, by hypothesis. Hence g o f is an IFγ*G irresolute mapping.

Theorem 4.11: Let f: (X, τ) → (Y, σ) be a mapping. Then the following conditions are equivalent if X and Y are IFγ*$T_{1/2}$ spaces:

(i) f is an IFγ*G irresolute mapping

(ii) $f^{-1}(B)$ is an IFγ*GOS in X for each IFγ*GOS in Y

(iii) $f^{-1}(\gamma int(B)) \subseteq \gamma int(f^{-1}(B))$ for each IFS B of Y

(iv) $\gamma cl(f^{-1}(B)) \subseteq f^{-1}(\gamma cl(B))$ for each IFS B of Y

Proof: (i) ⇒ (ii) is obvious.

(ii) ⇒ (iii) Let B be any IFS in Y and $\gamma int(B) \subseteq B$. Also $f^{-1}(\gamma int(B)) \subseteq f^{-1}(B)$. Since $\gamma int(B)$ is an IFγOS in Y, it is an IFγ*GOS in Y. Therefore $f^{-1}(\gamma int(B))$ is an IFγ*GOS in X, by hypothesis. Since X is an IFγ*$T_{1/2}$ space, $f^{-1}(\gamma int(B))$ is an IFγOS in X. Hence $f^{-1}(\gamma int(B)) = \gamma int(f^{-1}(\gamma int(B))) \subseteq \gamma int(f^{-1}(B))$.

(iii) ⇒ (iv) is obvious by taking complement in (iii).

(iv) ⇒ (i) Let B be an IFγ*GCS in Y. since Y is an IFγ*$T_{1/2}$ space, B is an IFγCS in Y and $\gamma cl(B) = B$. Hence $f^{-1}(B) = f^{-1}(\gamma cl(B)) \supseteq \gamma cl(f^{-1}(B)) \supseteq f^{-1}(B)$. Therefore $\gamma cl(f^{-1}(B)) = f^{-1}(B)$. This implies $f^{-1}(B)$ is an IFγCS and hence it is an IFγ*GCS in X. Thus f is an IFγ*G irresolute mapping.

Theorem 4.12: Let f: (X, τ) → (Y, σ) be an IFγ*G irresolute mapping. Then $f^{-1}(B) \subseteq \gamma int(f^{-1}(cl(int(B)) \cup int(cl(B))))$ for every IFγ*GOS B in Y, if X and Y are IFγ*$T_{1/2}$ spaces.

Proof: Let B be an IFγ*GOS in Y. Then by hypothesis $f^{-1}(B)$ is an IFγ*GOS in X. Since X is an IFγ*$T_{1/2}$ space, $f^{-1}(B)$ is an IFγOS in X. Therefore $\gamma int(f^{-1}(B)) = f^{-1}(B)$. Since Y is an IFγ*$T_{1/2}$ space, B is an IFγOS in Y and $B \subseteq (cl(int(B)) \cup int(cl(B)))$. Now, $f^{-1}(B) = \gamma int(f^{-1}(B))$ implies, $f^{-1}(B) \subseteq \gamma int(f^{-1}(cl(int(B)) \cup int(cl(B))))$.

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