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An EOQ Model with Ramp Type Demand Rate, Time Dependent Deterioration Rate and Shortages

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Abstract

In this paper, we have developed an order level inventory system for deteriorating items. The demand rate is quadratic function of time in beginning of cycle, which becomes linear as passage of time. Shortages are allowed and partially backlogged. Backlogging rate is variable which depends upon the duration of waiting time up to the arrival of next lot. Numerical example is used to illustrate the model. Sensitivity analysis is also performed to study the effect of change in various parameters on the behavior of the model.

Keywords: Inventory, Deterioration, Partial backlogging, Ramp type demand rate

1. INTRODUCTION

Inventory is an important part of our manufacturing, distribution and retail infrastructure where demand plays an important role in choosing the best inventory policy. Researchers were engaged to develop the inventory models assuming the demand of the items to be constant, linearly increasing or decreasing, exponential increasing or decreasing with time. Inventory models with time-dependent demand were studied by Dave (1981) and Maiti et al. (2009).

Later, it has been realized that the above demand patterns do not precisely depict the demand of certain items such as newly launched fashion items, garments, cosmetics, automobiles etc, for which the demand increases with time as they are launched into the market and after some time, it becomes constant. In order to consider demand of such types, the concept of ramp-type demand is introduced. Ramp-type demand depicts a demand which increases up to a certain time after which it stabilizes and becomes constant. Mandal and Pal (1998) have developed inventory models with ramp type demand rate for deteriorating items. Panda et al. (2008) have developed optimal replenishment policy for perishable seasonal products taking ramp-type time dependent demand rate. Avinadav et al. (2013) have developed considered demand function sensitive to price and time. Models for seasonal deteriorating products with ramp-type time-dependent demand are discussed by Wang and Huang (2014).

Deterioration is defined as decay, damage, spoilage, evaporation, obsolescence, pilferage and loss of utility or loss of marginal value of a commodity that reduces usefulness from original ones. Blood, fish, fruits and vegetables, alcohol, gasoline, radioactive chemicals, medicines, etc., lose their utility with respect to time. In this case, a discount price policy is implemented by the suppliers of these products to promote sales. Thus, decay or deterioration of physical goods in stock is a very realistic feature. Modelers felt the need to take this factor into consideration. Various types of order-level inventory models for items deteriorating at a constant rate were discussed by Shah and Jaiswal (1977) and Dave (1986). As time progressed, several researchers developed inventory models with variable deterioration rate. In this connection, researchers may consult the work by Covert and Philip (1973), Chakrabarti et al. (1998), Jalan et al. (1996)and Dye (2004), who have used Weibull distribution for representing deterioration rate. An inventory system with Markovian demands, phase type distributions for perishability and replenishment is developed by Chakravarthy (2011). Three parameter Weibull distribution is used by Sanni and Chukwu (2013) to represent deterioration rate.

When shortage for a product occurs, some customers will go away, while some would like to wait for backlogging after the next replenishment. But the willingness is diminishing with the length of the waiting time for the next replenishment. The longer the waiting time is, the smaller the backlogging rate would be. Thus practically, all shortages are not backlogged but only some part of shortages is backlogged. This phenomenon is called partial backlogging.

Chang and Dye (1999) developed an inventory model in which the demand rate is a time-continuous function and items deteriorate at a constant rate with partial backlogging rate which is the reciprocal of a linear function of the waiting time. Papachristos and Skouri (2000) developed an EOQ inventory model with time-dependent partial backlogging. Teng et al. (2003) then extended the backlogged demand to any decreasing function of the waiting time up to the next replenishment. The related analysis on inventory systems with partial backlogging have been performed by Teng and Yang (2004), Dye et al. (2006) etc. Singh and Singh (2007, 2009) studied inventory model with partial backlogging considering quadratic demand and power demand. San-Jose et al. (2015) have studied partial backlogging with non linear holding cost.

Most of these papers take the replenishment rate to be infinite.

In this paper, an effort has been made to analyse an EOQ model for time-dependent deteriorating items assuming the demand rate to be demand rate as a combination a linear and quadratic function of time. Such type of the demand pattern is generally seen in the case of any new brand of consumer goods coming to the market. The demand rate for such items increases with time up to certain time and then ultimately stabilizes and becomes constant. It is believed that such type of demand rate is quite realistic.

2. ASSUMPTIONS AND NOTATIONS:

To develop an inventory model with variable demand and partial backlogging, the following notations and assumptions are used:

i) Lead time is zero

ii) $c_1$is the inventory holding cost per unit per unit of time.

iii) c_2 the deterioration cost per unit per unit of time.

iv) c_3 is the shortage cost per unit per unit of time.

v) $c_4$is the unit cost of lost sales.

vi) Ordering cost is c'.

vii) Demand rate is a combination a linear and quadratic function of time defined by

$f(t) = a + bt + c\{t - (t - \mu)H(t - \mu)\}t$ where $H(t - \mu)$ is Heviside's function defined as follows: $H(t-\mu) = \begin{cases} 0, t < \mu \\ 1, t \geq \mu \end{cases}$

Thus demand can be written as $f(t) = \begin{cases} a + bt + ct^2, t < \mu \\ a + (b + c\mu)t, t \geq \mu \end{cases}$

$$= \begin{cases} a + bt + ct^2, t < \mu \\ a + kt, t \geq \mu \end{cases},$$

where $b + c\mu = k$

viii) $K = \beta f(t)$ is the production rate where $\beta > 1$ is a constant.

ix) Unsatisfied demand is backlogged at a rate $e^{-\lambda t}$, where t is the time up to next replenishment and λ is a positive constant.

x) R is the total cost per production cycle and $T$is the time for each cycle.

xi) K is the average cost per cycle so that $K = \frac{R}{T}$

xii) $Q(t)$ be the inventory level at time t.

xiii) A variable fraction $\theta(t) = \alpha t$ $(0 < \alpha \ll 1, t \geq 0)$ is the deterioration rate.

3. FORMULATION AND SOLUTION OF THE MODEL:

Initially the stock is zero. Production starts just after $t = 0$ and continues up to $t = t_1$ when the stock reaches a level S. Production is stopped at $t = t_1$. Inventory accumulated in $[0, t_1]$ after meeting the demands is used in$[t_1, t_2]$. The stock reaches the zero level at time t_2. Now shortages start to develop and accumulate to the level P at $t = t_3$. Production starts at time t_3. The running demands as well as the backlog for $[t_2, t_3]$ are satisfied in $[t_3, T]$. The inventory again reaches the zero level at time T. The cycle then repeats itself after a scheduling time T. Our objective is to determine the optimum values of K, t_1, t_2, t_3 and T with the assumptions stated above.

The situation is depicted in the following figure:

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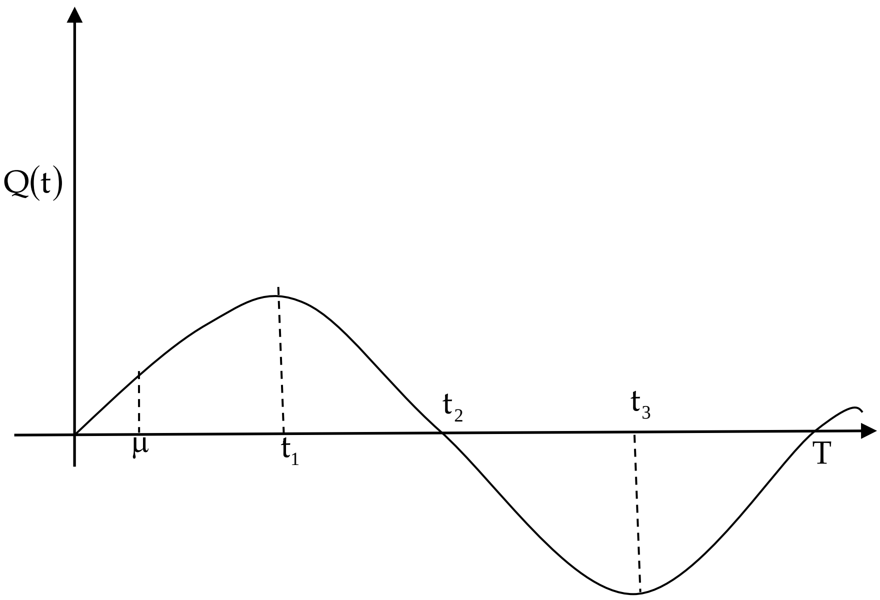

Figure 1

The differential equations governing the instantaneous states of $Q(t)$ in the interval $[0, T]$ are as follows:

$$\frac{dQ(t)}{dt} + \alpha t Q(t) = (\beta - 1)(a + bt + ct^2), 0 \le t \le \mu \quad (1)$$

with the condition $Q(0) = 0$

$$\frac{dQ}{dt} + \alpha t Q(t) = (\beta - 1)(a + kt), \mu \le t \le t_1 \quad (2)$$

with the condition $Q(t_1) = S$

$$\frac{dQ}{dt} + \alpha t Q(t) = -(a + kt), t_1 \le t \le t_2 \quad (3)$$

with the condition $Q(t_2) = 0$

$$\frac{dQ}{dt} = -(a + kt)e^{-\lambda t}, t_2 \le t \le t_3 \quad (4)$$

with the condition $Q(t_3) = -P$

$$\frac{dQ}{dt} = (\beta - 1)(a + kt), t_3 \le t \le T \quad (5)$$

with the condition $Q(T) = 0$

The solutions of equations (1) to (5) are given below:

$$Q(t) = -\frac{1}{120}(\beta - 1)\{a(40\alpha t^2 - 120) + b(15\alpha t^3 - 60t) + c(8\alpha t^4 - 40t^2)\},\ 0 \le t \le \mu \quad (6)$$

$$Q(t) = (\beta - 1)\left\{a\left(-\frac{\alpha t^3}{3} + \frac{1}{2}\alpha t^2 \mathrm{t}_1 + t - \frac{\alpha t_1^3}{6} - \mathrm{t}_1\right) + k\left(-\frac{\alpha t^4}{8} + \frac{1}{4}\alpha t^2 \mathrm{t}_1^2 + \frac{t^2}{2} - \frac{\alpha t_1^4}{8} - \frac{t_1^2}{2}\right)\right\} - \frac{1}{2}\alpha S t^2 + \frac{1}{2}\alpha S \mathrm{t}1^2 + S, \quad \mu \le t \le t_1 \quad (7)$$

$$Q(t) = \frac{1}{24}(t - t_2)\{4a(2\alpha t^2 - \alpha t t_2 - \alpha t_2^2 - 6) + 3k(\alpha t^3 + \alpha t^2 t_2 - \alpha t t_2^2 - 4t - \alpha t_2^3 - 4t_2)\},\ t_1 \le t \le t_2 \quad (8)$$

$$Q(t) = \frac{a(e^{-\lambda t} - e^{-\lambda t_3})}{\lambda} + \frac{k(e^{-\lambda t} + \lambda t e^{-\lambda t} - e^{-\lambda t_3} - \lambda t_3 e^{-\lambda t_3})}{\lambda^2} - P, \quad t_2 \le t \le t_3 \qquad (9)$$

$$Q(t) = \frac{1}{2}(\beta - 1)(t - T)\{2a + k(t + T)\}, \quad t_3 \le t \le T \qquad (10)$$

Using above relations, S is given by

$$S = -\frac{1}{120}(\beta - 1)\{a(40\alpha t_1^2 - 120) + b(15\alpha t_1^3 - 60t_1) + c(8\alpha t_1^4 - 40t_1^2)\} \quad (11)$$

$$P = \frac{(a\lambda + \lambda k t_2 + k)e^{-\lambda t_2}}{\lambda^2} - \frac{(a\lambda + \lambda k t_3 + k)e^{-\lambda t_3}}{\lambda^2} \qquad (12)$$

The inventory holding cost during the interval $(0, T)$ is given by

$$C_H = c_1\left[\int_0^{\mu} Q(t)dt + \int_{\mu}^{t_1} Q(t)\,dt + \int_{t_1}^{t_2} Q(t)dt\right]$$

$$= c_1\Big[\frac{1}{360}(\beta - 1)(t_1 - \mu)\{a(-30\alpha\mu^3 - 60\alpha\mu^2 + 180\mu - 30\alpha t_1^3 + 30\alpha\mu t_1^2 + 30\alpha\mu^2 t_1 - 60\alpha\mu t_1 - 180t_1 + 360) + b(15\alpha t_1^3 - 30\alpha\mu t_1^2 - 30\alpha\mu^2 t_1 + 180t_1) + c(16\alpha t_1^4 - 20\alpha\mu t_1^3 - 20\alpha\mu^2 t_1^2 + 120t_1^2) + k(-9\alpha\mu^4 + 60\mu^2 - 24\alpha t_1^4 + 21\alpha\mu t_1^3 + 21\alpha\mu^2 t_1^2 - 120t_1^2 - 9\alpha\mu^3 t_1 + 60\mu t_1)\} + a(\beta - 1)\mu - \frac{1}{9}(\beta - 1)\mu^3(a\alpha - c) - \frac{1}{120}(t_1 - t_2)^2(10a\alpha t_1^2 - 10a\alpha t_2^2 - 60a + 3\alpha k t_1^3 + 6\alpha k t_1^2 t_2 - \alpha k t_1 t_2^2 - 20kt_1 - 8\alpha k t_2^3 - 40kt_2) - \frac{1}{32}\alpha b(\beta - 1)\mu^4 + \frac{1}{4}b(\beta - 1)\mu^2 - \frac{1}{75}\alpha(\beta - 1)c\mu^5\Big] \qquad (13)$$

The cost due to deterioration of units in the period $(0, T)$ is given by

$$C_D = c_2\left[\int_0^{\mu} \alpha t Q(t)dt + \int_{\mu}^{t_1} \alpha t Q(t)\,dt + \int_{t_1}^{t_2} \alpha t Q(t)dt\right]$$

$$= \frac{1}{24}\alpha c_2\Big[2(\beta - 1)\,\mu^2\{6a + \mu(2b + c\mu)\} - \frac{1}{10}(\beta - 1)(t_1 - \mu)(-120at_1 + 40at_1^2 - 60bt_1^2 - 40ct_1^3 + 30kt_1^3 + 40at_1^3\alpha + 15bt_1^4\alpha + 8ct_1^5\alpha - 120a\mu + 40at_1\mu - 60bt_1\mu - 40ct_1^2\mu + 30kt_1^2\mu + 40at_1^2\alpha\mu + 15bt_1^3\alpha\mu + 8ct_1^4\alpha\mu - 80a\mu^2 - 30kt_1\mu^2 - 30k\mu^3) + (t_1 - t_2)^2\{4a(2t_1 + t_2) + 3k(t_1 + t_2)^2\}\Big] \qquad (14)$$

The cost due to shortages in the interval $(0, T)$ is given by

$$C_S = -c_3\left[\int_{t_2}^{t_3} Q(t)dt + \int_{t_3}^{T} Q(t)\,dt\right]$$

$$= \frac{1}{6}c_3(T - t_3)^2\{3a + k(2T + t_3)\}(-1 + \beta) - \frac{1}{\lambda^3}c_3\left[e^{-t_2\lambda}\{2k + (a + kt_2)\lambda\} + e^{-t_3\lambda}\{-2k + (-a + kt_2 - 2kt_3)\lambda + \left(at_2 - at_3 + kt_2t_3 - kt_3^2\right)\lambda^2\} + (t_2 - t_3)\lambda^3\left\{\frac{e^{-t_2\lambda}(k + a\lambda + kt_2\lambda)}{\lambda^2} - \frac{e^{-t_3\lambda}(k + a\lambda + k\,t_3\lambda)}{\lambda^2}\right\}\right] \quad (15)$$

The opportunity cost due to lost sales in the interval $(0, T)$ is given by

$$C_O = c_4\left[\int_{t_2}^{t_3}\left(1 - e^{-\lambda t}\right)f(t)dt\right]$$

$$= c_4\left[-at_2 - \frac{kt_2^2}{2} + at_3 + \frac{k\,t_3^2}{2} + e^{-t_2\lambda}\left(-\frac{k}{\lambda^2} - \frac{a}{\lambda} - \frac{kt_2}{\lambda}\right) + e^{-t_3\lambda}\left(\frac{k}{\lambda^2} + \frac{a}{\lambda} + \frac{kt_3}{\lambda}\right)\right] \quad (16)$$

The total cost R in the system in the interval $(0, T)$ is given by

$$R = c^{'} + C_H + C_D + C_S + C_O \quad (17)$$

In above relation (17), $c^{'}$ is constant, while C_H, C_D, C_S & C_O are given by the equations (13) to (16).

The average cost K per cycle is given by $K = \frac{R}{T}$ (18)

The optimum values of t_1, t_2, t_3 and T which minimize average cost K are obtained by using the equations:

$$\frac{\partial K}{\partial t_1} = 0,\ \frac{\partial K}{\partial t_2} = 0, \frac{\partial K}{\partial t_3} = 0 \text{ and } \frac{\partial K}{\partial T} = 0$$

Now,

$\frac{\partial K}{\partial t_1} = 0$ gives

$$c_2\left[\frac{1}{2}t_1(t_1 - t_2)(2a + kt_1 + kt_2)\alpha + \frac{1}{12}\alpha(\beta - 1)\{2at_1 - 6a\,t_1^2 + 9bt_1^2 + 8ct_1^3 - 6kt_1^3 + (6a - 3b - 4ct_1 + 6kt_1)\mu^2\}\right] + c_1\left[-\frac{1}{24}(t_1 - t_2)\{3k(t_1 + t_2)(-4 + t_1^2\alpha - t_2^2\alpha) + 4a(-6 + 2t_1^2\alpha - t_1t_2\alpha - t_2^2\alpha)\} - \frac{1}{360}(-1 + \beta)(-360a + 360at_1 - 360bt_1 -\right.$$

$360ct_1^2 + 360kt_1^2 + 120at_1^3\alpha - 60bt_1^3\alpha - 80ct_1^4\alpha + 120kt_1^4\alpha - 360a\mu + 180b\mu + 240ct_1\mu - 360kt_1\mu + 120at_1\alpha\mu - 180at_1^2\alpha\mu + 135bt_1^2\alpha\mu + 144ct_1^3\alpha\,\mu - 180kt_1^3\alpha\mu + 60a\alpha\mu^3 - 30b\alpha\mu^3\ - 40ct_1\alpha\mu^3 + 60kt_1\alpha\mu^3)\Big] = 0$ (19)

Also, $\frac{\partial K}{\partial t_2} = 0$ gives

$-c_4 e^{-t_2\lambda}\left(-1 + e^{t_2\lambda}\right)(a + kt_2) + c_3 e^{-t_2\lambda}(a + kt_2)(t_2 - t_3) - \frac{1}{2}c_2(a + kt_2)(t_1^2 - t_2^2)\alpha + c_1\left[(-t_1 + t_2)(a + kt_2) + \frac{1}{6}(t_1 - t_2)^2(t_1 + 2t_2)(a + kt_2)\alpha\right] = 0$
(20)

Similarly,

$\frac{\partial K}{\partial t_3} = 0$ gives

$c_4 e^{-t_3\lambda}\left(-1 + e^{t_3\lambda}\right)(a + kt_3) - c_3\left[\frac{1}{2}(T - t_3)\{2a + k(T + t_3)\}(-1 + \beta) - \frac{e^{-t_2\lambda}k}{\lambda^2} + \frac{e^{-t_3\lambda}k}{\lambda^2} - \frac{ae^{-t_2\lambda}}{\lambda} + \frac{ae^{-t_3\lambda}}{\lambda} - \frac{e^{-t_2\lambda}kt_2}{\lambda} + \frac{e^{-t_3\lambda}kt_3}{\lambda}\right] = 0$ (21)

Finally,

$\frac{\partial K}{\partial T} = 0$ gives,

$c' - c_3T(a + kT)(T - t_3)(-1 + \beta) + c_4\left[-at_2 - \frac{kt_2^2}{2} + at_3 + \frac{kt_3^2}{2} + e^{-t_2\lambda}\left(-\frac{k}{\lambda^2} - \frac{a}{\lambda} - \frac{kt_2}{\lambda}\right) + e^{-t_3\lambda}\left(\frac{k}{\lambda^2} + \frac{a}{\lambda} + \frac{kt_3}{\lambda}\right)\right] - c_3\left[-\frac{1}{6}(T - t_3)^2\{3a + k(2T + t_3)\}(-1 + \beta) + \frac{1}{\lambda^3}\left\{e^{-t_2\lambda}(2k + (a + kt_2)\lambda) + e^{-t_3\lambda}(-2k + (-a + kt_2 - 2kt_3)\lambda + (at_2 - at_3 + kt_2t_3 - k\,t_3^2)\lambda^2) + (t_2 - t_3)\lambda^3\left(\frac{e^{-t_2\lambda}(k + a\lambda + kt_2\lambda)}{\lambda^2} - \frac{e^{-t_3\lambda}(k + a\lambda + kt_3\lambda)}{\lambda^2}\right)\right\}\right] + c_1\left[-\frac{1}{120}(t_1 - t_2)^2(-60a - 20kt_1 - 40kt_2 + 10at_1^2\alpha + 3kt_1^3\alpha + 6kt_1^2t_2\alpha - 10at_2^2\alpha - kt_1t_2^2\alpha - 8kt_2^3\alpha) + a(-1 + \beta)\mu + \frac{1}{4}b(-1 + \beta)\mu^2 - \frac{1}{9}(-c + a\alpha)(-1 + \beta)\mu^3 - \frac{1}{32}b\alpha(-1 + \beta)\mu^4 - \frac{1}{75}c\alpha(-1 + \beta)\mu^5 + \frac{1}{360}(-1 + \beta)(t_1 - \mu)\{b(180t_1 + 15t_1^3\alpha - 30t_1^2\alpha\mu - 30t_1\alpha\mu^2) + c(120t_1^2 + 16t_1^4\alpha - 20t_1^3\alpha\mu - 20t_1^2\alpha\mu^2) + a(360 - 180t_1 - 30t_1^3\alpha + 180\mu - 60t_1\alpha\mu + 30t_1^2\alpha\mu - 60\alpha\mu^2 + 30t_1\alpha\mu^2 - 30\alpha\mu^3) + k(-120t_1^2 - 24t_1^4\alpha + 60t_1\mu + 21t_1^3\alpha\mu + 60\mu^2 + 21t_1^2\alpha\mu^2 - 9t_1\alpha\mu^3 - 9\alpha\mu^4)\}\right] + \frac{1}{24}c_2\alpha\Big[(t_1 - t_2)^2\{3k(t_1 + t_2)^2 + 4a(2t_1 + t_2)\} + 2(-1 + \beta)\mu^2\{6a +$

$$\mu(2b + c\mu)\} - \frac{1}{10}(-1+\beta)(t_1-\mu)[(40a(-3+t_1^2\alpha) + t_1\{8ct_1(-5+t_1^2\alpha) + 15b(-4+t_1^2\alpha)\})(t_1+\mu) + 10(t_1-\mu)(3k(t_1+\mu)^2 + 4a(t_1+2\mu))]\Big]$$

4. NUMERICAL EXAMPLE:

To illustrate the model numerically, we use the following parameter values:

$c_1 = 2.4$, $c_2 = 4$, $c_3 = 5$, $c_4 = 10$, $c' = 100, \mu = 1$, $\alpha = 0.02$, $\beta = 20$, $a = 30$, $b = 6$, $c = 5$, $k = b + c\mu$, $\lambda = 0.1$

Applying the subroutine FindRoot in Mathematica 8, we obtain the optimal solution for t_1, t_2, t_3 and T as follows:

$t_1 = 1.7694, t_2 = 4.4843, t_3 = 5.4343$ and $T = 5.5068$

Also, the optimal average cost for these parameters is 623.56

5. SENSITIVITY ANALYSIS:

Sensitivity analysis is performed by changing (increasing and decreasing) the parameters by 10%, 30% and 50%, and taking one parameter at a time, keeping the remaining parameters at their original values. Thus following table is formed:

Table1

Parameter	Initial value of the parameter	Change in %	Value after change	t_1	t_2	t_3	T	Average Cost	% Change in average cost
c'	100	+50	150	1.7666	4.5078	5.4759	5.5490	632.61	1.45
		+30	130	1.7677	4.4985	5.4593	5.5322	628.995	0.87
		+10	110	1.76886	4.48907	5.44265	5.51525	625.375	0.29
		−10	90	1.77001	4.47961	5.42591	5.49825	621.743	-0.29
		−30	70	1.77116	4.4701	5.4091	5.48116	618.099	-0.88
		−50	50	1.77232	4.46054	5.39221	5.46399	614.445	-1.46
c_1	2.4	+50	3.6	1.79652	4.25292	5.93991	6.03459	866.947	39.03
		+30	3.12	1.78719	4.33368	5.74073	5.82724	773.381	24.03
		+10	2.64	1.776	4.42912	5.53712	5.61454	674.903	8.23
		−10	2.16	1.762	4.5463	5.3315	5.3987	570.639	-8.49
		−30	1.68	1.74324	4.69956	5.13078	5.18633	459.395	-26.33
		−50	1.2	1.71464	4.92526	4.95684	4.99888	339.415	-45.57
c_2	4	+50	6	1.77083	4.47762	5.43253	5.50514	624.642	0.17
		+30	5.2	1.77027	4.4803	5.43323	5.50578	624.21	0.10
		+10	4.4	1.76972	4.483	5.43393	5.50643	623.777	0.03
		−10	3.6	1.76916	4.4857	5.43465	5.50709	623.343	-0.03

		−30	2.8	1.76859	4.48842	5.43537	5.50775	622.909	-0.10
		−50	2	1.76803	4.49115	5.4361	5.50843	622.475	-0.17
c_3	5	+50	7.5	1.75506	4.60147	5.29575	5.34575	632.764	1.48
		+30	6.5	1.75977	4.56339	5.34138	5.39843	629.738	0.99
		+10	5.5	1.76577	4.51446	5.39919	5.46565	625.897	0.37
		−10	4.5	1.77369	4.4492	5.47478	5.55449	620.856	-0.43
		−30	3.5	1.78463	4.35768	5.57772	5.67758	613.925	-1.55
		−50	2.5	1.80078	4.21965	5.72572	5.86027	603.732	-3.18
c_4	10	+50	15	1.75045	4.63845	5.11916	5.19709	645.325	3.49
		+30	13	1.75933	4.56695	5.23382	5.30981	638.248	2.36
		+10	11	1.76649	4.50855	5.36324	5.43697	628.997	0.87
		−10	9	1.77195	4.46357	5.51008	5.58121	617.579	-0.96
		−30	7	1.77568	4.43269	5.67782	5.74603	603.959	-3.14
		−50	5	1.77754	4.41716	5.87121	5.93617	588.052	-5.69
μ	1	+50	1.5	2.17215	4.90672	5.83261	5.90563	761.148	22.06
		+30	1.3	2.00938	4.74111	5.68029	5.75321	706.017	13.22
		+10	1.1	1.8489	4.57146	5.51924	5.59191	651.052	4.41
		−10	0.9	1.69042	4.39516	5.34559	5.4178	596.014	-4.42
		−30	0.7	1.53345	4.20834	5.1537	5.22515	540.584	-13.31
		−50	0.5	1.37739	4.00521	4.93508	5.00535	484.317	-22.33
α	0.002	+50	0.003	1.76992	4.46734	5.42569	5.49838	624.797	0.20
		+30	0.0026	1.76973	4.47409	5.42909	5.50169	624.304	0.12
		+10	0.0022	1.76954	4.48091	5.43255	5.50506	623.809	0.04
		−10	0.0018	1.76934	4.48781	5.43605	5.50847	623.311	-0.04
		−30	0.0014	1.76913	4.49479	5.4396	5.51194	622.81	-0.12
		−50	0.001	1.76893	4.50186	5.44321	5.51545	622.307	-0.20
β	20	+50	30	1.81185	5.05885	6.38971	6.44608	824.706	32.26
		+30	26	1.79786	4.84922	6.03395	6.09559	747.869	19.94
		+10	22	1.78021	4.61408	5.6447	5.71305	666.385	6.87
		−10	18	1.75688	4.3445	5.21093	5.2882	579.105	-7.13
		−30	14	1.72386	4.02515	4.71425	4.80418	484.276	-22.34
		−50	10	1.67138	3.62574	4.11948	4.22952	378.943	-39.23
a	30	+50	45	1.76683	4.7592	5.90315	5.98305	841.111	34.89
		+30	39	1.76776	4.66058	5.73349	5.81072	755.118	21.10
		+10	33	1.76883	4.54755	5.54102	5.6152	667.813	7.10
		−10	27	1.77012	4.41575	5.31915	5.38976	578.829	-7.17
		−30	21	1.77188	4.25839	5.05773	5.12408	487.618	-21.80
		−50	15	1.77466	4.0642	4.73991	4.80096	393.296	-36.93
b	6	+50	9	1.75069	4.30124	5.14413	5.2122	665.808	6.78
		+30	7.8	1.75769	4.36841	5.24994	5.31962	649.292	4.13
		+10	6.6	1.76534	4.44341	5.36894	5.44042	632.279	1.40
		−10	5.4	1.77374	4.52791	5.50414	5.57766	614.683	-1.42
		−30	4.2	1.78299	4.62417	5.65964	5.73549	596.392	-4.36
		−50	3	1.79324	4.73529	5.84119	5.91974	577.262	-7.42
c	5	+50	7.5	1.79685	4.36199	5.20959	5.27807	658.711	5.64
		+30	6.5	1.7858	4.40671	5.29216	5.36212	644.909	3.42
		+10	5.5	1.77486	4.45691	5.38419	5.45579	630.772	1.16
		−10	4.5	1.76403	4.51354	5.48745	5.56084	616.242	-1.17
		−30	3.5	1.75329	4.57783	5.60422	5.6796	601.253	-3.58
		−50	2.5	1.74262	4.65138	5.7375	5.81512	585.72	-6.07

λ	0.1	+50	0.15	1.76112	4.55243	5.30032	5.37561	637.56	2.25
		+30	0.13	1.76487	4.52184	5.35243	5.42668	632.64	1.46
		+10	0.11	1.76807	4.49561	5.40634	5.47944	626.829	0.52
		−10	0.09	1.77064	4.47446	5.46308	5.5349	620.031	-0.57
		−30	0.07	1.77247	4.45927	5.52387	5.59427	612.132	-1.83
		−50	0.05	1.77346	4.45112	5.59019	5.659	603.002	-3.30

From Table 1, the following points are noted:

(i) It is seen that the percentage change in the optimal cost is almost equal for both positive and negative changes of all the parameters except β and c_3.

(ii) It is observed that the model is more sensitive for a negative change than an equal positive change in the parameter c_3 andβ.

(iii) The optimal average cost increases with the increase and decreases with decrease in the values of the parameters.

(iv) Model is highly sensitive to changes in c_1, μ, β & a and moderately sensitive to changes in c_4, b & c . It has low sensitivity to c', c_2, c_3, α & λ.

(v) From the above points, it is clear that much care is to be taken to estimate c_1, μ, β & a

REFERENCES

[1] Avinadav, T., Herbon, A., Spiegel, U. 2013. Optimal inventory policy for a perishable item with demand function sensitive to price and time . Int. J. Production Economics 144, 497–506

[2] Chakrabarti, T., Giri, B.C., Chaudhuri, K.S., 1998. An EOQ model for items Weibull distribution deterioration shortages and trended demand-an extension of Philip's model. Computers and Operations Research 25 (7/8), 649–657.

[3] Chakravarthy, S.R., 2011. An inventory system with Markovian demands, phase type distributions for perishability and replenishment. Opsearch 47(4),266–283

[4] Chang, H.J., Dye, C.Y., 1999. An EOQ model for deteriorating items with time varying demand and partial backlogging. Journal of the Operational Research Society 50 (11), 1176-182.

[5] Covert, R.P., Philip, G.C., 1973. An EOQ model for items with Weibull distribution deterioration. AIIE Transactions 5, 323–326.

[6] Dave, U., 1986. An order level inventory model for deteriorating items with variable instantaneous demand and discrete opportunities for replenishment. Opsearch 23, 244–249.

[7] Dave, U., Patel, L.K., 1981. (T, Si) policy inventory model for deteriorating items with time proportional demand. Journal of the Operational Research Society 32, 137–142.

[8] Dye,C.U., 2004. A Note on “An EOQ Model for Items with Weibull Distributed Deterioration, Shortages and Power Demand Pattern”. Information and Management Sciences. 15(2), 81-84

[9] Dye, C.Y., Chang, H.J., Teng, J.T., 2006. A deteriorating inventory model with time-varying demand and shortage-dependent partial backlogging. European Journal of Operational Research 172, 417–429.

[10] Jalan, A.K., Giri, R.R., Chaudhuri, K.S., 1996. EOQ model for items with Weibull distribution deterioration shortages and trended demand. International Journal of Systems Science 27, 851–855.

[11] Maiti, A.K., Maiti, M.K., Maiti, M., 2009. Inventory model with stochastic lead-time and price dependent demand incorporating advance payment. Applied Mathematical Modelling 33, 2433–2443

[12] Mandal, B., Pal, A.K., 1998. Order level inventory system with ramp type demand rate for deteriorating items. Journal of Interdisciplinary Mathematics 1 (1), 49–66.

[13] Panda, S., Senapati, S., Basu, M., 2008. Optimal replenishment policy for perishable seasonal products in a season with ramp-type time dependent demand.Computers & Industrial Engineering 54, 301–314

[14] Papachristos, S., Skouri, K., 2000. An optimal replenishment policy for deteriorating items with time-varying demand and partial-exponential type-backlogging. Operations Research Letters 27 (4), 175-184.

[15] San-Jose, L.A., Siciliab,J., García-Lagunac, J., 2015. Analysis of an EOQ inventory model with partial backordering and non-linear unit holding cost. Omega 54, 147–157.

[16] Sanni, S.S., Chukwu, W.I.E., 2013. An Economic order quantity model for items with three-parameter Weibull distribution deterioration, ramp-type demand and shortages. Applied Mathematical Modelling 37, 9698–9706.

[17] Shah, Y.K., Jaiswal, M.C., 1977. An order-level inventory model for a system with constant rate of deterioration. Opsearch 14, 174–184.

[18] Singh, T. J., Singh, S.R., 2007. Perishable inventory model with quadratic demand, partial backlogging and permissible delay in payments. International Review of Pure and Applied Mathematics 3, 199-212.

[19] Singh, T. J., Singh, S.R., 2009. An EOQ model for perishable items with power demand and partial backlogging. International Journal of Operations and Quantitative Management 15, 65-72.

[20] Teng, J.T., Yang, H.L., Ouyang, L.Y., 2003. On an EOQ model for deteriorating items with time-varying demand and partial backlogging. Journal of the Operational Research Society 54 (4), 432-436.

[21] Teng, J.T., Yang, H.L., 2004. Deterministic economic order quantity models with partial backlogging when demand and cost are fluctuating with time. Journal of the Operational Research Society 55 (5), 495-503.

[22] Wang, C., Huang, R., 2014. Pricing for seasonal deteriorating products with price and ramp-type time dependent demand. Computers & Industrial Engineering 77, 29-34