Global Journal of Pure and Applied Mathematics.
ISSN 0973-1768 Volume 13, Number 7 (2017), pp. 3395-3401

http://www.ripublication.com

Cordial Double-Staircase Graphs

K. Ameenal Bibi, P. Rekha and T. Ranjani
P.G and Research Department of Mathematics
D.K.M College for women (Autonomous), Vellore, Tamil Nadu, India.

Abstract

Let G= (V, E) be finite, simple, connected and undirected graph with vertex set V and edge set E. Let p and q denote the order and size of the graph G. A vertex labeling f: V(G) → {0, 1} induces an edge labeling f^*: E(G) → {0, 1}. For an edge $e=uv$, the mapping $f^*$is given by $f^*(e)=|f(u) – f(v)|$, then $v_f(i)$= number of vertices of G having label i under f. $e_{f^*}(i)$ = number of vertices of G having label i under f^*.

The function f is said to be a cordial labeling of G if the number of vertices labeled 0 and the number of vertices labeled 1 differ by at most one, the number of edges labeled 0 and the number of edges labeled 1 differ by at most one.

In this paper, we have established that Double-Staircase Graph of order k admits Cordial Labeling.

AMS Subject Classification (2010): 05C78

Keywords: Staircase graph, Double-Staircase graph, k-step staircase graph, Cordial labeling.

1. INTRODUCTION:

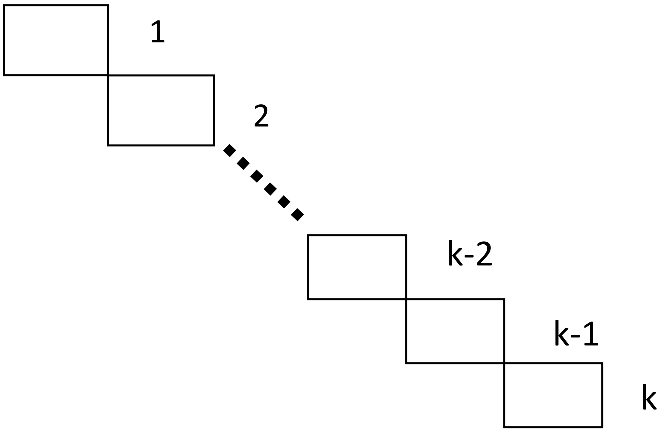
In 1967, A. Rosa introduced graph labeling. A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions.

A. Solairaju and M. Antony Arockiasamy proved that any staircase graph G(S , k) of order k admits cordial labeling.[4] [11].

The notation of cordial labeling was introduced by Ibrahim Cahit, Turkey. A Cordial labeling of a graph G with vertex set V is a function of f from the vertices of G to {0, 1} such that for each edge *uv*, assign the label |f (u) – f (v)|. The number of vertices labeled ‘0’ and the number of vertices labeled ‘1’ differ by at most one and also the number of edges labeled ‘0’ and the number of edges labeled ‘1’ differ by at most one.

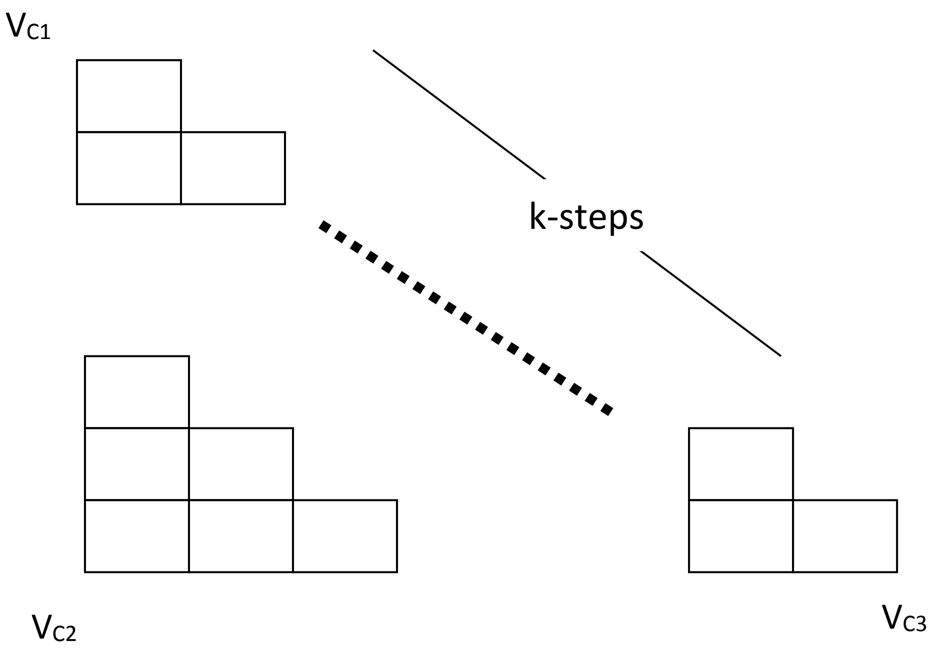
DEFINITION 1.1

$k^*(C_4)$ is a graph, consisting of k copies of C_4 with a vertex common in between any two consecutive copies of C_4 except first and last copies of C_4.



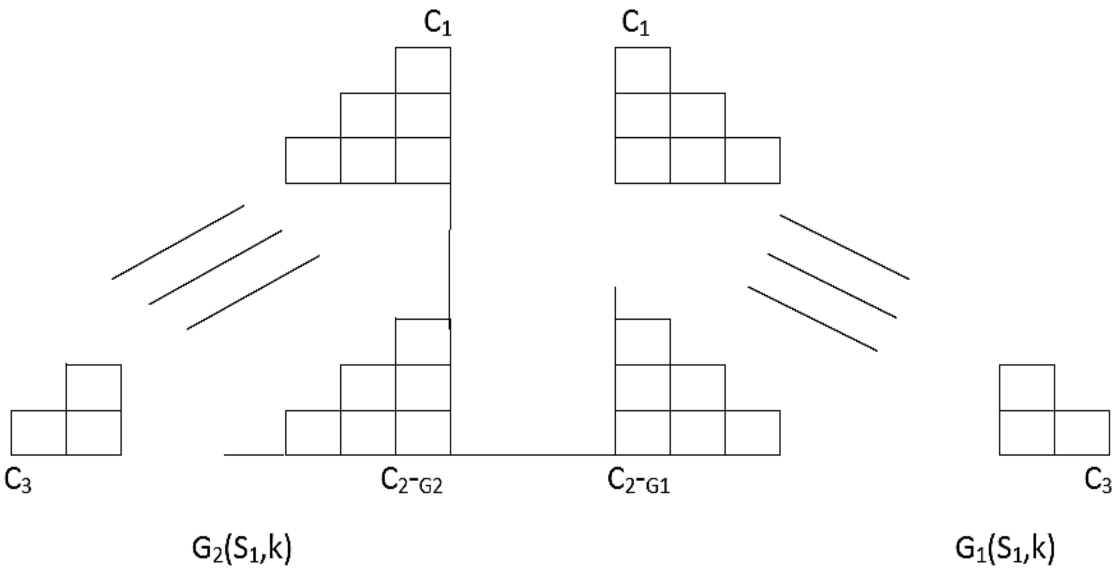
DEFINITION 1.2

A Staircase graph of G order k (k≥2 and k∈ Z) is a graph obtained by merging (2k-2) edges of k^*C_4 with (2k-2) edges in the shortest path V_{c1} V_{c3} and $(k-1)^{th}$ staircase.



DEFINITION 1.3

A double-staircase graph G of order k is obtained by joining an edge between $C_{2-G_1} - C_{2-G_2}$. [(ie) C_2 of $G_1(S_1 , k)$ is joined with C_2 of $G_2(S_1 , k)$ by an edge]. As there are 3 corner vertices in each of the staircase graph joining which we get a double – staircase graph of order k.

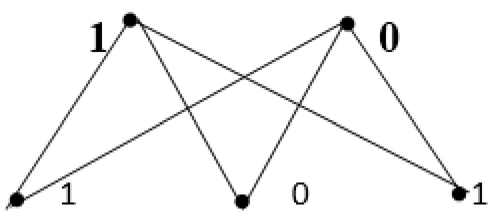


G(S_2,k) Double – Staircase Graph of order k.

DEFINITION 1.5

A binary vertex labeling f of a graph G is called a cordial labeling if $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$.

EXAMPLE 1.6



THEOREM 2.1

Double – Staircase graph G of order k admits cordial labeling.

PROOF:

Let G (S_2, k) be a double – staircase graph of order k.

Let $f: V \rightarrow \{0,1\}$ be a mapping from the set of vertices of G to {0, 1}.

$f^{*}: V \rightarrow \{0,1\}$ be a mapping from set of edges of G to {0,1} and for each edge $uv \in E$, assign the label $|f(u) - |f(v)|$.

Here, the staircase G_1(S, k) is split into $k+1$ horizontal path $P_{m1}, P_{m2,\ldots\ldots} P_{mk}, P_{mk+1}$ where P_{mi} represents the shortest horizontal path between v_{i1} and $v_{ij}, 1 \leq j \leq k+1$. The pattern of vertex labeling proposed for any staircase graph of k-steps (for odd and even k) is obtained as follows

For each path $P_{mi\,,}$ $1 \leq i \leq k+1$, the label 1 is assigned to all the vertices of $P_{mi\,,}$ $if\ i\ is\ odd$ and the label 0 is assigned to all the vertices of $P_{ni} if\ i\ is\ even.$

Similarly, the staircase G_2(S , k) is split into $k+1$ horizontal path $P_{n1}, P_{n2,\ldots\ldots} P_{nk}, P_{nk+1}$ where P_{ni} represents the shortest horizontal path between v_{i1} and $v_{ij}, 1 \leq j \leq k+1$. The pattern of vertex labeling proposed for any staircase graph of k steps (for odd and even k) is obtained as follows

For each path $P_{ni\,,}$ $1 \leq i \leq k+1$, the label 0 is assigned to all the vertices of $P_{ni\,,}$ $if\ i\ is\ odd$ and the label 1 is assigned to all the vertices of P_{ni} $if\ i\ is\ even.$

The above rule need to be slightly altered in final path $P_{mk-1\,,} P_{mk,} P_{mk-1\,,}$ $P_{nk-1\,,}$ $P_{nk,}$ $P_{nk+1\,,}$ in such a way that the number of vertices labeled as 0 and the number of vertices labeled as 1, differ atmost by one.

Simultaneously, the resulting number of edges labeled as 1 and the number of edges labeled as 0 also differ at most by one.

Let G_1(S , k) and G_2(S , k) be two staircase graphs of same order obtained by joining an edge between $C_{2-G_1} - C_{2-G_2}$

Let the total number of vertices for the double –staircase graph $p = \frac{2k^2+10k+4}{2}$ and total number of edges for the double – staircase graph is $q = 2k^2 + 6k + 1.$

We defined already that f : V → {0,1}. Let p contains 0's and 1's. Split the values of p equally and denote the 1st set of values as 'r' and 2nd set of values as't'.

If r is even, again split the set of r vertices into two equal sets and assign 0's and 1's to the number of vertices.

If r is odd, then split the set of r vertices into two sets containing $r_1\ and\ r_2$ number of vertices respectively. Take $r_1 = \frac{r-1}{2}$ and assign these r_1 vertices with 0's and take r_2 = r-$r_1$and assign these r_2 vertices with 1's.

In this way, we obtain G_1(S , k) graph.

Now to find $G_2(S , k)$:

Consider t, If t contains even number of vertices split t equally into two sets and assign 0's and 1's to the number of vertices.

If t contains odd number of vertices then split t as t_1 and t_2.

Take $t_1 = \frac{t-1}{2}$ and assign these t_1 vertices with 1's and take $t_2 = t - t_1$ and assign these t_2 vertices with 0's.

After assigning the values for the vertices, the edge values should be assigned with at most difference 1.

$$|e_f(0) - e_f(1)| \leq 1$$

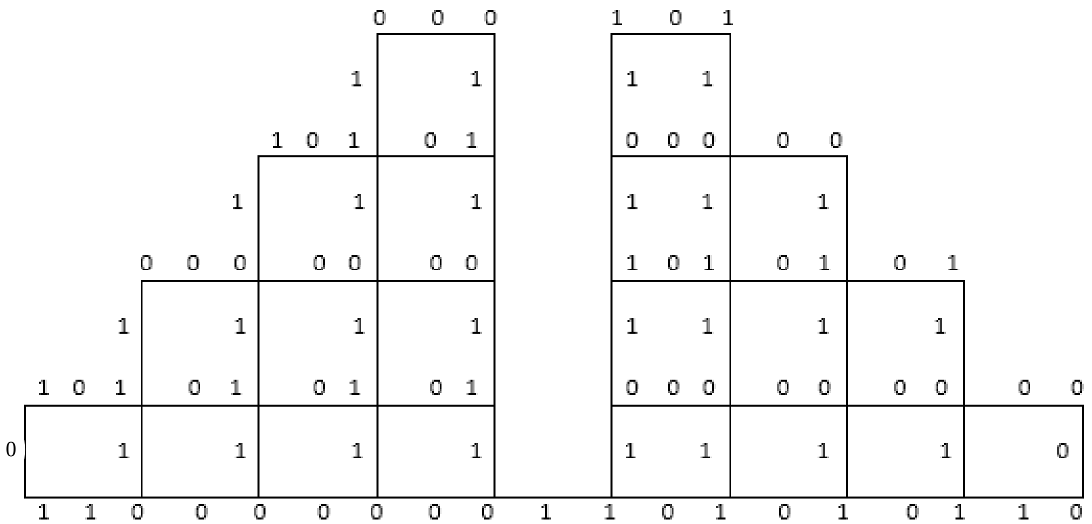
Always the number of 1 assigned to the edges must be greater then number of 0's assigned to the edges with atmost difference 1.

The above pattern of labeling established the cordiality of $G(S_2 , k)$.

Thus, the double – staircase graph of order k admits cordial labeling.

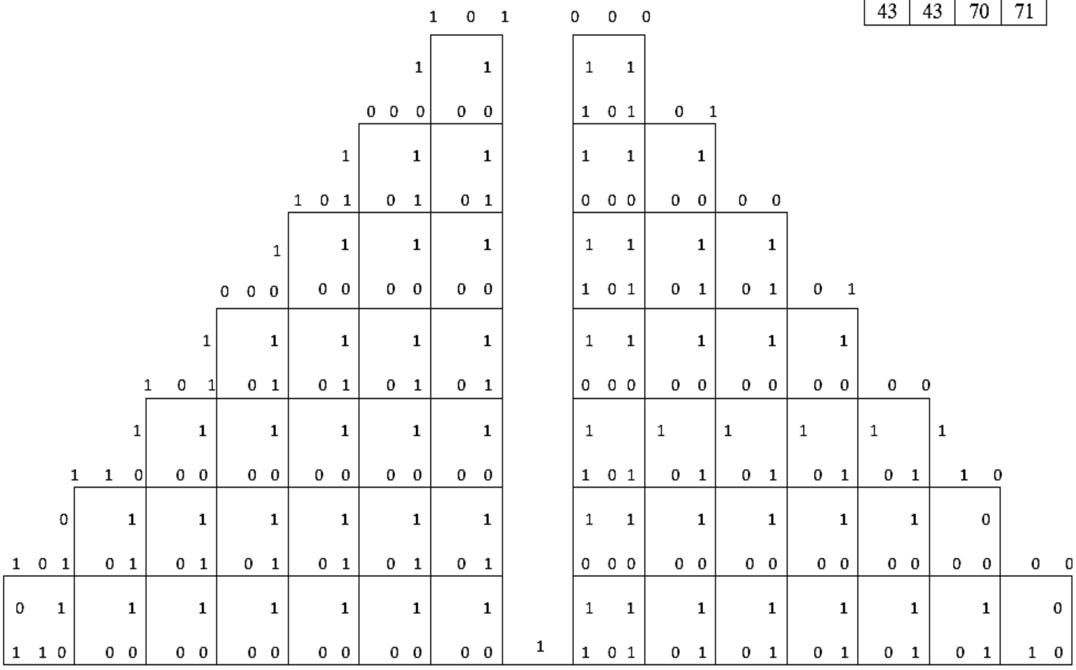
Example 2.2: G (S_2, 4) Double staircase graph G of order 4

p		q	
0	1	0	1
19	19	28	29



Example 2.3: G (S_2, 7) Double staircase graph G of order 7

p		q	
0	1	0	1
43	43	70	71



CONCLUSION:

In this paper, we established that the double-staircase graph of order k admits Cordial labeling. We have given few examples of graphs of odd and even order.

REFERENCES

[1] Acharya.B.D, Rao.S.B and S.Arumugan, Embeddings and NP- Complete problems for graceful graphs, in Labeling of Discrete Structures and Applications, Narosa Publishing house, New Delhi,2008,57-62.

[2] Barrientos.C, Graceful labelings of Cyclic Snakes, Ars Combin., 60(2001)85-96.

[3] Bhat.V.N – Nayak and A.Selvam, Gracefulness of n-Cone C_mVK_nC, Ars Combin66(2003) 283-298.

[4] Cahit.I, Cordial graphs: a weaker version of graceful and harmonious graphs, Ars Combin.,23(1987)201-207.

[5] Cahit .I, On Cordial and 3- equitable labeling of graphs, Util.Math.,37(1990)189-198.

[6] Gallian.J.A, A Survey: recent results , Conjectures and open problems on labeling graphs.J.Graph Theory,13(1989)491-504.

[7] Golomb.S.W,How to number a graph in Graph Theory and Computing, R.C.Rend,ed.,Academic press, Newyork (1972)23-27.

[8] Graham.R.L, and N.J.A. Sloane, On additive bases and Harmonious graph, SIAM J.Alg.Discrete Math.,1(1980)382-404.

[9] Khan.N, Cordial labeling of Cycles, Annals Pure Appl.Math.1,No.2(2012).

[10] Liu.Z and Zhu.B, A necessary and sufficient condition for a 3- regular graph to be cordial, Ars Combin.,84(2007) 225-230.

[11] Michael Raj.A, Thesis on labelings of staircase Graphs, Sacred Heart College, Tirupattur.

[12] Rosa.A, On Certain Valuation of the vertices of a graph, Theory of graphs(international Symposium, Rome, July1966) ,Gordon and Breach,N.Y and Dunod Paris (1967)349-355.

[13] Solairaju.A and Antony Arockiasamy .M, Gracefulness of double – staircase graphs, Int.J.Contemp. Math. Sciences, Vol.5, 2010, no.49, 2433-2441.

[14] West.D.B, Introduction to graph theory, Prentice – Hall Inc,(2000).