Global Journal of Pure and Applied Mathematics.
ISSN 0973-1768 Volume 13, Number 7 (2017), pp. 3113-3120

http://www.ripublication.com

Lict Double Domination in Graphs

[1]M. H. Muddebihal and [2]Suhas. P. Gade

[1]Department of Mathematics Gulbarga University, Kulburgi-585106, Karnataka, India.
[2] Department of Mathematics, Sangameshwar College, Solapur-413001, Maharashtra, India.

Abstract

For any graph $G = (V, E)$, lict graph $n(G)$ of a graph G is the graph whose vertex set is the union of the set of edges and the set of cut vertices of G in which two vertices are adjacent if and only if the corresponding edges are adjacent or the corresponding members of G are incident. A subset D^d of $V[n(G)]$ is double dominating set of $n(G)$ if for every vertex $v \in V[n(G)], |N(v) \cap D^d| \geq 2$, that is v is in D^d and has at least one neighbour in D^d or v is in $V[n(G)] - D^d$ and has at least two neighbours in D^d. The lict double dominating number $\gamma_{ddn}(G)$ is a mimimum cardinality of lict double dominating set. In this paper many bounds on $\gamma_{ddn}(G)$ are obtained and its exact values for some standard graph are found in terms of parameter of G. Also its relationship with other domination parameters is investigated.

Subject classification number: $AMS - 05C69, 05C70$.

Keyword: Lict Graph/ Line graph/Dominating set/Total dominating set/Double dominating set.

INTRODUCTION

The graphs considered here are simple and finite. Let G be a graph with $V = V(G)$ is the vertex set of G and $E = E(G)$ is the edge set of G. The neighbourhood of a vertex $v \in V$ is defined by $N(v) = \{ u \in V/uv \in E\}$. The close neighbourhood of a vertex v is $N[v] = N(v) \cup \{v\}$. The order $|V(G)|$ of G is denoted by p. The degree of v is

$d(v) = |N(v)|$. The maximum degree of a graph G is denoted by $\Delta(G)$ and the minimum degree is denoted by $\delta(G)$. The minimum number of color in any colouring of a graph G such that no two adjacent vertices have same color is called the chromatic number of G and is denoted by $\chi(G)$. A vertex cover in a graph G is a set of vertices that covers all the edges of G. The vertex covering number $\alpha_0(G)$ is a minimum cardinality of a vertex cover in G. An edge cover of a graph G without isolated vertices is a set of edges of G that covers all vertices is a set of edges of G that covers all the vertices of G. The edge covering number $\alpha_1(G)$ of a graph G is the minimum cardinality of an edge cover of G. A set of vertices/edges in a graph G is called an independent set if no two vertices /edges in the the set are adjacent. The vertex independent number $\beta_0(G)$ is the maximum cardinality of an independent set of vertices. The edge independent number $\beta_1(G)$ of a graph G is the maximum cardinality of an independent set of edges. A total dominating set of G is a subset S of V such that each vertex in V is adjacent to a vertex of S. The total domination number, denoted by $\gamma_t(G)$ is the minimum cardinality of a total dominating set. A lict graph $n(G)$ of a graph G is the graph whose vertex set is the union of the set of edges and the set of cutvertices of G in which two vertices are adjacent if and only if the corresponding edges are adjacent or the corresponding members of G are incident. Let $G = (V, E)$ be a graph. A set D of vertices in a graph G is called a dominating set of G if every vertex in $V - D$ is adjacent to some vertex in D. The domination number of G, denoted by $\gamma(G)$ is the minimum cardinality of a dominating set. A set D subset of $V[n(G)]$ is said to be a dominating set of $n(G)$, if every vertex not in D is adjacent to a vertex in D of $n(G)$. The domination number of $n(G)$ is denoted by $\gamma[n(G)]$ is the minimum cardinality of a dominating set. A subset D^d of $V[n(G)]$ is double dominating set of $n(G)$ if for every vertex $v \in V[n(G)]$, $|N(v) \cap D^d| \geq 2$, that is v is in D^d and has at least one neighbour in D^d or v is in $V[n(G)] - D^d$ and has at least two neighbours in D^d and it is denoted by $\gamma_{ddn}(G)$. In this paper many bounds on $\gamma_{ddn}(G)$ are obtained and its exact values for some standard graph are found in terms of parameter of G. Also its relationship with other domination parameters is investigated. Further domination related graph valued functions were studied in [4, 5, 6]. We need the following theorems.

Theorem A [1] Let G be a connected graph of order n, Then $\gamma'(G) \leq \left\lfloor \frac{n}{2} \right\rfloor$.

Theorem B [2] For any graph G, $\kappa(G) \leq \lambda(G) \leq \delta(G)$.

Theorem C [3] For any path P_n, the edge covering number is $\alpha_1(\mathrm{P}_n) = \begin{cases} \frac{n}{2} \text{ if n is even} \\ \frac{n+1}{2} \text{ if n is odd.} \end{cases}$

Theorem D [3] For any path P_n, the vertex covering number is $\alpha_0(P_n) = \begin{cases} \frac{n}{2} \text{ if n is even} \\ \frac{n-1}{2} \text{ if n is odd.} \end{cases}$

Theorem E [3] For any graph G of order p,

(i) $\chi(G) \geq \omega(G)$.

(ii) $\chi(G) \geq \frac{q}{\beta_0}(G)$.

Theorem F[7] Let G be a connected graph, $\chi(G) \leq 1 + \Delta(G)$.

Upper Bounds for $\boldsymbol{\gamma_{ddn}(G)}$

Theorem 1. For any connected (p, q) graph G with $p \geq 3$, $\gamma_{ddn}(G) \leq p - 1$.

Proof. Let T be a spanning tree of G. If $p = 2$, then $n(G)$ has an isolated vertex. Hence $p \geq 3$. Let $I = \{ e_1, e_2, e_3, \dots, e_n \}$ be the set of all end edges of T and $I' = E(T) - I$. Then there exist a maximal independent set of edges $J = \{ e_1, e_2, e_3, \dots, e_k \} \subseteq I'$, in I' such that J forms an edge dominating set of T. Further if $J = \emptyset$, then $J = \{e\} \subseteq I$ forms an edge dominating set of T. Now without loss of generality, the corresponding edges of J forms a vertex set $D_1 = \{ v_1, v_2, v_3, \dots, v_k \}$ in $n(T)$ which is also a dominating set of $n(T)$. Let $V_1 = V[n(T)] - D_1$and $V_1 \in N(D_1)$. Clearly $D^d = D_1 \cup D_2$ form a double dominating set in $n(T)$, where $D_2 \subseteq V_1$. It follows that $|D_1 \cup D_2| \leq p - 1$ and hence $\gamma_{ddn}(G) \leq p - 1$.

Theorem 2. For any connected (p, q) graph G, $\gamma_{ddn} \leq \gamma_t(G) + \Delta(G) - 1$.

Proof. Let $D = \{ v_1, v_2, v_3, \dots, v_k \}$ be a dominating set of G and $V_1 = V(G) - D$, $V_1 \in N(D)$. Let $H \subseteq V_1$ be the minimum set of vertices which are adjacent to D then $D \cup H$ is a total dominating set of G. Let $v \in D \cup H$, $S \subseteq V(G) - DUH$ be the set of all vertices adjacent to v. Then $D \cup H \cup S$ becomes a maximal domination set of G. Now, let $S_1 = \left\{ e_1, e_2, e_3, \dots, e_j \right\}$ be the minimal set of edges which are incident to the vertices of D. Now without loss of generality, let $D_1 = \{v_1, v_2, v_3, \dots, v_i \}$ be a dominating set of $n(G)$. Further if $V_2 = V[n(G)] - D_1$ and $D_2 = \left\{v_1, v_2, v_3, \dots, v_j \right\} \subseteq$

V_2 then $D^d = D_1 \cup D_2$ form a double dominating set $n(G)$. Clearly, it follows that $|D^d| \leq |D \cup H \cup S| \leq |D \cup H| \cup |S| - 1$ and hence $\gamma_{ddn} \leq \gamma_t(G) + \Delta(G) - 1$.

Theorem 3. For any tree T with $p \geq 3$, then $\gamma_{ddn} \leq \left\lceil \frac{p}{2} \right\rceil + 1$.

Proof. Let $S = \{ e_1, e_2, e_3, \dots, e_k \}$ be an edge dominating set of T. By definition of $n(T)$, $V[n(T)] = E(T) \cup C(T)$, corresponding to the edges of S, we obtain a vertex set $D_1 = \{ v_1, v_2, v_3, \dots, v_k \}$ which is a dominating set of $n(T)$, since $D_1 \in V[n(G)]$, suppose $V_1 \subseteq V[n(T)] - D_1$ be the set of vertices which are neighbours of the elements of D_1. Further $D_2 \subseteq V_2$ and $D_2 \in N(D_1)$. Then $D^d = D_1 \cup D_2$ becomes double dominating set of $n(T)$ such that any vertex $v \in V[n(T)] - D^d$ has at least two neighbours in $D_1 \cup D_2$. Also by Theorem A, $\gamma'(G) \leq \left\lfloor \frac{p}{2} \right\rfloor$ clearly it follows that $|D^d| \leq \left\lceil \frac{p}{2} \right\rceil + 1$ and hence $\gamma_{ddn} \leq \left\lceil \frac{p}{2} \right\rceil + 1$.

Theorem 4. For any connected(p, q) graph , $\gamma_{ddn}(G) + \chi(G) \leq p + \Delta(G)$. Equality holds if G is isomorphic to C_4, C_5.

Proof. By Theorem 1, $\gamma_{ddn}(G) \leq p - 1$ and by Theorem F, $\chi(G) \leq 1 + \Delta(G)$. Clearly it follows that, $\gamma_{ddn}(G) + \chi \leq p + \Delta(G)$. If $G \cong C_4, C_5, C_6$ then $\gamma_{ddn}(G) = p - 1$ and $\chi(G) = 3$. Hence $\gamma_{ddn}(G) + \chi(G) = p + \Delta(G)$.

Theorem 5. For any connected(p, q) graph G, $\gamma_{ddn} + \kappa \leq p + \delta - 1$, where κ denotes the connectivity of G. Equality hold if G is isomorphic to C_4, C_5, C_6 or p_3, p_4.

Proof. By Theorem 1, $\gamma_{ddn}(G) \leq p - 1$ and by Theorem B, $\kappa \leq \lambda \leq \delta$. Clearly it follows that $\gamma_{ddn}(G) + \kappa \leq p + \delta - 1$. If $G \cong C_4, C_5, C_6$ or p_3, p_4, then $\gamma_{ddn}(G) = p - 1$ and $k = 2$. Hence $\gamma_{ddn} + \kappa = p + \delta - 1$.

Now we proceed to construct an upper bound to γ_{ddn} by connecting edge connectivity of a graph G.

Corollary 1. For any connected (p, q) graph $G, \gamma_{ddn}(G) + \lambda \leq p + \delta - 1$, where λ denotes the edge connectivity of G. Equality holds, if G is isomorphic to $C_3, C_4, C_5, C_6, P_3, P_4$, P_5.

Proof. The result follows from Theorem 1 and Theorem B.

Theorem 6. For any (p, q) tree T with $p \geq 3$, $\gamma_{ddn}(T) \leq q$.

Proof. Let T be a tree with $E = \left\{ e_1, e_2, e_3, \dots, e_q \right\}$ and $C = \{ c_1, c_2, c_3, \dots, c_i \}$ $i < q$ be the set of edges and cutvertices in G. In $n(G)$, $V[n(G)] = E(G) \cup C(G)$. Further if there exists a vertex set $V_1 \subseteq V[n(G)] - C$, $\{ V_1 \subseteq E(G)\ in\ n(G)\}$. Then $D^d = V_1 \cup C_1$, $C_1 < C$ where C_1 is a set of cutvertices in $n(G)$. Also every vertex of $n(G)$ are adjacent to at least two vertices of $n(G)$ are adjacent to at least two vertices of D^d. Clearly D^d forms a double dominating set of $n(G)$. Therefore it follow that $|D^d| \leq E(G)$. Hence $\gamma_{ddn}(T) \leq q$.

Theorem 7. For any path P_n of order n, $\gamma_{ddn} \leq \begin{cases} 2\alpha_0(P_n) - 1\ n\ is\ even \\ 2\alpha_0(P_n)\ n\ is\ odd. \end{cases}$

Proof. Let P_n be the path with $n \geq 3$ vertices. Consider $V = \{v_1, v_2, v_3, \dots, v_n\}$ be the vertices and $E = \{(v_i, v_{i+1}\}\ i = 1,2,3, \dots$ be the edge set of path P_n. By the Theorem D, we have the following cases.

Case($\boldsymbol{i}$): Suppose n is even. Then $\alpha_0(P_n) = \frac{n}{2} \Longrightarrow n = 2\alpha_0(P_n)$. Since $\gamma_{ddn}(P_n) \leq n - 1$, we have $\gamma_{ddn}(P_n) \leq 2\alpha_0(P_n) - 1$.

Case($\boldsymbol{ii}$): Suppose n is an odd. Then $\alpha_0(P_n) = \frac{n-1}{2} \Rightarrow n - 1 = 2\alpha_0(P_n) - 1$. Since $\gamma_{ddn}(P_n) \leq n - 1$, we have $\gamma_{ddn}(P_n) \leq 2\alpha_0(P_n)$.

Theorem 8. For any path P_n of order n, $\gamma_{ddn} \leq \begin{cases} 2\alpha_1(P_n) - 1\ \ n\ is\ even \\ 2\alpha_1(P_n) - 2\ \ n\ is\ odd. \end{cases}$

Proof: Let P_n be the path with $n \geq 3$ vertices. Consider $V = \{v_1, v_2, v_3, \dots, v_n\}$ be the vertices and $E = \{(v_i, v_{i+1}\}\ i = 1,2,3, \dots$ be the edge set of path P_n. We have the following cases.

Case($\boldsymbol{i}$): Suppose n is even, by the Theorem C, we have, $\alpha_1(P_n) = \frac{n}{2} \Longrightarrow n = 2\alpha_1(P_n)$. Since $\gamma_{ddn}(P_n) \leq n - 1$, we have $\gamma_{ddn}(P_n) \leq 2\alpha_1(P_n) - 1$.

Case($\boldsymbol{ii}$): Suppose n is an odd, by Theorem C, we have, $\alpha_1(P_n) = \frac{n+1}{2} \Longrightarrow n + 1 = \frac{n+1}{2} \Longrightarrow n = 2\alpha_1(P_n) - 1$. Since $\gamma_{ddn}(P_n) \leq p - 1$, we have $\gamma_{ddn} \leq 2\alpha_1(P_n) - 2$.

Theorem 9. For any tree T with k number of cutvertices $\gamma_{ddn}(T) \leq k + 1$, further equality holds if $T = K_{1,p}\ p \geq 3$.

Proof. Let $V = \{v_1, v_2, v_3, \dots, v_k\} \subset V(T)$ be the set of all cutvertices of a tree T with $|C| = k$, since the number of vertices and the number of pendant vertices. If for every cutvertex $u \in C, u \neq v$ such that u is adjacent to v. Otherwise, let $e_1 \in E(G)$ such that e_1 is incident with C, so that $\gamma_{ddn}(T) \leq \{ C \cup e_1 \} = |C| + 1 = k + 1$. For equality, let $T = K_{1,p}$ with a cutvertex $k = 1$, then $D^d = \{ K \cup e \}$ is a double dominating set of $n(T)$ with cardinality $k + 1$. Hence $\gamma_{ddn}(T) \leq k + 1$.

Lower Bounds for $\gamma_{ddn}(G)$

Theorem 10. For any tree T of order $p \geq 3$, $\gamma_{ddn}(T) \geq \chi(T)$, and equality holds for all star graphs $K_{1,p}$.

Proof. For any tree T, we have $\chi(T) = 2$ and $2 \leq \gamma_{ddn}(T) \leq p - 1$. Hence $\gamma_{ddn}(T) \geq \chi(T)$. For $T = K_{1,p}$, clearly $\chi(T) = 2$ and $\gamma_{ddn}(T) = 2$. Hence the proof.

Theorem 11. For any tree T of order $p \geq 3$, $\gamma_{ddn}(T) \geq \omega(T)$.

Proof. The result follows from Theorem 10 and Theorem E.

Theorem 12. For any tree T of order $p \geq 3$, $\gamma_{ddn}(T) \geq \frac{q}{\beta_0}(T)$.

Proof. For any tree T, we have $\chi(T) \geq \frac{q}{\beta_0}(T)$ and $\gamma_{ddn}(T) \geq \chi(T)$. Hence $\gamma_{ddn}(T) \geq \frac{q}{\beta_0}(T)$. Hence the proof.

Theorem 13. For any graph G of order p, $\gamma_{ddn}(T) \geq p - m$, where m is the number of end vertices.

Proof. Let $V_1 = \{v_1, v_2, v_3, \dots, v_m\}$ be the set of all end vertices in T with $|V_1| = m$. Further $E = \{e_1, e_2, e_3, \dots, e_q\}$ and $C = \{c_1, c_2, c_3, \dots, c_i\}$ be the set of edges and cut vertices in G. In $n(G)$, $V[(G)] = E(G) \cup C(G)$. Let $D^d = \{v_1, v_2, v_3, \dots, v_n\} \subseteq V[n(G)]$ be the double dominating set such that $|V[n(G)] - D^d| \geq 1$, then $\{V[n(G)] - D^d\}$ contains at least one vertex which gives $|n - m| \leq |D^d|$. Hence $\gamma_{ddn}(T) \geq p - m$.

Theorem 14. For any nontrival tree (p, q) tree T with k number of cut vertices, then $p - k \leq \gamma_{ddn}(G)$.

Proof. Let $S = X \cup C$ be the set of vertices of $n(T)$ where $X = \{v_1, v_2, v_3, \dots, v_i\}$ and $C = \{c_1, c_2, c_3, \dots, c_j\}$, $j < i$ are the vertices of $n(T)$ corresponding the edges and cutvertices of T respectively. Now consider the set $D^d = X' = \{v_1, v_2, v_3, \dots, v_i\} \subseteq X \subseteq V[n(T)]$ be the minimal set of vertices which covers all the vertices in $n(T)$. Suppose any vertex $v \in V[n(T)] - X'$ has at least two neighbours in X' then D^d itself is a double dominating set of $n(T)$. Clearly it follows that $|V[n(T)] - C| \leq |D^d|$. Hence $p - k \leq \gamma_{ddn}(T)$.

Corollary 2. For any path $P_n, n \geq 3$, $\gamma_{ddn}(P_n) \geq p - k$ where k be the cutvertices.

Proof. From Theorem 14 the result follows.

Theorem 15. For any connected(p,q) graph G, $\gamma_{ddl}(G) \leq \gamma_{ddn}(G)$, equality holds if G a block graph.

Proof. Since $V[L(G)] \subseteq V[n(G)]$ by definition, then the result follows. If G is a block, then $V[L(G)] = V[n(G)]$ and $L(G) \cong n(G)$. Hence the equality holds.

Theorem 16. If T is a tree which is not a star, then $\gamma_{ddn}(T) \geq \beta_0$.

Proof. Suppose $T = K_{1,p}, p \geq 3$. Then $\beta_0 = p > \gamma_{ddn}(T)$. Let $K = \{v_1, v_2, v_3, \dots, v_n\} \subseteq V(G)$ be the maximum set of vertices such that $d(v_i, v_j) \geq 2$ and $N(v_i) \cap N(v_j) = x, \forall\, v_i, v_j \in K$ so that $x \in V(G) - K$. Clearly $|K| = \beta_0(G)$. Let $E_1 = \{e_1, e_2, e_3, \dots, e_n\}, E_2 = \{e_1, e_2, e_3, \dots, e_m\}, C = \{c_1, c_2, c_3, \dots, c_k\}$ be the set of end edges, nonend edges and cut vertices of G. By the definition of $n(G)$, $V[n(G)] = E_1 \cup E_2 \cup C$ and each block of $n(G)$ is complete. Suppose $E_1' = \{e_1, e_2, e_3, \dots, e_i\} \subseteq E_1, E_2' = \{e_1, e_2, e_3, \dots, e_j\} \subseteq E_2$ be the set of vertices and cutvertices corresponding to the edges of G. Then $E_1' \cup E_2'$ covers all the vertices of $n(G)$ such that $\forall\, v_i \in E_1' \cup E_2'$ covers at least two vertices of $V[n(G)] - \{E_1' \cup E_2'\}$. Then $\{E_1' \cup E_2'\}$ forms $\gamma_{ddn}(T)$ set. Otherwise $E_2' \cup C_1$ where $C_1 \subset C$ gives $\gamma_{ddn}(T) -$ set. Hence in all the cases with $|E_1' \cup E_2'| \geq |K|$ or $|E_2' \cup C_1| \geq |K|$ gives $\gamma_{ddn}(T) \geq \beta_0(T)$.

Thorem17. For any connected graph $G, n(G) \neq k_n, n > 4$ vertices $\gamma_{ddn}(G) \geq \left\lceil \frac{n}{2} \right\rceil$.

Proof. We consider the following cases,

Case$(\boldsymbol{i})$: Suppose G is a tree with $V = \{v_1, v_2, v_3, \dots, v_n\}$ be the set of all vertices in T. Then $V_1 = \{v_1, v_2, v_3, \dots, v_i\}$ be the set of all end vertices in T and let $E_1 = \{e_1, e_2, e_3, \dots, e_i\}$ be the set of all non end edges in T and also $E_2 = \{e_1, e_2, e_3, \dots, e_j\}$ be the set of all end edges of T. Let C be the set of all cut vertices in T. $V[n(T)] = E(T) \cup C(T) = E_1 \cup E_2 \cup C$. Suppose D^d be a $\gamma_{ddn}(T)$ -set of T such that $D^d = E_2' \cup E_1'$ where $E_2' \subset E_2, E_1' \cup E_1$ which gives $|E_2' \cup E_1'| = \gamma_{ddn}(T) \geq \frac{V \cup V_1}{2}$ implies that $\gamma_{ddn}(G) \geq \left\lceil \frac{n}{2} \right\rceil$.

Case$(\boldsymbol{ii})$: Suppose G is not a tree. Then there exists at least one edge joining two distinct vertices of a tree T, which from a cycle. From case(i) $|V[n(G)| \geq |E_2' \cup E_1' \cup C_1| + 1$, where $C_1 \subset C$, it follows that $|E_2' \cup E_1'| + 1 \geq \left|\frac{V \cup V_1}{2}\right| + 1$, which implies $\gamma_{ddn}(G) \geq \left\lceil \frac{n}{2} \right\rceil$.

Theorem 18. For any connected (p, q) graph , $\gamma_{ddn} \geq \left\lceil \frac{p}{\Delta(G)} \right\rceil$. Equality holds if $G \cong K_{1,p}, p \geq 2$.

Proof. Let D be a dominating set of $n(G)$ and $V_1 = V[n(G)] - D$ such that $V_1 \in N(D)$. Let $D_2 \subseteq V_1$ and $D_2 \in N(D)$, then $D^d = D_1 \cup D_2$ is a double dominating set of $n(G)$. Further, let $C = \{ v_1, v_2, v_3, \dots, v_k \}$ be the set of all non end vertices in G, then there exists at least one vertex v of maximum degree $\Delta(G)$ in C, such that $|D_1 \cup D_2| \cdot \Delta(G) \geq p$. It follows that $\gamma_{ddn} \geq \left\lceil \frac{p}{\Delta(G)} \right\rceil$. Suppose G is isomorphic to $K_{1,p}, p \geq 1$. Then $n(G) \cong K_{p+1}$, clearly $\gamma_{ddn}(G) = 2$. Since for any given graph $G \cong K_{1,p}$, $p = \Delta(G) + 1$ and $\left\lceil \frac{p}{\Delta(G)} \right\rceil = 2$. Hence it follows that , $\gamma_{ddn} = \left\lceil \frac{p}{\Delta(G)} \right\rceil$.

REFERENCES

[1] Araya Chaemchan, 2010, "The edge domination number of connected graphs", Australasian journal of Combinatorics, Vol.48, 185-189.

[2] F. Harary, 1969, Graph Theory, Addison-wesely, Reading, MA.

[3] Gray Chartrand and Ping Zhan, 2006, Introduction to Graph Theory, Tata McGraw-Hill Pub.Co.Ltd.

[4] Girish.V.R. and P. Usha, 2014, "Total Domination in lict graph", International J.Math. Combin. Vol.1, 19-27.

[5] M.H. Muddebihal and Srinivasa. G, 2012, "Bounds on double domination in squares of graphs", International journal of research in Engineering and Technology, Vol.2, 454-458.

[6] M.H. Muddebihal and Suhas P.Gade, 2015, "Line double domination in graphs", International journal of Mathematics and Computer Application Research, Vol.5, 75-80.

[7] R.L. Brooks, 1941, "On colouring the nodes of a network", Mathematical Proceedings of the Cambridge Philosophical Society, Vol.37, 194-197.

[8] V.R. Kulli and M.H. Muddebihal, 2006, "Lict Graph and Litact Graph of a Graph", Journal of Analysis and Computation, Vol.2, 133-143.