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Computation model of radial bearing taking into account the dependence of the viscosity of lubricant on pressure and temperature

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Abstract

Method and realization of exact self-similar solution of the radial sliding bearing is introduced in the paper, based on the equation of motion of a viscous incompressible fluid for the case of a "thin layer", the equation of continuity and expression that reflects the regularity of dissipation velocity change of the lubricating material energy, taking into account the simultaneous dependence of viscosity of the lubricant on pressure and temperature. The field of velocities and pressures in the lubricating layer is found, and analytical expressions for the bearing capacity and friction force are obtained. The influence of parameters, characterizing the dependence of viscosity on pressure and temperature, as well as the effect of the adapted profile on the performance of tribosystems are assessed.

Keywords: Hydrodynamics, adapted profile of the bearing surface, dependence of viscosity of the lubricant on pressure and temperature.

1. INTRODUCTION

Tribosystems of modern machines operate under high load and speed modes. In these conditions, the use of liquid friction provided by liquid lubricants is the most promising. Despite the considerable experience in the use of sliding bearings operating in the hydrodynamic lubrication mode, and the efforts of many scientists who have devoted time to their calculation [1-6], even now, hydrodynamic calculations of bearings in the design cause certain difficulties. This is due to the presence of a large number of variable factors, the ambiguity in the choice of boundary conditions, the randomness of variation in the patterns of change in mechanical, physical and chemical related

processes. Theoretically, these data do not allow either generalizations or extrapolations.
The attempt to develop the hydrodynamic lubrication mode calculation for different bearing designs, applying self-similar variables and combining the developed models into a single computational complex for engineering practice application is of particular importance and scientific interest.
One of the main parameters that must be taken into account in the calculation of sliding bearings is their design, which determines the geometry of the contact surface, the method of supplying the lubricant and the loading modes. All the existing methods of hydrodynamic calculation of radial bearings of infinite length differ by the coordinate system, the type of the basic equation, the way of its integration, and the type of boundary conditions [7-10]. This, in turn, establishes the specifics of the statement of the problem and the method of hydrodynamic calculations.
The key to the solution of the problem lies in increasing bearing capacity of the bearings as a result of application of the supporting profile of bearing bush adapted to the conditions of hydrodynamics and, in addition, refinement of the calculation models of bearings on the basis of simultaneous consideration of the viscosity of the liquid lubricant against pressure and temperature. The latter determines the novelty and urgency of the solution obtained.
The scientific novelty of the proposed solution and refinement of the computation model is in the simultaneous consideration of viscosity of the liquid lubricant on pressure and temperature, in addition, the use of adapted profile of the bearing surface provides its increased bearing capacity. Simultaneous accounting of a complex of variable factors made it possible to significantly refine the computational model and to approximate the obtained results to real.

2. ALLOCATION OF TASK

The model of steady motion of a viscous incompressible liquid lubricant in the working gap of the infinite radial bearing is considered (Fig. 1). The shaft rotates at a nominal speed Ω, and the bearing bush with the adapted profile of the bearing surface is stationary. It is assumed that the space between the shaft and the bearing is completely filled with lubricant. We believe that viscosity of the lubricant is simultaneously dependent on pressure and temperature.

$$\mu' = \mu_0 e^{\alpha' p' - \beta' T'}, \tag{1}$$

where μ_0 is the characteristic viscosity, p' is hydrodynamic pressure, T' is temperature, α', β' is constant experimental value, μ' is lubricant dynamic viscosity coefficient.

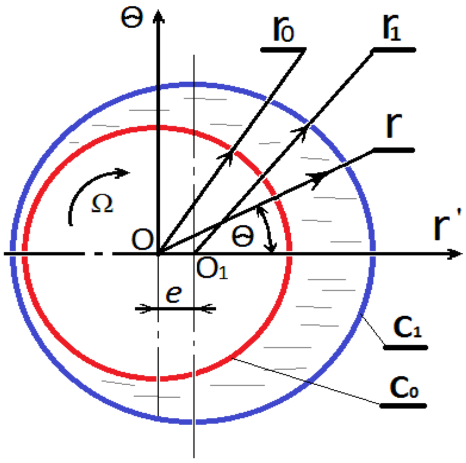

Fig. 1. Computation Scheme

NOMENCLATURE

μ_0 – characteristic viscosity, Ns/m^2 ;

p' – hydrodynamic pressure, Pa;

T' – temperature, $^\circ C$;

α', β' – constant experimental value,

μ' – lubricant dynamic viscosity coefficient, Ns/m^2 ;

r_0 – shaft radius, m;

r_1 – bearing radius, m;

e – eccentricity;

δ – radial gap, m;

ω, a – parameters, characterizing the adapted profile of the bearing;

u', v' – components of lubricating material's velocity vector;

T^* – characteristic temperature, $^\circ C$;

α – parameter, characterizing the dependence of viscosity on pressure,
β – parameter, characterizing the dependence of viscosity on temperature,

Q – lubricant consumption per unit time, $\frac{m^3}{c}$;

C_p – heat capacity at constant pressure, J/kg·degrees;

h – oil film thickness, m;

Ω – angle speed, c^{-1};

η – eccentricity ratio of bearing bush;

η_1 – eccentricity ratio of adapted profile of the bearing.

In the polar coordinate system r', θ with pole in the center of the shaft, equations of contour of the shaft and of the bearing bush will be written as follows:

$$r' = r_0, \quad r' = r_1 + e\cos\theta - a\sin\omega\theta, \tag{2}$$

where r_0 is the shaft radius; r_1 is the bearing radius; e is eccentricity; $\frac{e}{\delta}, \frac{a}{\delta}$ is the small value of one order of smallness $(\delta = r_1 - r_0)$; ω, a are parameters of contact profile subject to defining.

3. BASIC EQUATIONS AND BOUNDARY CONDITIONS.

The dimensionless equations of motion for a viscous incompressible fluid for the case of a "thin layer" and the continuity equation are taken as the initial equations, taking into account the simultaneous dependence of the lubricant viscosity on pressure and temperature:

$$\frac{\partial^2 v}{\partial r^2} = \frac{1}{e^{\alpha p - \beta T}} \cdot \frac{dp}{d\theta}; \qquad \frac{\partial u}{dr} + \frac{\partial v}{\partial \theta} = 0. \tag{3}$$

Dimensional values $u', v', p', r', \mu', \alpha', \beta', T'$ are connected with non-dimensional $u, v, p, r, \mu, \alpha, \beta, T$ by the following correlations:

$$u' = \Omega r_0 u; \quad v' = \Omega r_0 v; \quad p' = p^* p; \quad \mu' = \mu_0 \mu;$$
$$T' = T^* T; \quad p^* = \frac{\mu_0 \Omega r_0^2}{\delta^2}; \quad \alpha = p^* \alpha'; \quad \beta = T^* \beta'; \tag{4}$$

where u', v' are components of lubricating material's velocity vector; T^* is characteristic temperature, α is parameter, characterizing the dependence of viscosity on pressure, β is parameter, characterizing the dependence of viscosity on temperature.

The equation system (3) is solved under the following boundary conditions:

$$u = 0, \quad v = 0, \quad \text{при} \quad r = 1 + \eta\cos\theta - \eta_1 \sin\omega\theta;$$
$$u = 0, \quad v = 1, \quad \text{при} \quad r = 0; \tag{5}$$
$$p(0) = p(2\pi) = \frac{p_a}{p^*}, \quad \eta = \frac{e}{\delta}; \quad \eta_1 = \frac{a}{\delta}.$$

Conditions (5) mean adhesion of the lubricant to the surface of the shaft and bearing bush, as well as pressure equality in initial and final cross-sections.

4. ACCURATE SELF-SIMILAR SOLUTION.

The accurate self-similar solution of the problem (3) taking into account (4) and (5) is performed by integration after introduction of self-similar variable $\xi = \dfrac{r}{h(\theta)}$:

$$v = \frac{\partial\psi}{\partial r} + V(r,\theta), \qquad u = -\frac{\partial\psi}{\partial\theta} + U(r,\theta),$$
$$\psi(r,\theta) = \tilde{\psi}(\xi), \qquad V(r,\theta) = \tilde{v}(\xi), \qquad U(r,\theta) = -\tilde{u}(\xi)\cdot h'(\theta), \tag{6}$$
$$\frac{1}{e^{\alpha p - \beta T}}\frac{dp}{d\theta} = \frac{\tilde{C}_1}{h^2(\theta)} + \frac{\tilde{C}_2}{h^3(\theta)}, \qquad h(\theta) = 1 + \eta\cos\theta - \eta_1\sin\omega\theta.$$

Adding (6) to the equation (3) taking into account the boundary conditions (5) we get the following differential equations:

$$\tilde{\psi}''' = \tilde{C}_2, \qquad \tilde{v}'' = \tilde{C}_1, \qquad \tilde{u}'(\xi) + \xi\tilde{v}'(\xi) = 0, \tag{7}$$

And boundary conditions:

$$\tilde{\psi}'(0) = \psi'(1) = 0, \qquad \tilde{u}(1) = \tilde{v}(1) = 0, \qquad \tilde{u}(0) = 0, \qquad \tilde{v}(0) = 1$$
$$\int_0^1 \tilde{v}(\xi)\,d\xi = 0. \tag{8}$$

Solution of the equation (7) taking into account the boundary conditions (8) is found by direct integration. At the result we get:

$$\tilde{\psi}'(\xi)=\frac{\tilde{C}_2}{2}\left(\xi^2-\xi\right),\qquad \tilde{v}(\xi)=\tilde{C}_1\frac{\xi^2}{2}-\left(\frac{\tilde{C}_1}{2}+1\right)\xi+1,$$

$$\tilde{C}_1=6,\quad \tilde{C}_2=\frac{-\tilde{C}_1\int\limits_0^{2\pi}\frac{\mu(\theta)}{h^2(\theta)}d\theta}{\int\limits_0^{2\pi}\frac{\mu(\theta)}{h^3(\theta)}d\theta}=-\frac{\tilde{C}_1\tilde{J}_2(\theta)}{\tilde{J}_3(\theta)},\tag{9}$$

where $\tilde{J}_k(\theta)=\int\limits_0^{2\pi}\frac{\mu(\theta)}{h^k(\theta)}d\theta$

5. DEFINITION OF HYDRODYNAMIC PRESSURE.

The non-dimensional hydrodynamic pressure in the lubricating layer is defined from the equation:

$$\frac{1}{\mu(\theta)}\frac{dp}{d\theta}=\frac{\tilde{C}_1}{h^2(\theta)}+\frac{\tilde{C}_2}{h^3(\theta)}\tag{10}$$

In order to solve the equation (10), let's at first define $\mu(\theta)$ as function, depending on θ.

Defining $\mu(\theta)$ we use the expression, reflecting the regularity of change of lubricating material energy dissipation speed:

$$\frac{dH'}{d\theta}=\frac{2\mu_0\mu\Omega^2r_0^2h(\theta)}{\delta}\int\limits_0^1\left(\frac{\tilde{\psi}''(\xi)}{h^2(\theta)}+\frac{\tilde{v}'(\xi)}{h(\theta)}\right)^2d\xi\tag{11}$$

Then the temperature rise is defined by the expression:

$$\frac{dT'}{d\theta}=\frac{dH'}{d\theta}\cdot\frac{1}{C_p\theta}=\frac{1}{C_p\theta}\cdot\frac{2\mu_0\mu\Omega^2r_0^2h(\theta)}{\delta}\int\limits_0^1\left(\frac{\tilde{\psi}''(\xi)}{h^2(\theta)}+\frac{\tilde{v}'(\xi)}{h(\theta)}\right)^2d\xi,\tag{12}$$

where Q is lubricant consumption per unit time; C_p is heat capacity at constant pressure; h is oil film thickness.

$$Q=\Omega r_0\delta\int\limits_0^1\psi'(\xi)d\xi=-\Omega r_0\delta\frac{\tilde{C}_2}{12}\tag{13}$$

Let's differentiate the expression $\mu = e^{\alpha p - \beta T}$ according to θ, we get:

$$\frac{d\mu}{d\theta} = \mu(\theta)\left(\alpha \frac{dp}{d\theta} - \beta \frac{dT}{d\theta}\right) = \mu(\theta)\alpha \frac{dp}{d\theta} + \frac{\mu(\theta)\beta 24\mu_0 \Omega r_0 h(\theta)}{T^* C_p \cdot \delta^2 \tilde{C}_2} \cdot \int_0^1 \left(\frac{\tilde{\psi}''(\xi)}{h^2(\theta)} + \frac{\tilde{v}'(\xi)}{h(\theta)}\right)^2 d\xi. \tag{14}$$

Taking into account the equation (10) for definition $\mu(\theta)$ we come to the following differential equation:

$$\frac{1}{\mu^2(\theta)} \frac{d\mu}{d\theta} = \frac{\alpha \tilde{C}_1}{h^2(\theta)} + \frac{\alpha \tilde{C}_2}{h^3(\theta)} + \frac{24\mu_0 \beta \Omega r_0 h(\theta)}{T^* C_p \delta^2 \tilde{C}_2} \cdot \int_0^1 \left(\frac{\tilde{\psi}''(\xi)}{h^2(\theta)} + \frac{\tilde{v}'(\xi)}{h(\theta)}\right)^2 d\xi. \tag{15}$$

Integrating this equation we get:

$$\frac{1}{\mu(\theta)} = 1 - \alpha \tilde{C}_1 J_2(\theta) - \alpha \tilde{C}_2 J_3(\theta) - \tilde{K}\left[\Delta_1 J_3(\theta) + \Delta_2 J_2(\theta) + \Delta_3 J_1(\theta)\right], \tag{16}$$

where $\tilde{K} = \frac{24\mu_0 \beta \Omega r_0}{T^* C_p \delta^2 \tilde{C}_2}$, $\quad \Delta_1 = \int_0^1 \left(\tilde{\psi}''(\xi)\right)^2 d\xi,$

$$\Delta_2 = 2\int_0^1 \left(\tilde{\psi}''(\xi) \cdot \tilde{v}'(\xi)\right) d\xi, \quad \Delta_3 = \int_0^1 \left(\tilde{v}'(\xi)\right)^2 d\xi, \quad J_\kappa(\theta) = \int_0^\theta \frac{d\theta}{h^\kappa(\theta)}.$$

Solving the equation (16) in relation to $\mu(\theta)$, we get:

$$\mu(\theta) = \frac{1}{1 - \tilde{K}\left[\Delta_1 J_3(\theta) + \Delta_2 J_2(\theta) + \Delta_3 J_1(\theta)\right] - \alpha \tilde{C}_1 J_2(\theta) - \alpha \tilde{C}_2 J_3(\theta)}. \tag{17}$$

The function $\mu(\theta)$ will be substituted in the following by its averaged integral value:

$$\tilde{\mu} = \int_0^{2\pi} \mu(\theta) d\theta =$$

$$= \int_0^{2\pi} \left[\frac{1}{1 - \tilde{K}\left[\Delta_1 J_3(\theta) + \Delta_2 J_2(\theta) + \Delta_3 J_1(\theta)\right] - \alpha\left(\tilde{C}_1 J_2(\theta) + \tilde{C}_2 J_3(\theta)\right)}\right] d\theta. \tag{18}$$

Solving the obtained equations for $\Delta_1, \Delta_2, \Delta_3, \tilde{C}_2, J_3(\theta), J_2(\theta), J_1(\theta)$ within the accuracy to the members of the second order of smallness $O(\eta^2), O(\eta_1^2), O(\eta, \eta_1)$, we get the following expressions:

$$\tilde{C}_2 = -6\left(1+\frac{\eta_1}{2\pi\omega}\left(\cos 2\pi\omega - 1\right)\right),\ \Delta_1 = \frac{\tilde{C}_2^2}{12} = 3\left(1+\frac{\eta_1}{\pi\omega}\left(\cos 2\pi\omega - 1\right)\right),$$

$$\Delta_2 = \frac{1}{6}\tilde{C}_2\cdot\tilde{C}_1 = \tilde{C}_2 = -6\left(1+\frac{\eta_1}{2\pi\omega}\left(\cos 2\pi\omega - 1\right)\right),\quad \Delta_3 = 4,$$

$$\int_0^{2\pi} J_2(\theta)d\theta = 2\pi^2 - \frac{2\eta_1}{\omega}\left(\frac{\sin 2\pi\omega}{\omega} - 2\pi\right),\quad \int_0^{2\pi} J_3(\theta)d\theta = 2\pi^2 - \frac{3\eta_1}{\omega}\left(\frac{\sin 2\pi\omega}{\omega} - 2\pi\right),$$

$$\int_0^{2\pi} J_1(\theta)d\theta = 2\pi^2 - \frac{\eta_1}{\omega}\left(\frac{\sin 2\pi\omega}{\omega} - 2\pi\right).$$

Than for the averaged integral value $\tilde{\mu}$ we get the following expression:

$$\tilde{\mu} = 1 + K\left(-\frac{\pi^2}{3} + \frac{\eta_1}{6\omega}\left(\frac{\sin 2\pi\omega}{\omega} - 2\pi\right) + \frac{2\eta_1\pi}{3\omega}\left(\cos 2\pi\omega - 1\right)\right) + $$
$$+\frac{6\eta_1}{\omega}\alpha\left(\frac{\sin 2\pi\omega}{\omega} - \pi\cos 2\pi\omega - 2\pi\right), \tag{19}$$

где $K = \frac{\mu_0\beta\Omega r_0}{T^* C_p \delta^2}$.

Taking into account (10) and (19) the non-dimensional hydrodynamic pressure is defined by the expression:

$$p = \tilde{\mu}\left[\tilde{C}_1 J_2(\theta) + \tilde{C}_2 J_3(\theta)\right] + \frac{p_a}{p^*} =$$
$$= 6\tilde{\mu}\left(\eta\sin\theta + \frac{\eta_1}{\omega}\left(\cos\omega\theta - 1\right) - \frac{\eta_1\theta}{2\pi\omega}\left(\cos 2\pi\omega - 1\right)\right) + \frac{p_a}{p^*}. \tag{20}$$

6. RESULTS OF RESEARCH AND THEIR DISCUSSION

Let's come to defining of basic working characteristics of the bearing.

For component of sustaining force and friction force vector we get the following expression:

$$R_x = \frac{\tilde{\mu}\mu_0\Omega r_0^3}{\delta^2}\int_0^{2\pi}\left(p - \frac{p_a}{p^*}\right)\sin\theta d\theta = -\frac{\tilde{\mu}\mu_0\Omega r_0^3}{\delta^2}6\left(\eta\pi + \eta_1\left(1-\omega^2\right)\left(\cos 2\pi\omega - 1\right)\right),$$

$$R_y = \frac{\tilde{\mu}\mu_0\Omega r_0^3}{\delta^2}\int_0^{2\pi}\left(p - \frac{p_a}{p^*}\right)\cos s\theta d\theta = -\frac{\tilde{\mu}\mu_0\Omega r_0^3}{\delta^2}6\eta_1\sin 2\pi\omega,$$
$$\tag{21}$$

$$L_{mp} = \frac{\tilde{\mu}\mu_0\Omega r_0}{\delta}\left[\int_0^{2\pi} -\frac{\tilde{C}_2}{2}\frac{d\theta}{h^2(\theta)} - \int_0^{2\pi}\left(\frac{\tilde{C}_1}{2}+1\right)\frac{d\theta}{h(\theta)}\right] = \frac{\tilde{\mu}\mu_0\Omega r_0}{\delta}\left[-2\pi + \frac{\eta_1}{\omega}\left(\cos 2\pi\omega - 1\right)\right]. \quad (22)$$

Input parameters for calculation of load-carrying ability and friction force, defined by the expressions (21), (22):

$$\eta = \eta_1 = 0,3 \div 1; \quad \omega = 0 \div 1; \quad K = 0,1 \div 3;$$

$$\beta = 0 \div 1; \quad T = 30 \div 100^\circ C; \quad \mu_0 = 0,00595 \ \frac{\text{Н}\cdot\text{с}}{\text{м}^2};$$

$$P_a = 0,08 \div 0,101325 \quad \text{мПа}.$$

$$\Omega = 100 \div 1800 \ \text{с}^{-1}; \quad \delta = 0,05\cdot 10^{-3} \div 0,07\cdot 10^{-3} \ \text{м}; \quad r_0 = 0,01995 \div 0,04933 \ \text{м}.$$

The diagrams, shown in the figures №$2 \div 7$ at the result of numerical computations:

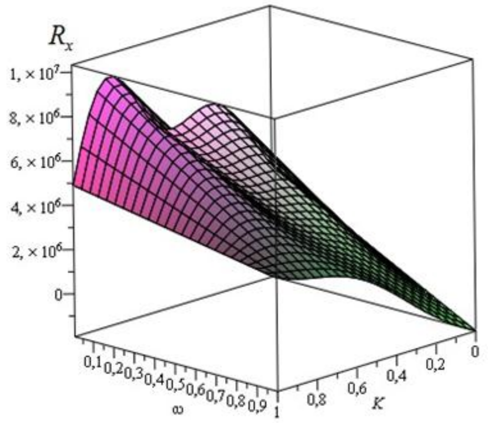

Fig. 2. Dependence of sustaining force vector component on parameter ω, characterizing the adapted profile of the bearing surface of bearing bush, and the heat parameter *K*.

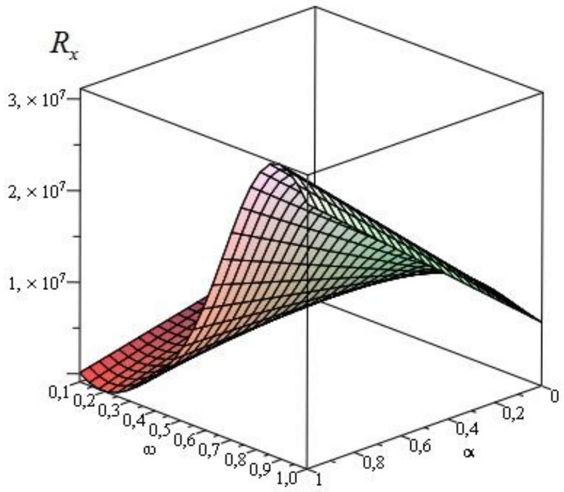

Fig. 3. Dependence of sustaining force vector component on parameter ω, characterizing the adapted profile of the bearing surface of bearing bush, and parameter *α*, characterizing the dependence of lubricant viscosity on pressure.

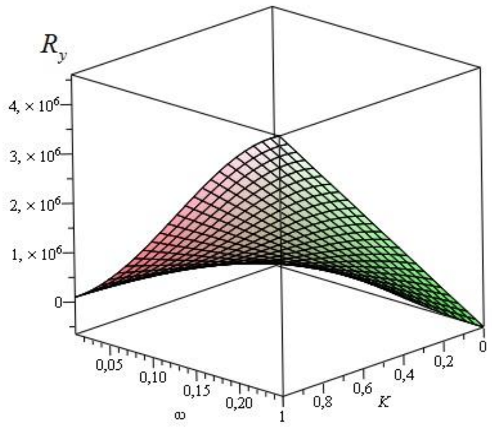

Fig. 4. Dependence of sustaining force vector component on parameter ω, characterizing the adapted profile of the bearing surface of bearing bush, and the heat parameter *K*.

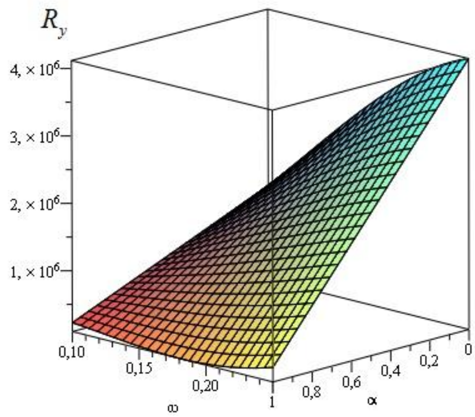

Fig. 5. Dependence of sustaining force vector component on parameter ω, characterizing the adapted profile of the bearing surface of bearing bush, and parameter *α*, characterizing the dependence of lubricant viscosity on pressure.

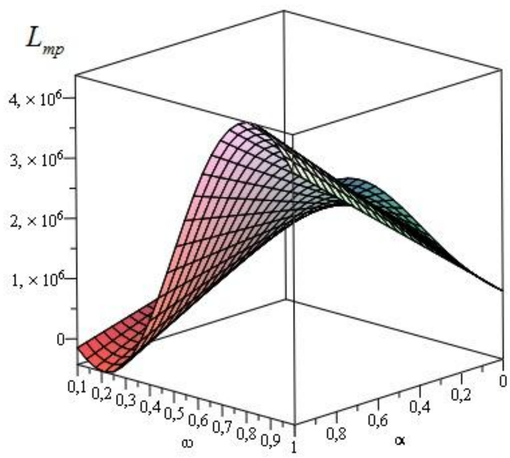

Рис. 6. Dependence of the friction force on parameter ω, characterizing the adapted profile of the bearing surface of bearing bush, and parameter *α*, characterizing the dependence of lubricant viscosity on pressure.

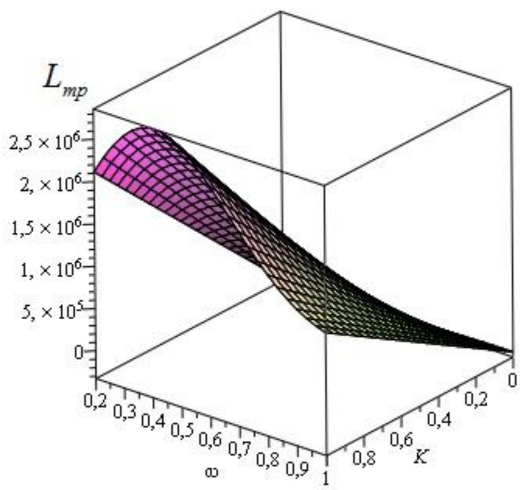

Рис. 7. Dependence of the friction force on parameter ω, characterizing the adapted profile of the bearing surface of bearing bush, and the heat parameter *K*.

8. CONCLUSIONS

1. 1. As a result of theoretical studies, the main regularities of the influence of viscosity characteristics of liquid lubricants on temperature and pressure in the lubricating layer were established.
2. It was established that the decrease in the thermal parameter *K*, the viscosity characteristics of the lubricant on pressure and temperature intensively increase.
3. The bearing capacity increases in case of decrease in the parameter α due to the dependence of viscosity on pressure.
4. The influence of the adapted profile of the bearing bush on the main bearing performance is characterized by the presence of maximum and minimum values at $\omega = \frac{\pi}{2}$ and $\omega = \frac{3\pi}{2}$, which, naturally, leads to the decrease of friction forces in these areas and to increase in the bearing capacity of the bearing at a maximum and vice versa at a minimum.

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