Global Journal of Pure and Applied Mathematics.
ISSN 0973-1768 Volume 13, Number 7 (2017), pp. 2895-2906

http://www.ripublication.com

A Simulation Study on M/M/1 and M/M/C Queueing Model in a Multi Speciality Hospital

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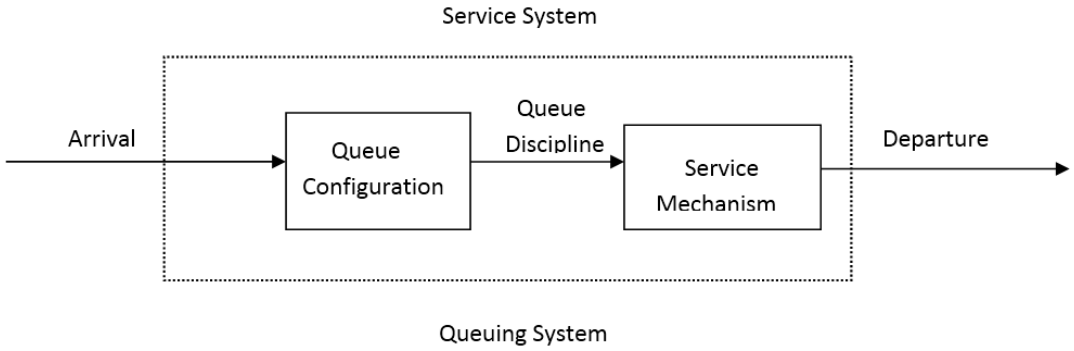
Abstract

In this paper, we analyse the parallel service in a single server and multi server queueing model using Monte Carlo Simulation. When a hospital has more than one department (General, Cardiologist, Gynaecologist Paediatrician, Dentist) the behaviour of the multi speciality hospital has been analysed both in simulation and analytical method. The numerical example illustrates the feasibility of the model.

Keywords: Average arrival, Average service, M/M/1 and M/M/C queueing model, Monte Carlo Simulation, Analytical method, Queue length.

INTRODUCTION

A common situation occurring in everyday life is that of queueing or waiting in a line. Queues form at bus stops, ticket booths, doctors clinics, bank counters, traffic signals and so on. Queues are also found in industry, in shops where the machines wait to be repaired; at a tool crib where the machines wait to receive tools, in a warehouse where parts are to be used and in telephone exchange where incoming calls wait to be matured. A queue is formed when either units requiring services – commonly referred to as customers, wait for service or the service facilities stand idle and wait for customers. A.K.Erlang (1909) [1] Danish Engineer who is called the father of Queuing theory, has published his articles relating to the study of congestion in telephone traffic.



The essential features of queueing systems are (i) Input source (ii) Queue configuration (iii) Queue discipline (iv)Service mechanism. Input source: Customers requiring service are generated at different times by an input source, commonly known as population. For example, Patients arriving at the hospital are normally of these categories: Walk-in patients with appointments and emergency patients.

Each patient class places different demands on service facility, but the waiting expectations of each category differ significantly. Queue Configuration: The queue configuration refers to the number of queues, and their respective lengths. The numbers of queues depend upon the layout of a service system. A service system is unable to accommodate more than the required number of customers at a time. Such types of situations are referred to as finite source queue. Examples of finite source queues are cinema halls, restaurants etc. On the other hand, if a service system is able to accommodate any number of customers at a time, then it is referred to as infinite source queue. For example, in a sales department, in worship places etc. Queue Discipline: Queue discipline refers to the logical ordering of customers in a queue and determines which customer will be chosen for service when a server becomes free. Common queue disciplines include first-in-first-out (FIFO); last-in first-out (LIFO); service in priority (PR). Service Mechanism: The Service Mechanism is concerned with the manner in which customers are serviced and leave the system. It is characterized by (i) The capacity of service facility (ii) The distribution of service times (iii) Server's behaviour (iv) Management policies. Operating Characteristics of Queueing System: Expected waiting time in queue: It is the average time spent by a customer in the queue before the commencement of his service and can be used to evaluate the quality of service. Expected waiting time in system: It is an average amount representing the total time spent by a customer in the system. Expected number of customers in the queue: It is the number of customers waiting to be serviced. Expected number of customers in the system: It is the number of customers either waiting in queue or being serviced. It can be used for finding the mean customer time spent in the system. The server utilization factor or busy period: It is the proportion of the time that a server actually spends with the customers.

Simulation was introduced by Mr.John von neuman and Mr.Stanishlaw ulam. They gave the first important application in the behaviour of Neutrons in a Nuclear Shielding Problem with remarkable success. Simulation is a method of solving

Decision-Making problems by designing, constructing and manipulating a model of the real system. Punitha.S (2014) [2] has analyzed the basic ideas of simulation method and calculates the queue length, customer waiting time and average service time in toll gate. Simulation is the process of designing a model of a real system and conducting experiments with this model for the purpose of understanding the behaviour of the operation for the system. Syed Shujauddin Sameer(2014) [3] has applied simulation method successfully for understanding queueing behaviour and had helped to increase the performance of the system. A simulation model initially developed by Takakuwa (2008) [4] for the entire departments of the outpatient hospital ward of a university hospital has been redesigned considering actual working time of the doctors. Simulation is a numerical solution method that seeks optimal alternatives through a trial and error process. The Simulation approach can be used to study almost any problem that involves uncertainty, that is one or more decision variables can be represented by a probability distribution, like decision making under risk. Simulation approach requires an analogous physical model to represent mathematical and logical relationship among variables of the problem under study. After having constructed the desired model, the Simulation approach evaluate each alternative by generating a series of values of random variable on paper over a period of time within the given set of conditions or criteria. This process of generating series of values one after another to understand the behaviour of the system is called the executing model. Simulation is the use of a system model that has the designed characteristics of reality in order to produce the essence of actual operation.

Ishan P Lade, Sandeep A Chowriwar and Pranay B Sawaitul (2013) [5] have described the use of queueing systems to decrease the waiting time of patients in the queue and in the system using simulation method. The random numbers so generated are known as pseudorandom numbers: such a pseudorandom number contains a bounded (fixed) number of digits, implying that continuous uniform distribution is approximated by a discrete one. Pseudorandom numbers are made use of it in simulation studies. The waiting time of the patients in the queue and in the medical centre in the single server queueing system using simulation model in a medical centre has been analyzed in the paper [6]. Monte Carlo method needs large number of pseudorandom numbers and computers are in their generation. Simulation approaches are popular and powerful. It should be emphasized that simulation is just one of possible experiments for an interested system. Simulation analysis requires a careful process of designing an abstract model of a real system for a given set of conditions. Kim,B.J (2011) [7] has discussed on the basis of the sensitivity of toll plaza using a discrete event simulation method in a conceptualization of traffic flow. Banumathi.P (2016) [8] has analyzed the queue length of each queue and waiting time of a customer in M/M/1 queueing system using simulation model in railway system.

MODEL DESCRIPTION

The main purpose of this paper is to apply Monte Carlo Simulation method in a multi speciality hospital in order to find the queue lengths of five different doctors like

General, Cardiologist, Gynaecologist Paediatrician, and Dentist. The collected data regarding number of patients arriving and service given to them by five doctors has been utilized in the Monte Carlo Simulation method to find the queue length of all five doctors and is compared to the analytical method as shown in table 6(a). The queue lengths for two of the departments (General and Gynaecology) are seen to be higher than that of other departments, so the multi server system (additional doctors) is introduced for those two departments as shown in table 6(b). It can be noted from the above mentioned table that the queue lengths have become null. The comparison between simulation and analytical method has been given in the bar diagram. The queue lengths of the various departments are presented by the simulation tables 6(a) and 6(b). In a single and multi channel queueing simulation, arrival and service are generated from the distributions by using the random variables [9].

TABLE 1: ARRIVAL DISTRIBUTION

S.NO	NO.OF PATIENTS	PROBABILITY
1	79	0.11
2	174	0.23
3	249	0.33
4	131	0.18
5	112	0.15

TABLE 2: TAG NUMER TABLE FOR ARRIVAL DISTRIBUTION

S.NO	NO.OF PATIENTS	PROBABILITY	CUMMULATIVE PROBABILITY	TAG NUMBERS
1	79	0.11	0.11	1 to 10
2	174	0.23	0.34	11 to 33
3	249	0.33	0.67	34 to 66
4	131	0.18	0.85	67 to 84
5	112	0.15	1	85 to 100

TABLE 3: SERVICE DISTRIBUTION

S.NO	NO.OF PATIENTS	PROBABILITY
1	214	0.35
2	152	0.25
3	106	0.17
4	87	0.14
5	60	0.09

TABLE: 4 TAG NUMER TABLE FOR SERVICE DISTRIBUTION

S.NO	NO.OF PATIENTS	PROBABILITY	CUMMULATIVE PROBABILITY	TAG NUMBER
1	214	0.35	0.35	0 to 34
2	152	0.25	0.6	35 to 59
3	106	0.17	0.77	60 to 76
4	87	0.14	0.91	77 to 90
5	60	0.09	1	91 to 100

TABLE 5: TAG NUMER TABLE FOR SERVER CHOOSEN

SERVER	PROBABILITY	CUMMULATIVE PROBABILITY	TAG NUMBER
GENERAL	0.2	0.2	1 to 19
CARDIOLOGIST	0.2	0.4	20 to 39
GYNAECOLOGIST	0.2	0.6	40 to 59
PAEDIATRICIAN	0.2	0.8	60 to 79
DENTIST	0.2	1	80 to 100

TABLE 6(a): SIMULATION TABLE

S.NO	RANDOM NUMBER	SERVER	RANDOM NUMBER FOR ARRIVAL	ARRIVAL	RANDOM NUMBER FOR SERVICE	SERVICE	GENERAL			CARDIOLOGIST			GYNAECOLOGIST			PAEDIATRICIAN			DENTIST		
							NUMBER OF PATIENTS FOR SERVICE	NUMBER OF PATIENTS TREATED	NUMBER OF PATIENTS WAITING	NUMBER OF PATIENTS FOR SERVICE	NUMBER OF PATIENTS TREATED	NUMBER OF PATIENTS WAITING	NUMBER OF PATIENTS FOR SERVICE	NUMBER OF PATIENTS TREATED	NUMBER OF PATIENTS WAITING	NUMBER OF PATIENTS FOR SERVICE	NUMBER OF PATIENTS TREATED	NUMBER OF PATIENTS WAITING	NUMBER OF PATIENTS FOR SERVICE	NUMBER OF PATIENTS TREATED	NUMBER OF PATIENTS WAITING
1	36	CARDIOLOGIST	65	249	14	214				249	214	35									
2	54	GYNAECOLOGIST	71	131	51	152							131	131	-						
3	7	GENERAL	20	174	51	152	174	152	22												
4	45	GYNAECOLOGIST	17	174	9	214							174	174	-						
5	60	PAEDIATRICIAN	48	249	71	106										249	106	143			
6	9	GENERAL	89	112	25	214	134	134	-												
7	93	DENTIST	18	174	52	152													174	152	22
8	41	GYNAECOLOGIST	83	131	92	60							131	60	71						
9	17	GENERAL	8	79	37	152	79	79	-												
10	10	GENERAL	90	112	25	214	112	112	-												
11	39	CARDIOLOGIST	5	79	32	214				114	114	-									
12	94	DENTIST	89	112	4	214													134	134	-
13	95	DENTIST	18	174	50	152													174	152	22
14	61	PAEDIATRICIAN	8	79	8	214										222	214	8			
15	75	PAEDIATRICIAN	26	174	70	106										182	106	76			
16	83	DENTIST	47	249	89	87													271	87	184
17	43	GYNAECOLOGIST	94	112	68	106							183	106	77						
18	35	CARDIOLOGIST	6	79	72	106				79	79	-									
19	6	GENERAL	72	131	99	60	131	60	71												
20	76	PAEDIATRICIAN	40	249	29	214										325	214	111			
21	9	GENERAL	62	249	95	60	320	60	260												
22	52	GYNAECOLOGIST	47	249	82	87							326	87	239						
23	71	PAEDIATRICIAN	68	131	65	106										242	106	136			
24	38	CARDIOLOGIST	60	249	85	87				249	87	162									
25	69	PAEDIATRICIAN	88	112	68	106										248	106	142			
26	80	DENTIST	17	174	1	214													358	214	144
27	92	DENTIST	36	249	85	87													393	87	306
28	44	GYNAECOLOGIST	77	131	27	214							370	214	156						
29	67	PAEDIATRICIAN	43	249	62	106										391	106	285			
30	47	GYNAECOLOGIST	28	174	21	214							330	214	116						
31	39	CARDIOLOGIST	31	174	94	60				336	60	276									

32	65	PAEDIATRICIAN	6	79	20	214										364	214	150			
33	5	GENERAL	58	249	52	152	509	152	357												
34	59	GYNAECOLOGIST	39	249	45	152							365	152	213						
35	99	DENTIST	71	131	1	214													437	214	223
36	4	GENERAL	22	174	73	106	531	106	425												
37	79	PAEDIATRICIAN	76	131	58	152										281	152	129			
38	60	PAEDIATRICIAN	81	131	24	214										260	214	46			
39	61	PAEDIATRICIAN	88	112	72	106										158	106	52			
40	73	PAEDIATRICIAN	94	112	84	87										164	87	77			
41	58	GYNAECOLOGIST	53	249	35	152							462	152	310						
42	47	GYNAECOLOGIST	79	131	41	152							441	152	289						
43	23	CARDIOLOGIST	9	79	32	214				355	214	141									
44	69	PAEDIATRICIAN	62	249	75	106										326	106	220			
45	35	CARDIOLOGIST	82	131	20	214				272	214	58									
46	21	CARDIOLOGIST	18	174	97	60				232	60	172									
47	41	GYNAECOLOGIST	20	174	47	152							463	152	311						
48	14	GENERAL	37	249	99	60	674	60	614												
49	59	GYNAECOLOGIST	65	249	22	214							560	214	346						
50	28	CARDIOLOGIST	71	131	16	214				303	214	89									
		TOTAL	-	8348	-	7379			614			89			346			220			223

SIMULATION CALCULATION
Average queue length on general = 12.28
Average queue length on Cardiologist = 1.78
Average queue length on Gynaecologist = 6.92
Average queue length on Paediatrician = 4.4
Average queue length on Dentist = 4.46
Average queue length = 5.9

ANALYTICAL CALCULATION
Average number of arrival = 171.26
Average number of service = 148.5
Arrival rate $\lambda = 0.0058$
Service rate $\mu = 0.0067$
Average queue length = 5.6

NUMERICAL STUDY

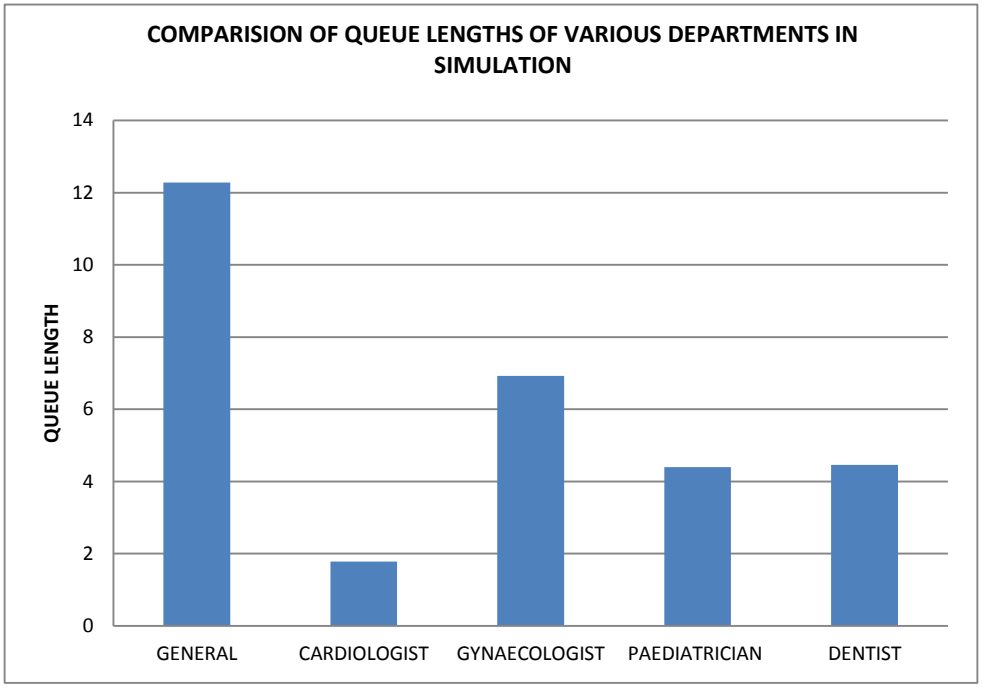


Figure 1

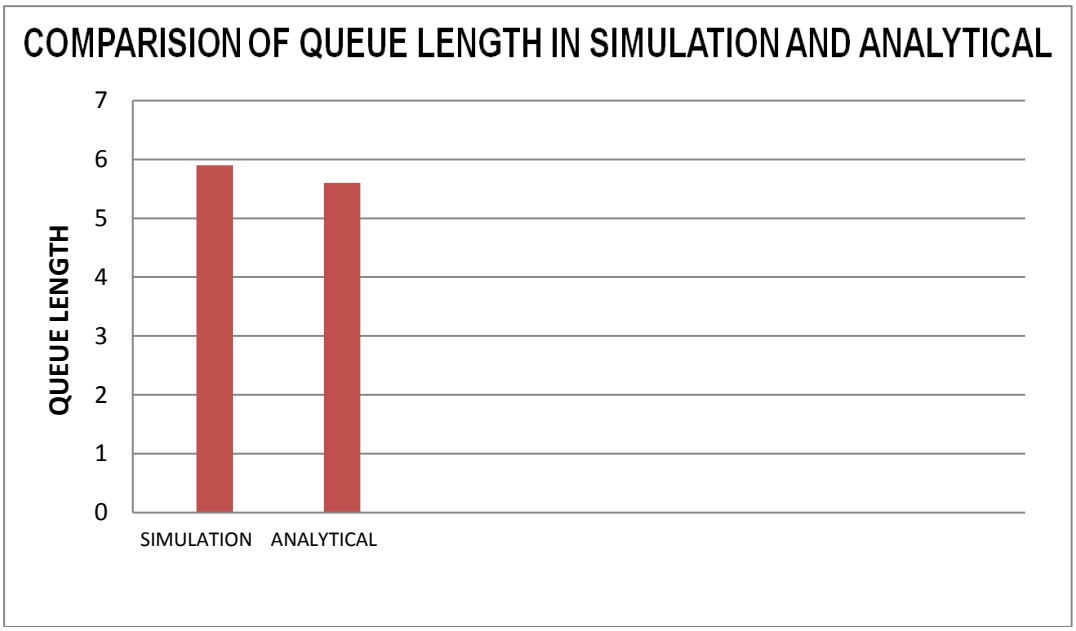


Figure 2

TABLE 6(b): SIMULATION TABLE

S.No	1	2	3	4	5	6	GENERAL 1			GENERAL 2					CARDIOLOGIST			GYNAECOLOGIST 1			GYNAECOLOGIST 2					PAEDIATRICIAN			DENTIST		
							7	8	9	5	6	7	8	9	7	8	9	7	8	9	5	6	7	8	9	7	8	9	7	8	9
1	45	GY	69	131	57	152				88	87							131	131	-	82	87									
2	61	PA	37	249	06	214				74	106										59	152				249	214	35			
3	60	PA	01	79	51	152				96	60										40	152				114	114	-			
4	50	GY	92	112	67	106				30	214							112	106	-	47	152	6	6	-						
5	80	DE	47	249	34	214				05	214										85	87							249	214	35
6	56	GY	94	112	28	214				31	214							112	112	-	41	152									
7	04	GE	07	79	86	87	79	79	-	11	214										72	106									
8	06	GE	19	174	04	214	174	174	-	55	152										66	106									
9	25	CA	29	174	25	214				61	214				174	174	-				76	106									
10	31	CA	87	112	40	152				20	214				112	112	-				34	214									
11	14	GE	66	249	60	106	249	106	-	83	87	143	87	56							25	214									
12	66	PA	78	131	96	60				44	152										73	106				131	60	71			
13	90	DE	56	249	59	152				67	106										87	87							284	152	132
14	53	GY	07	79	74	106				14	214							79	79	-	59	152									
15	48	GY	43	249	49	152				98	60							249	152	-	38	152	97	97	-						
16	88	DE	36	249	85	87				59	152										35	152							381	87	294
17	92	DE	16	174	47	152				70	106										47	152							468	152	316
18	07	GE	74	131	21	214	187	187	-	55	152										07	214									
19	12	GE	82	131	16	214	131	131	-	49	152										70	106									
20	56	GY	31	174	94	60				47	152							174	60	-	25	214	114	114	-						
21	37	CA	17	174	33	214				05	214				174	174	-				94	60									
22	25	CA	04	79	55	152				82	87				79	79	-				46	152									
23	46	GY	24	174	79	87				39	152							174	87	-	84	87	87	87	-						
24	74	PA	89	112	55	152				78	87										25	214				183	152	31			
25	34	CA	11	174	25	214				01	214				174	174	-				79	87									
26	45	GY	61	249	06	214				92	60							249	214	-	07	214	35	35	-						
27	29	CA	23	174	12	214				43	152				174	174	-				78	87									
28	11	GE	86	112	95	60	112	60	-	75	106	52	52	-							59	152									
29	82	DE	41	249	01	214				42	152										26	214							565	214	351
30	7	PA	6	24	7	10				2	21										03	214				280	106	174			

	3		4	9	5	6				8	4																				
31	15	GE	50	249	20	214	249	214	-	39	152	35	35	-							29	214								
32	70	PA	65	249	30	214				87	87										12	214				423	214	209			
33	33	CA	86	112	9	214				69	106				112	112	-				58	152									
34	87	DE	99	112	23	214				71	106										68	106						463	214	249	
35	99	DE	91	112	50	152				07	214										55	152						361	152	209	
36	12	GE	73	131	39	152	131	131	-	12	214										80	87									
37	56	GY	21	174	46	152				52	152							174	152	-	92	60	22	22	-						
38	80	DE	11	174	58	152				86	87										18	214						383	152	231	
39	72	PA	26	174	13	214				93	60										96	60				383	214	169			
40	24	CA	36	249	31	214				55	152				249	214	35				68	106									
41	07	GE	20	174	60	106	174	106	-	65	106	68	68	-							43	152									
42	93	DE	88	112	13	214				79	87										23	214						343	214	129	
43	35	CA	55	249	49	214				47	152				284	214	70				08	60									
44	80	DE	83	131	59	214				11	214										67	106						260	214	46	
45	17	GE	45	249	64	106	249	106	-	39	152	143	143	-							42	152									
46	04	GE	12	174	43	152	174	152	-	29	214	22	22	-							77	87									
47	45	GY	10	79	31	214				15	214							79	79	-	61	106									
48	20	CA	22	174	70	106				31	214				244	106	138				92	60									
49	03	GE	77	131	66	106	131	106	-	12	214	25	25	-							46	152									
50	88	DE	46	249	75	106				97	60										91	60						295	106	189	
									-					-			138								-			169			189

1 - RANDON NUMBER FOR SERVER
2 - SERVER
3 - RANDOM NUMBER FOR ARRIVAL
4 - ARRIVAL
5 - RANDOM NUMBER FOR SERVICE
6 - SERVICE
7 - NUMBER OF PATIENTS FOR SERVICE
8 - NUMBER OF PATIENTS TREATED
9 - NUMBER OF PATIENTS WAITING

GE – GENERAL

CA – CARDIOLOGIST

GY – GYNAECOLOGIST

PA – PAEDIATRICIAN

DE – DENTIST

SIMULATION CALCULATION

Average queue length on Cardiologist = 2.76

Average queue length on Paediatrician = 3.38

Average queue length on Dentist = 3.78

Average queue length on General & Gynaecologist = 0

ANALYTICAL CALCULATION

Average number of arrival = 171.26

Average number of service = 148.5

Arrival rate $\lambda = 0.0058$

Service rate $\mu = 0.0067$

Average queue length on General & Gynaecologist = 0.2

CONCLUSION

The simulation table 6(a) is presented for queueing system with parallel services for five departments served by one doctor each respectively. The collected data for the arrival and service distribution has been analyzed using Monte Carlo Simulation method, which is compared to the analytical method. The simulation method almost coincides with analytical method. Also various departments' queue lengths have been analyzed in simulation method. The queue lengths for two of the departments (General and Gynaecology) are seen to be higher than other departments, as shown in table 6(a). So the multi server system has been introduced for those two departments to decrease the queue length, as seen in table 6(b). From the above discussion, we can deduce that this study will guide us to develope an efficient procedure to take managerial decisions in a multi speciality hospital.

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