Global Journal of Pure and Applied Mathematics.
ISSN 0973-1768 Volume 13, Number 7 (2017), pp. 3543-3548

http://www.ripublication.com

Efficient Numerical Solution for Seventh Order Differential Equation by using Septic B-Spline Collocation Method with non-uniform length

Dr. Y. Rajashekhar Reddy

Department of mathematics, college of Engineering Jagatai, Nachupally (kondagattu), Jawaharlal Nehru Technological University, India.

Abstract

In this paper, recursive form of septic B-spline Collocation method is proposed for calculating numerical solution of seventh Order differential equation. Recursive form of B-spline is used with non uniform length whereas expanded form of B-spline is useful for uniform length only. The performance of the method is tested for without fixing the length of subintervals. The numerical result shows that the present method is a successful numerical technique to find solution for higher order differential equation with mixed boundary conditions.

Keywords: Septic B-Spline Collocation, Non-uniform length, Mixed boundary conditions, Maximum Relative Error

1. INTRODUCTION

B-spline collocation method is widely used method for solving ordinary differential equations with various types of boundary value problems. Many researchers [6] used this method to find solution of linear and nonlinear problems. This paper deals with the development of collocation method using recursive form of B-spline as base function for the numerical solution of seventh order ordinary linear differential equation with mixed boundary conditions. The seventh order linear differential equation with boundary conditions is given as

$$P_1(x)\frac{d^7U}{dx^7}+P2(x)\frac{d^6U}{dx^6}+P3(x)\frac{d^5U}{dx^5}+P_4(x)\frac{d^4U}{dx^4}+P5(x)\frac{d^3U}{dx^3}+P_6(x)\frac{d^2U}{dx^2}+P_7(x)\frac{dU}{dx}+P_8(x)U=Q(x)$$

$$x\in(a,b) \quad , \tag{1}$$

with the boundary conditions

i) $U(a)=d1, U(b)=d2 \;\; U'(a)=d3, U'(b)=d4, U''(a)=d5, U''(b)=d6, U'''(a)=d7$

where $a,b,d_1,d_2,d_3,d_4,d_5,d_6,d_7$ are constants. $P_1(x),P_2(x)$, $P_3(x),P_4(x)$, $P_5(x),P_6(x)$, $P_7(x),P_8(x),Q(x)$ are function of x.

2. THE NUMERICAL SCHEME

Let $[a,b]$ be the domain of the governing differential equation and is partitioned as $X=\{a=x_0,x_1,x_2 \ldots\ldots x_{n-1},x_n=b\}$ without any restriction on length of n sub domains. Let $N_i(x)$ be septic B- splines with the knots at the points x_i, i=0,1,…n. The set $\{N_{-7}, N_{-6}, N_{-5}, \ldots, N_6, N_7\}$ forms a basis for functions defined over [a,b].

Let $U^h(x)=\sum_{i=-7}^{n+7} C_i N_{i,p}(x)$

(2)

where C_i's are constants to be determined and $N_{i,p}(x)$ are the septic B-spline functions , be the approximate global solution to the exact solution $U(x)$ of the considered seventh order linear differential equation (1).

A zero degree and other than zero degree B-spline basis functions are defined at x_i recursively over the knot vector space $X=\{x_1,x_2,x_3 \ldots\ldots x_{n-1},x_n\}$ as

$i)\;\; if \;\; p=0$

$$N_{i,p}(x)=1 \qquad if \quad x\in(x_i,x_{i+i})$$

$$N_{i,p}(x)=0 \qquad if \quad x\notin(x_i,x_{i+i})$$

$ii)\;\; if \;\; p\geq 1$

$$N_{i,p}(x)=\frac{x-x_i}{x_{i+p}-x_i}N_{i,p-1}(x)+\frac{x_{i+p+1}-x}{x_{i+p+1}-x_{i+1}}N_{i+1,p-1}(x) \tag{3}$$

where p is the degree of the B-spline basis function and x is the parameter belongs to X .When evaluating these functions, ratios of the form 0/0 are defined as zero

Derivatives of B-splines

If p=7,
we have

$$N'_{i,p}(x) = \frac{x - x_i}{x_{i+p} - x_i} N'_{i,p-1}(x) + \frac{N_{i,p-1}(x)}{x_{i+p} - x_i} + \frac{x_{i+p+1} - x}{x_{i+p+1} - x_{i+1}} N'_{i+1,p-1}(x) - \frac{N_{i+1,p-1}(x)}{x_{i+p+1} - x_{i+1}}$$

$$N^{vii}{}_{i,p}(x) = 7\frac{N^{vi}{}_{i,p-1}(x)}{x_{i+p} - x_i} - 7\frac{N^{vi}{}_{i+1,p-1}(x)}{x_{i+p+1} - x_{i+1}} \quad \ldots\ldots\ldots \quad (4)$$

$$(U^h)^{vii}(x) = \sum_{i=-7}^{n+7} C_i N^{vii}{}_{i,p}(x) \quad \ldots\ldots\ldots\ldots \quad (5)$$

The x_i's are known as nodes, the nodes are treated as knots in collocation B-spline method where the B-spline basis functions are defined and these nodes are used to make the residue equal to zero to determine unknowns C_i's in (2).Seven extra knots are taken into consideration besides the domain of problem when evaluating the septic B-spline basis functions at the nodes which are within the considered domain.

Substituting the equations (2) and (5) in equation (1) for U (x) and derivatives of U (x).Then system of (n+1) linear equations are obtained in (n+8) constants. Applying the boundary conditions to equation (2) , seven more equations are generated in constants. Finally, we have (n+8) equations in (n+8) constants.

Solving the system of equations for constants and substituting these constants in equation (2) then assumed solution becomes the known approximation solution for equation (1) at corresponding the collocation points.

This is implemented using the Matlab programming.

3. NUMERICAL EXAMPLE

Consider the non homogeneous linear seventh order differential equation [1]

$\frac{d^7 y}{dx^7} - xy = e^x\,(x^2 - 2x - 6)$ with the boundary conditions

$$y(0)=1,\ y(1)=0, y'(0)=1,\ y'(1)=-e,\ y''(0)=-1, y''(1)=-2e,$$
$$y'''(0)=-2$$

The exact solution is $y=(1-x)\,e^x$

Table 2: Number of collocation points and max relative error

Number of collocation points	11	21	31	41
Max relative error	1.2903e-006	6.6459e-007	4.4688e-007	3.1832e-007

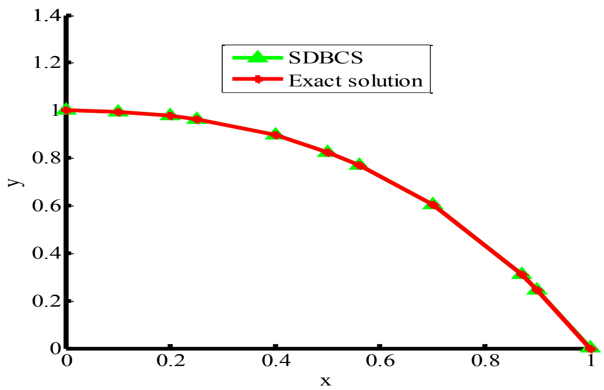

Figure 1: Comparison of Solutions SDBCS and Exact solution

Table 1: Solution values of SBCS and Exact solution

nodes	SDBCS	Exact solution
0	1.0000	1
.1	.9947	.9947
.2	.9771	.9771
.25	.9630	.9630
.4	.8951	.8951
.5	.8244	.8244
.56	.7703	7703
.6	.6041	.6041
.87	.3103	.3103
.9	.2460	.2460
1	1.000	1

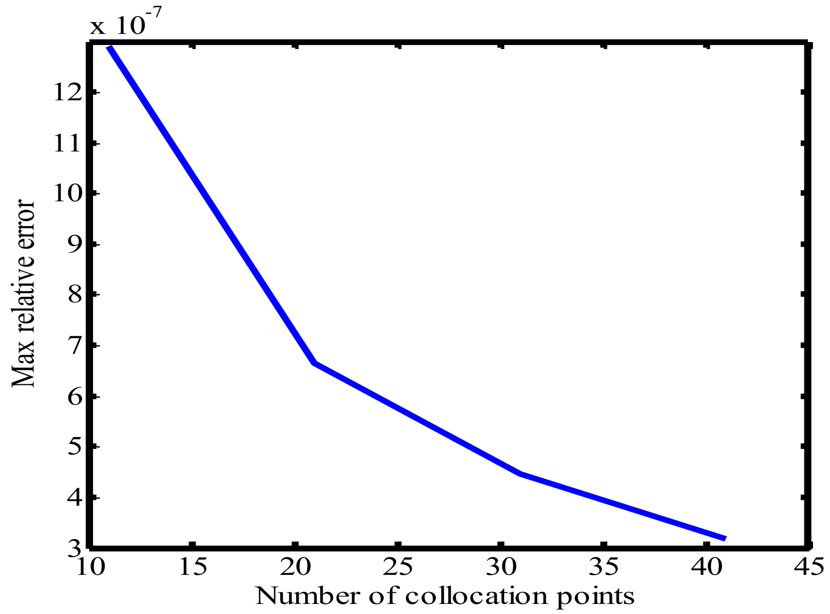

Figure 2: Number of collocation points and max relative error

CONCLUSION

The numerical results obtained by the proposed method are in good agreement with the exact solutions which are available in the literature. The strength of present septic B-spline collocation method lies in its easy applicability accurate and efficient to solve the seventh order linear differential equation.

REFERENCES

[1] I. J. SCHOENBERG, Contributions to the problem of approximation of equidistant data by analytic functions, Quart. Appl. Math. 4 (1946), 45-99; 112-141.

[2] H. B. CURRY AND I. J. SCHOENBERG, On spline distributions and their limits: the Polya distributions, Abstr. Bull. Amer. Math. Sot. 53 (1947), 1114.

[3] J.H.Ahlberg, E.N.Nielson and J.L.Walsh, The Theory of Splines and Their Applications, Academic Press, Newyork.1967.

[4] C.deBoor, A Practical Guide to Splines, Springer. 1978.

[5] K.H¨ollig, Finite Element Methods with B-Splines, The SIAM series on Frontiers in Applied Mathematics. 2003.

[6] P.Kalyani, M.N.Lemma. (2016) Solutions of Seventh Order Boundary Value Problems using Ninth Degree Spline Functions and Comparison with Eight Degree Spline Solutions. Journal of Applied Mathematics and Physics,4, 249-261.