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Effect of micro-inertia on reflection/refraction of plane waves at the orthotropic and thermoelastic micropolar materials with voids

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Abstract

The problem of reflection/refraction of elastic waves at the plane interface between orthotropic micropolar elastic half-space and micropolar thermoelastic half-space with voids is investigated. We obtain amplitude and energy ratios of those reflected and refracted waves due to the incident longitudinal wave. It is observed that these ratios are functions of angle of incidence, micropolar, thermal and voids parameters. The effect of micro-inertia on the amplitude and energy ratios are studied numerically and analytically for a particular model.

AMS subject classification:
Keywords: Longitudinal wave, transverse wave, amplitude and energy ratios, orthotropy, micropolarity, voids.

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Introduction

Micropolar theory is an extension of elasticity with extra independent degrees of freedom for local rotation. The theory explains certain static and dynamic effects, i.e. new types of waves and coupled stress of the materials. In this theory, the motions of the particles are expressed in terms of displacement and micro-rotation vector. Eringen [1] introduced the linear theory of micropolar elasticity and explained the micro-rotational motion and spin inertia that can support coupled stress and body couples in the materials. Smith [2] discussed the problem of waves in micropolar elastic solid and obtained the velocity of surface wave. Parfitt and Eringen [3] studied the problem of propagation of plane waves in a micropolar half-space and the reflections from a stress free flat surface. Eringen [4] derived equations of motion, constitutive equations and boundary conditions for a class of micropolar elastic solids which can stretch and contract. Lord and Shulman [5] formulated a generalized dynamical theory of thermoelasticity using a form of the heat transport equation which includes the time needed for acceleration of the heat flow. Eringen [6] discussed the foundation of micropolar thermoelasticity. Green and Lindsay [7] introduced the theory of generalized thermoelasticity. Iesan [8] derived the uniqueness and existence theorems in the orthotropic micropolar elastic solid and also reduced the boundary value problems to integral equations for which the Fredholm's basic theorems are valid.

Goodman and Cowin [9] developed the concept of distributed body and the continuum theory for granular materials with interstitial voids as a kinematic variable. Cowin and Nunziato [10] established a linear theory of elastic materials with voids for the treatment of porous solids, they formulated the basic equations and boundary value problems. Marin [11] proved the reciprocal theorem, uniqueness results and minimum principle that satisfies the constitutive equations, geometrical equations and a kinematically admissible state in micropolar materials with voids. Passarella [12] derived constitutive and field equations for micropolar porous thermoelasticity. Singh [13] investigated the reflection and refraction of micropolar thermoelastic waves at an interface between liquid and solid half-spaces. Kumar and Choudhary [14] discussed the plane strain problem in homogeneous micropolar orthotropic elastic solids and obtained the disturbance due to time harmonic concentrated source using eigen-value approach. Mondal and Acharya [15] studied the effects of voids and micropolar character of the material on the propagation of surface waves in micropolar elastic medium with voids. Ciarletta et al. [16] obtained the fundamental solution of steady oscillations in the linear theory of micropolar thermoelasticity for materials with voids. Ciarletta et al. [17] investigated the problem of plane waves and vibrations in the micropolar thermoelastic materials with voids. They proved the existence theorems of non-trivial solutions and eigenfrequencies of the interior homogeneous boundary value problems of steady vibration.

Singh [18] studied the problem of wave propagation in an orthotropic micropolar elastic solid and obtained the reflection coefficients of the reflected waves. Passarella et al. [19] derived a uniqueness theorem with no positive definiteness assumption on the elastic constitutive coefficients of heat-flux dependent micropolar porous thermoelastic media. They also proved a reciprocal relation and a variational principle under nonhomogeneous initial conditions. Singh and Lianngenga [20] investigated the problem of plane waves in micropolar thermoelastic materials with voids and obtained the phase velocities of the waves propagating in such medium. Various problems of waves and vibrations are in open literatures, i.e. Chanrasekhariah [21], Kumar and Choudhary [22], Vinh and Ogden [23], Sharma and Kumar [24], Passarella et al. [25], Singh [26, 27], Kumar and Kaur [28] and others.

In this paper, we investigate the problem of reflection/refraction of elastic waves from a plane interface between two half-spaces of orthotropic and micropolar thermoelastic materials with voids. We obtain the amplitude and energy ratios of the reflected and refracted waves for the incident longitudinal wave. The effect of micro-inertia on the amplitude and energy ratios are discussed for a particular model.

Basic Equations

Let us consider the cartesian co-ordinate system with $x-$axis lying horizontally and $y-$axis vertically with positive direction pointing downward. We assume that an orthotropic micropolar elastic half-space, $M = \{(x, y); x \in R, y \in (0, \infty)\}$ and another half-space of micropolar thermoelastic materials with voids, $\bar{M} = \{(x, y); x \in R, y \in (-\infty, 0)\}$ are weld contact and are separated by $y = 0$.

Orthotropic micropolar elastic half-space, M

The equations of motion for homogeneous orthotropic micorpolar solid in $xy-$plane, without body couples and forces, are written as [18]

$$A_{11}u_{1,11} + (A_{12} + A_{78})u_{2,12} + A_{88}u_{1,22} - K_1\phi_{3,2} = \varrho\ddot{u}_1, \tag{1}$$

$$(A_{12} + A_{78})u_{1,12} + A_{77}u_{2,11} + A_{22}u_{2,22} - K_2\phi_{3,1} = \varrho\ddot{u}_2, \tag{2}$$

$$B_{66}\phi_{3,11} + B_{44}\phi_{3,22} - K_3\phi_3 + K_1u_{1,2} + K_2u_{2,1} = \varrho f\ddot{\phi}_3, \tag{3}$$

where A_{11}, A_{12}, A_{22}, A_{77}, A_{78}, A_{88}, A_{44}, B_{44} and B_{66} are characteristic constant of the material, f is micro-inertia, ϱ is density, $K_1 = A_{78} - A_{88}$, $K_2 = A_{77} - A_{78}$ and $K_3 = K_2 - K_1$. Here, the displacement vector and micro-rotation vector are respectively represented by $\mathbf{u} = (u_1, u_2, 0)$ and $\boldsymbol{\phi} = (0, 0, \phi_3)$. The subscripts preceded by coma indicate coordinate derivatives and superposed dots mean time derivatives.

The constitutive relations in such an anisotropic materials are given by

$$\begin{aligned} t_{22} &= A_{12}u_{1,1} + A_{22}u_{2,2}, \\ t_{21} &= A_{78}u_{2,1} + A_{88}u_{1,2} + (A_{88} - A_{78})\phi_3, \\ m_{23} &= B_{44}\phi_{3,2} \end{aligned} \tag{4}$$

where t_{22}, t_{21} and m_{23} are tractions due to normal force stresses, tangential force stresses and tangential couple stresses respectively.

Half-space of micropolar thermoelastic materials with voids, $\bar{M}$

The equation of motions of homogeneous and isotropic micropolar thermoelastic materials with voids in the absence of body forces and body couples are [20]

$$\begin{aligned}(\lambda + 2\mu + \kappa)\nabla^2 u' + s\psi - m\Theta &= \rho \ddot{u}', & (5)\\ s\nabla^2 u' - (a\nabla^2 - \zeta)\psi + \xi\Theta &= -\rho_2 \ddot{\psi}, & (6)\\ k_0\nabla^2\Theta - \Theta_0(1 + \tau\frac{\partial}{\partial t})(d\dot{\Theta} + m\nabla^2\dot{u}' - \xi\dot{\psi}) &= 0, & (7)\\ (\alpha + \beta + \gamma)\nabla^2\phi' - 2\kappa\phi' &= \rho_1\ddot{\phi}', & (8)\\ (\mu + \kappa)\nabla^2\mathbf{u}'' + \kappa\nabla\times\boldsymbol{\phi}'' &= \rho\ddot{\mathbf{u}}'', & (9)\\ (\gamma\nabla^2 - 2\kappa)\boldsymbol{\phi}'' + \kappa\nabla\times\mathbf{u}'' &= \rho_1\ddot{\boldsymbol{\phi}}'', & (10)\end{aligned}$$

where λ, μ are lame's parameters, $\alpha, \beta, \gamma, \kappa$ are micropolar parameters, s, a, ζ, ξ are voids parameters and m, k_0, d, τ are thermal parameters, ρ is mass density, $\rho_1 (= \rho J)$ and $\rho_2 (= \rho\chi)$ are inertial coefficients, J, χ are micro-inertia and equilibrated inertia respectively. Here, ψ is the void volume fraction and Θ is temperature measured from a reference temperature Θ_0. The displacement ($\bar{\mathbf{u}}$) and micro-rotation ($\bar{\boldsymbol{\phi}}$) vectors are represented as

$$\bar{\mathbf{u}} = \nabla u' + \nabla\times\boldsymbol{u}'', \ \bar{\boldsymbol{\phi}} = \nabla\phi' + \nabla\times\boldsymbol{\phi}''. \quad (11)$$

Equations (5)-(7) are couple in u', ψ and Θ, while Equations (9) and (10) are also couple in $\mathbf{u}''$ and $\boldsymbol{\phi}''$. Equation (8) is uncouple longitudinal waves or micropolar waves (Parfitt and Eringen, 1969). The constitutive relations for half-space, $\bar{M}$ in the absence of body forces and body couples are given as [12]

$$\begin{aligned}\bar{T}_{ij} &= \lambda\bar{u}_{k,k}\delta_{ij} + \mu(\bar{u}_{i,j} + \bar{u}_{j,i}) + \kappa(\bar{u}_{i,j} + \epsilon_{ijk}\bar{\phi}_k) + s\psi\delta_{ij} - m\Theta\delta_{ij},\\ \bar{M}_{ij} &= \alpha\bar{\phi}_{k,k}\delta_{ij} + \beta\bar{\phi}_{j,i} + \gamma\bar{\phi}_{i,j}, \ \bar{h}_i = a\psi_{,i}, \ (i, j = 1, 2) \quad (12)\end{aligned}$$

where $\bar{T}_{ij}$ and $\bar{M}_{ij}$ are the stress and couple stress tensors, $\bar{h}_i$ is the equilibrated stress vector.

Wave Propagation

A train of longitudinal wave with amplitude, A_0 is incident at the plane interface making an angle θ_0 with the normal. This wave gives three reflected coupled waves in the half-space, M and five refracted waves (three coupled longitudinal waves and two coupled shear waves) in half-space, $\bar{M}$. The complete geometry of the problem is shown in Figure 1.

The structures of the various reflected and refracted waves are given below.

For the reflected waves in the half-space, M:

$$\{u_1, u_2, \phi_3\} = \sum_{i=0}^{3}\{1, \eta_i, \imath k_i\pi_i\}k_i A_i \exp(Q_i), \quad (13)$$

where ϕ_3 is the $z-$component of $\boldsymbol{\phi}$.

For the refracted waves in the half-space, $\bar{M}$:

$$\{u', \psi, \Theta\} = \sum_{i=4}^{6}\{1, \eta_i, \pi_i\}A_i \exp(Q_i), \tag{14}$$

$$\{u'', \Phi_3\} = \sum_{i=7}^{8}\{1, \eta_i\}A_i \exp(Q_i), \tag{15}$$

where u'' and Φ_3 are the $z-$component of $\mathbf{u}''$ and $\boldsymbol{\phi}''$ respectively, $Q_i = \imath k_i(xp_1^{(i)} + yp_2^{(i)} - v_i t)$, $\mathbf{p}^{(i)} = (p_1^{(i)}, p_2^{(i)}, 0)$ is propagation vector, v_i is the phase velocity and k_i is the wavenumber and A_i is the amplitude constant at angle θ_i with wavenumber k_i.

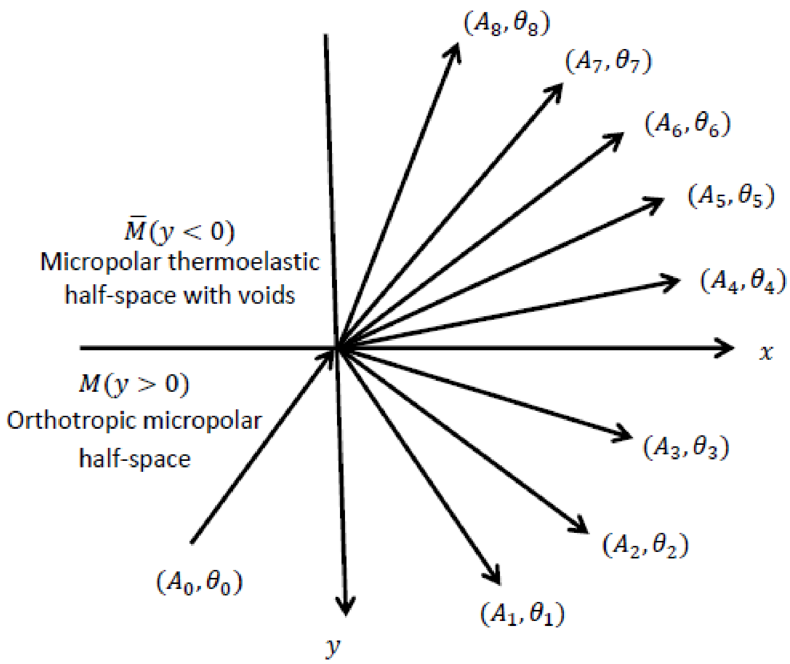

Figure 1: Geometry of the problem

The coupling constants, η_i and π_i are defined as

$$\eta_i = \begin{cases} \{a_5^2(a_1^2 - v_i^2) - a_2^2 a_3^2\}/\{a_3^2(a_4^2 - v_i^2) - a_2^2 a_5^2\}, & (i = 0, 1, 2, 3) \\ \{c_4^2 c_k^2 k_i^4 - (c_4^2 c_d^2 - c_7^2 c_m^2)k_i^2\}/\{c_5^2 c_k^2 k_i^4 & \\ -(\omega^2 c_k^2 - c_6^2 c_k^2 + c_5^2 c_d^2)k_i^2 - c_7^2 c_\xi^2 - c_6^2 c_d^2 + \omega^2 c_d^2\}, & (i = 4, 5, 6) \\ c_1^2/\{c_2^2 + c_3^2/k_i^2 - v_i^2\}, & (i = 7, 8) \end{cases}$$

$$\pi_i = \begin{cases} \{a_7^2\eta_i + a_6^2\}/\{a_9^2/v_i^2 + a_8^2 - 1\}, & (i = 0, 1, 2, 3) \\ \{c_m^2 c_5^2 k_i^4 + (c_m^2 c_6^2 + c_4^2 c_\xi^2 - c_m^2\omega^2)k_i^2\}/\{-c_5^2 c_k^2 k_i^4 \\ +(\omega^2 c_k^2 - c_6^2 c_k^2 + c_5^2 c_d^2)k_i^2 + c_7^2 c_\xi^2 + c_6^2 c_d^2 - \omega^2 c_d^2\}, & (i = 6, 7, 8). \end{cases}$$

where

$$\begin{aligned}
&a_1^2 = (A_{11}p_1^2 + A_{88}p_2^2)/\varrho,\ a_2^2 = p_1 p_2(A_{12} + A_{78})/\varrho, \\
&a_3^2 = K_1 p_2/\varrho,\ a_4^2 = (A_{77}p_1^2 + A_{22}p_2^2)/\varrho, \\
&a_5^2 = K_2 p_1/\varrho,\ a_6^2 = K_1 p_2/\varrho j\omega^2,\ a_7^2 = K_2 p_1/\varrho j\omega^2, \\
&a_8^2 = K_3/\varrho j\omega^2,\ a_9^2 = (B_{66}p_1^2 + B_{44}p_2^2)/\varrho j \\
&c_1^2 = (\alpha + \beta)/\rho_1,\ c_2^2 = \gamma/\rho_1,\ c_3^2 = 2\kappa/\rho_1, \\
&c_4^2 = s/\rho_2,\ c_5^2 = a/\rho_2,\ c_6^2 = \zeta/\rho_2,\ c_7^2 = \xi/\rho_2, \\
&c_m^2 = m/\rho_1,\ c_\xi^2 = \xi/\rho_1,\ c_d^2 = d/\rho_1,\ c_k^2 = k_0/\Theta_0\rho_1\omega(\omega\tau + \iota).
\end{aligned}$$

The Snell's law in this problem may be written as

$$p_0^{(0)}k_0 = p_1^{(i)}k_i,\ (i = 1, 2, \ldots, 8). \tag{16}$$

It is also noted that θ_i $(i = 1, 2, 3)$ correspond to angles of reflected waves in the half-space, M and θ_i $(i = 4, 5, 6, 7, 8)$ correspond to angles of refracted waves in the half-space, $\bar{M}$.

Boundary Conditions

The continuity of tractions due to force stresses and couple stresses at $y = 0$ may be written as

$$t_{22} = \bar{T}_{22},\ t_{21} = \bar{T}_{21},\ m_{23} = \bar{M}_{23}. \tag{17}$$

The displacement components, micro-rotation vectors, temperature gradient and the equilibrated stress vector at the interface, $y = 0$ are also continuous as

$$u_1 = \bar{u}_1,\ u_2 = \bar{u}_2,\ \phi_3 = \Phi_3,\ \Theta_{,2} = 0,\ h_2 = 0. \tag{18}$$

With the help of Equations (4) and (12), we re-write Equation (17) as

$$\begin{aligned} A_{12}\frac{\partial u_1}{\partial x} + A_{22}\frac{\partial u_2}{\partial y} &= \lambda\frac{\partial^2 u'}{\partial x^2} + (\lambda + 2\mu + \kappa)\frac{\partial^2 u'}{\partial y^2} \\ &+(2\mu + \kappa)\frac{\partial^2 u''}{\partial x \partial y} + s\psi - m\Theta, \end{aligned} \tag{19}$$

$$\begin{aligned} A_{78}\frac{\partial u_2}{\partial x} + A_{88}\frac{\partial u_1}{\partial y} + (A_{88} - A_{78})\phi_3 &= (2\mu + \kappa)\frac{\partial^2 u'}{\partial x \partial y} \\ -(\mu + \kappa)\frac{\partial^2 u''}{\partial y^2} &+ \mu\frac{\partial^2 u''}{\partial x^2} - \kappa\Phi_3, \end{aligned} \tag{20}$$

$$B_{44}\frac{\partial \phi_3}{\partial y} = \gamma\frac{\partial \Phi_3}{\partial y}. \tag{21}$$

Using Equations (13)-(16) into the boundary conditions (18)-(21), we have a system of eight equations and they are given by

$$\sum_{j=1}^{8} a_{ij} Z_j = b_i, \ (i = 1, 2, \ldots, 8). \tag{22}$$

The non-zero, a_{ij} are given as

$$a_{1j} = \begin{cases} (A_{12} p_1^{(j)} + A_{22} p_2^{(j)}) \imath k_j^2, & (j = 0, 1, 2, 3) \\ \{\lambda + p_2^{(j)^2} (2\mu + \kappa) - (s\eta_j - m\pi_j)/k_j^2\} k_j^2, & (i = 4, 5, 6) \\ (2\mu + \kappa) p_1^{(j)} p_2^{(j)} k_j^2, & (i = 7, 8) \end{cases}$$

$$a_{2j} = \begin{cases} (A_{78} \eta_j p_1^{(j)} + A_{88} p_2^{(j)} - K_1 \pi_j) \imath k_j^2, & (j = 0, 1, 2, 3) \\ (2\mu + \kappa) p_1^{(j)} p_2^{(j)} k_j^2, & (i = 4, 5, 6) \\ -\{\mu (p_2^{(j)^2} - p_1^{(j)^2}) + \kappa p_1^{(j)^2} - \kappa \eta_j / k_j^2\}, & (i = 7, 8) \end{cases}$$

$$a_{3j} = \begin{cases} B_{44} \pi_j p_2^{(j)} k_j^3, & (j = 0, 1, 2, 3) \\ \imath \gamma \eta_j p_2^{(j)} k_j, & (j = 7, 8) \end{cases}$$

$$a_{4j} = \begin{cases} k_j, & (j = 0, 1, 2, 3) \\ -\imath p_1^{(j)} k_j, & (j = 4, 5, 6) \\ \imath p_2^{(j)} k_j, & (j = 7, 8), \end{cases}$$

$$a_{5j} = \begin{cases} \imath \eta_j k_j, & (j = 0, 1, 2, 3) \\ p_2^{(j)} k_j, & (j = 4, 5, 6) \\ p_1^{(j)} k_j, & (j = 7, 8) \end{cases}$$

$$a_{6j} = \begin{cases} \pi_j k_j^2, & (j = 0, 1, 2, 3) \\ \imath \eta_j, & (7, 8) \end{cases}$$

$$a_{7j} = p_2^{(j)} \eta_j k_j, \ (j = 4, 5, 6), \ a_{8j} = p_2^{(j)} \pi_j k_j, \ (j = 4, 5, 6),$$

$$b_i = -a_{i0}, \ (i = 1, 2, \ldots, 8)$$

and $Z_i (= A_i / A_0)$ are the amplitude ratios of the reflected and refracted waves for the incident longitudinal wave. It may be noted that Z_i, $(i = 1, 2, 3)$ correspond to the amplitude ratios of reflected waves, while Z_i, $(i = 4, 5, 6, 7, 8)$ correspond to the amplitude ratios of the refracted waves.

Energy Partition

Let us consider energy partition among the reflected and refracted waves at the plane interface, $y = 0$. The rate of energy transmission per unit area at $y = 0$ is given by

$$P^* = \langle t_{22}, \dot{u}_2\rangle + \langle t_{21}, \dot{u}_1\rangle + \langle m_{23}, \dot{\phi}_3\rangle + \langle h_2, \dot{\psi}\rangle. \tag{23}$$

The energy of the incident, reflected and refracted waves are given as

$$P_i = l_i \omega k_i^3 A_i^2 \exp(2Q_i), \ (i = 0, 1, 2, 3, 4, 5, 6, 7, 8) \tag{24}$$

where

$$l_i = \begin{cases} (A_{12} + A_{78}\eta_i)p_1^{(i)} + (A_{22}\eta_i + A_{88} - B_{44}\pi_i^2 k_i^2)p_2^{(i)} - K_1\pi_i, & (i = 0, 1, 2, 3) \\ \{\lambda + 2\mu + \kappa - (s\eta_i + a\eta_i^2 - m\pi_i)/k_i^2\}p_2^{(i)}, & (i = 4, 5, 6) \\ \{\mu + \kappa - \eta_i(\kappa + \gamma\eta_i)/k_i^2\}p_2^{(i)}, & (i = 7, 8) \end{cases}$$

It may be noted that $i = 0$ represents for the energy of incident wave, $i = 1, 2, 3$ represent for the energy of reflected waves and $i = 4, 5, 6, 7, 8$ represent for the energy of refracted waves.

The energy ratios of the reflected and refracted waves are

$$E_i = \frac{P_i}{P_0}, \ (i = 1, 2, \ldots, 8) \tag{25}$$

Here, the energy ratios, E_i, $(i = 1, 2, 3)$ correspond for the reflected waves and E_i, $(i = 4, 5, 6, 7, 8)$ correspond for the refracted waves. We come to know that these ratios are functions of the angle of propagation, elastic, micropalar, thermal and void parameters.

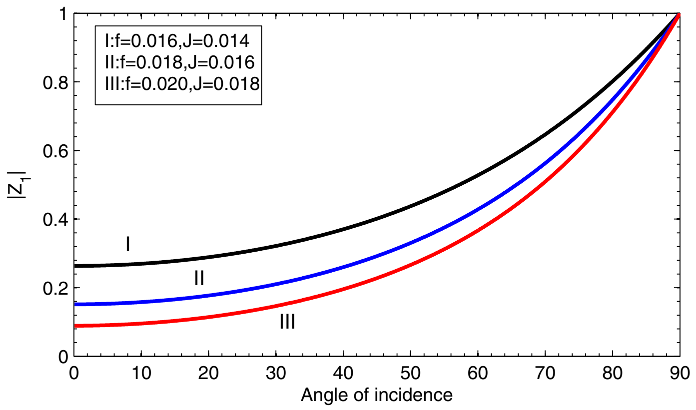

Figure 2: Variation of $|Z_1|$ with θ_0 at different values of $f \& J$.

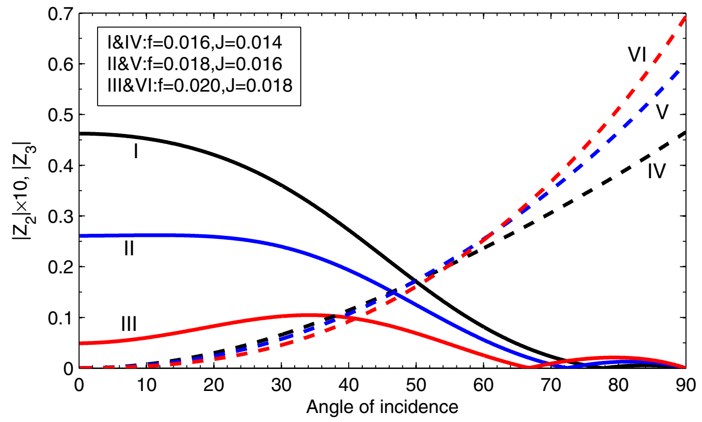

Figure 3: Variation of $|Z_2|$ (I, II, III) & $|Z_3|$ (IV, V, VI) with θ_0 at different values of f&J.

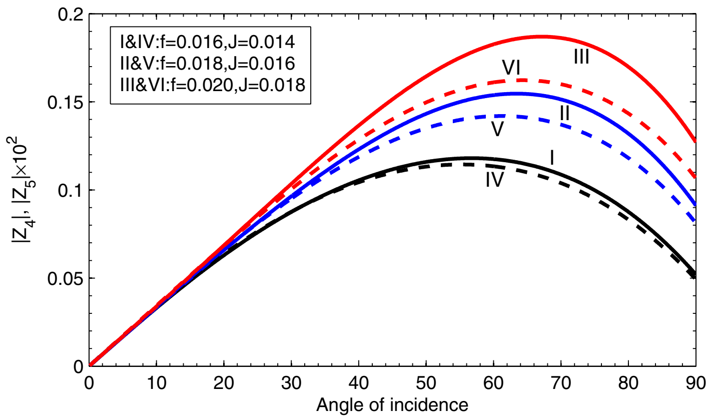

Figure 4: Variation of $|Z_4|$ (I, II, III) & $|Z_5|$ (IV, V, VI) with θ_0 at different values of f&J.

Numerical Results and Discussion

We are interested in the computation of amplitude and energy ratios of reflected and refracted waves for the incident longitudinal wave. We have developed programs on MATLAB for the computation of amplitude and energy ratios and will discuss the effects of micro-inertia parameters, f and J. The following relevant parameters are considered.

For the orthotropic micropolar half-space, M(modified values of Singh, 2007):
$\varrho = 2290\ Kg/m^3$, $A_{11} = 1.165 \times 10^{11}\ N/m^2$, $A_{22} = 1.265 \times 10^{11}\ N/m^2$,
$A_{12} = 7.69 \times 10^{10}\ N/m^2$, $A_{77} = 1.669 \times 10^{10}\ N/m^2$, $A_{78} = 1.59 \times 10^{10}\ N/m^2$,
$A_{88} = 2.29 \times 10^{10}\ N/m^2$, $B_{44} = 4.9 \times 10^4\ N$, $B_{66} = 4.8 \times 10^4\ N$.
For the half-space, $\bar{M}$ of thermoelastic micropolar materials with voids (Singh and Lianngenga, 2016): $\rho = 2190\ Kg/m^3$, $\lambda = 7.59 \times 10^{10}\ N/m^2$, $\mu = 1.89 \times 10^{10}\ N/m^2$,

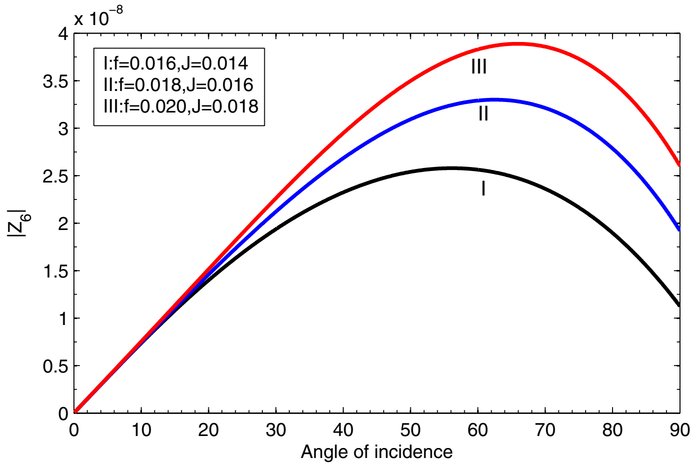

Figure 5: Variation of $|Z_6|$ with θ_0 at different values of $f \& J$.

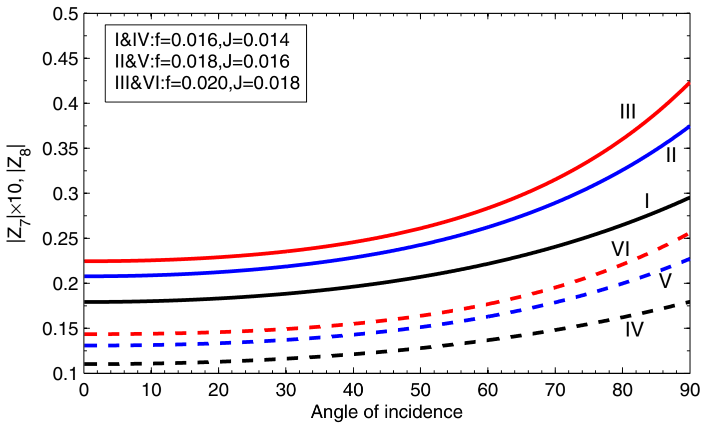

Figure 6: Variation of $|Z_7|$ (I, II, III) & $|Z_8|$ (IV, V, VI) with θ_0 at different values of $f \& J$.

$\kappa = 1.49 \times 10^8\ N/m^2$, $\chi = 0.00753\ m^2$, $\zeta = 1.49 \times 10^{10}\ N/m^2$, $s = 1.05 \times 10^{10}\ N/m^2$, $a = 6.68 \times 10^{-10}\ N/m^2$,
$\gamma = 2.68 \times 10^5\ N$, $\xi = 1.475 \times 10^6\ N/m^2$, $d = 2.16 \times 10^6\ N/m^2$, $k_0 = 1.7 \times 10^2\ Jm^{-1}s^{-1}K^{-1}$, $\nu = 0.02\ K^{-1}$, $\tau = 0.12\ s$, $\Theta_0 = 293\ K$, $\omega = 5\ s^{-1}$.
The variation of amplitude ratios with angle of incidence are depicted at Figures 2-6 and those of energy ratios are shown in Figures 7-11. In all the figures, we take
Curve I & IV: $(f, J) = (0.016, 0.014) \times 10^{-4}m^2$,
Curve II & V: $(f, J) = (0.018, 0.016) \times 10^{-4}m^2$ and
Curve III & VI: $(f, J) = (0.020, 0.018) \times 10^{-4}m^2$.

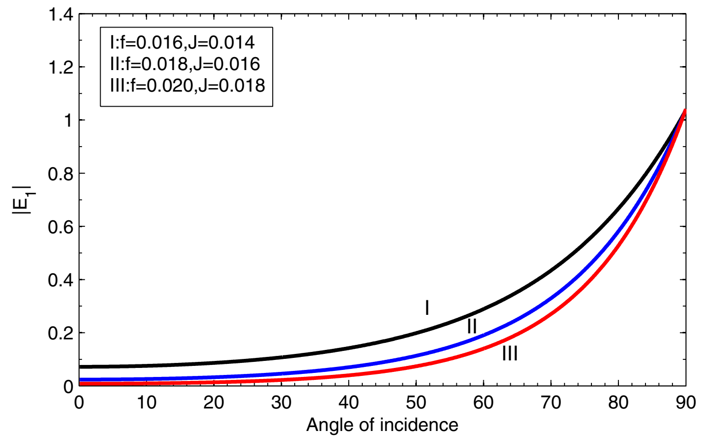

Figure 7: Variation of $|E_1|$ with θ_0 at different values of $f\&J$.

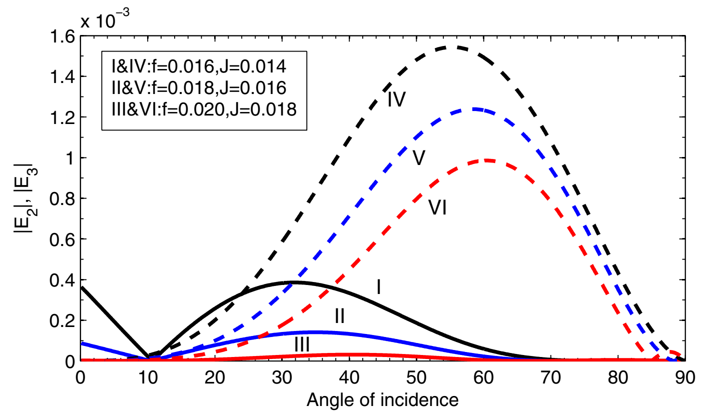

Figure 8: Variation of $|E_2|$ (I, II, III) & $|E_3|$ (IV, V, VI) with θ_0 at different values of $f\&\ J$.

Effect of micro-inertia on amplitude ratios

In Figure 2, the value of $|Z_1|$ increases from a certain value with the increase of angle of incidence, θ_0 attaining the maximum value at the grazing angle of incidence. The values of $|Z_1|$ decrease with the increase of micro-inertia and the minimum effect of micro-inertia is found near $\theta_0 = 90^0$. In Figure 3, Curve I shows the decreasing nature of $|Z_2|$ thereby making local minimums at $\theta_0 = 77^0$ and $\theta_0 = 90^0$, but Curves II and III represent increasing $|Z_2|$ with the increase of θ_0 upto the maximum value at $\theta_0 = 15^0$ and $\theta_0 = 34^0$ respectively. In this figure, we come to know that the value $|Z_3|$(Curves IV, V, VI) increases with the increase of θ_0. The minimum and maximum effects of micro-inertia on $|Z_2|$ are observed near grazing and normal angle of incidence. But in the case of $|Z_3|$, the minimum effect is observed near normal angle of incidence. The amplitude ratios, $|Z_4|$ (Curves I, II, III) and $|Z_5|$ (Curves IV, V, VI) in Figure 4, and $|Z_6|$

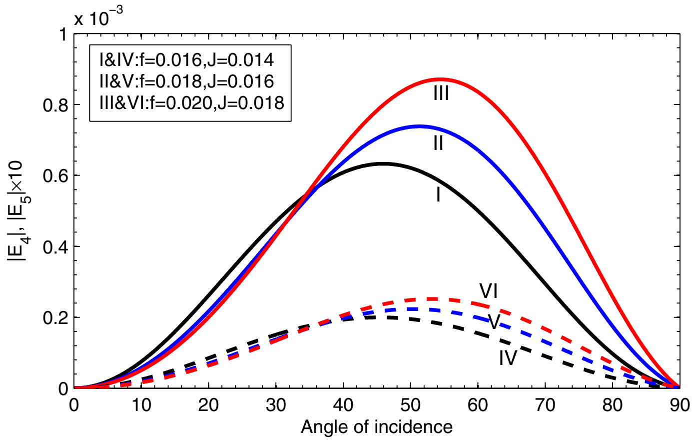

Figure 9: Variation of $|E_4|$ (I, II, III) & $|E_5|$ (IV, V, VI) with θ_0 at different values of $f \& J$.

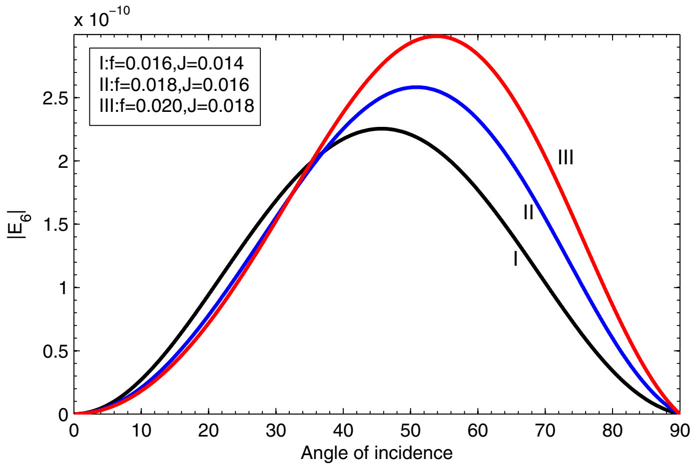

Figure 10: Variation of $|E_6|$ with θ_0 at different values of $f \& J$.

in Figure 5 have similar nature. They increase to the maximum value and then decrease with the increase of θ_0. The values of these amplitude ratios increase with the increase of micro-inertia. The minimum effect is observed near the normal angle of incidence. The similar nature of $|Z_7|$ and $|Z_8|$ is observed in Figure 6 and they increase with the increase of angle of incidence. The values of these amplitude ratios increase with the increase of micro-inertia.

Effect of micro-inertia on energy ratios

In Figures 7, $|E_1|$ starts from certain values and increases with the increase of angle of incidence. The minimum effect of micro-inertia is found near grazing angle of incidence. The Energy ratio, $|E_2|$ (Curves I, II, III) in Figure 8, decreases upto $\theta_0 = 11^0$ and thereafter, it increases to the maximum values with the increase of θ_0. It may be noted

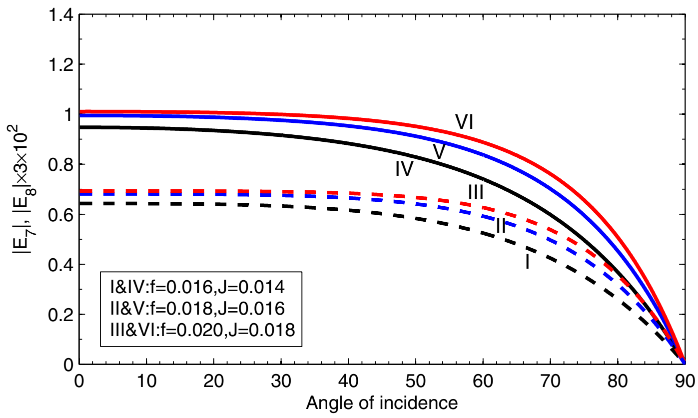

Figure 11: Variation of $|E_7|$ (I, II, III) & $|E_8|$ (IV, V, VI) with θ_0 at different values of $f \& J$.

that it decreases to the minimum value near grazing angle of incidence. In this figure, we have seen that $|E_3|$ starts with very small values and it increases to the maximum value which leads to the decrease with the increase of θ_0. The values of these energy ratios decrease with the increase of micro-inertia. The energy ratios, $|E_4|$(Curves I, II, III), $|E_5|$(Curves IV, V, VI) and $|E_6|$ have similar nature in Figures 9 and 10. They initially start from zero and increase to the maximum value, which then decrease with the increase of θ_0. The minimum effect of micro-inertia is observed near normal and grazing angle of incidence. The energy ratios, $|E_7|$ and $|E_8|$ in Figure 11 decrease with the increase of θ_0. The values of these ratios increase with the increase of micro-inertia. The sum of the energy ratios due to reflected and refracted waves is close to unity.

Conclusions

The problem of the effect of micro-inertia on the reflection and refraction of elastic waves at a plane interface between two half-spaces of an orthotropic micropolar materials and micropolar thermoelastic materials with voids has been investigated. The amplitude and energy ratios of the reflected and refracted waves due to incident longitudinal wave are obtained. These ratios are computed numerically for different values of micro-inertia and study the effects. We may summarize with the following concluding remarks:

(i) The amplitude and energy ratios are functions of angle of incidence, micropolar, thermal and voids parameters.

(ii) The amplitude ratio, $|Z_4|$, $|Z_5|$, $|Z_6|$, $|Z_7|$ and $|Z_8|$ increase with increasing micro-inertia, while $|Z_1|$ decreases with the increase of micro-inertia.

(iii) The energy ratios, $|E_7|$ and $|E_8|$ increase with the increase of micro-inertia, while $|E_1|$, $|E_2|$ and $|E_3|$ decrease with the increase of micro-inertia.

(iv) The amplitude ratios, $|Z_3|$, $|Z_4|$, $|Z_5|$ and $|Z_6|$ have minimum effect of micro-inertia near normal angle of incidence, while $|Z_1|$ and $|Z_2|$ have minimum effect near grazing angle of incidence.

(v) The energy ratios, $|E_1|$, $|E_2|$, $|E_7|$ and $|E_8|$ have minimum effect of micro-inertia near grazing angle of incidence, while $|E_4|$, $|E_5|$ and $|E_6|$ have minimum effect of micro-inertia near the normal and grazing angle of incidence.

(vi) The sum of energy ratios is close to unity.

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