Global Journal of Pure and Applied Mathematics.
ISSN 0973-1768 Volume 13, Number 7 (2017), pp. 3627-3639

https://dx.doi.org/10.37622/GJPAM/13.7.2017.3627-3639

Some Properties of θ-Semigeneralized Pre Continuous Functions

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Abstract

In this paper, we continue the further study of perception of θ-sgp-closed sets and introduce θ-sgp-continuous and θ-sgp-irresolute functions via θ-sgp-closed sets. Also discussed their relationship with other forms of continuous functions in topological spaces.

Keywords: θ-sgp-closed set, θ-sgp-continuous, θ-sgp-irresolute.
2010 Mathematics Subject Classification: 54C05

1. INTRODUCTION

Levine[13], in 1970 introduced the perception of g-closed sets in topological spaces and a class topological spaces called $T_{1/2}$-space. This perception of g-closed sets has been broadly studied in late years by many topologists. Bhattacharyya and Lahiri[4] investigated another concept to g-closed sets called sg-closed set in 1987. Dontchev and Maki[9] contributed some other new generalization of Levin's g-closed set by applying θ-closure operator called θ-g-closed set. T. Noiri et al.[14] introduced and studied gp-closed sets. Dontchev[8] and Gnanambal[10] introduced gsp-closed and gpr-closed sets in topological spaces respectively.

Continuous functions perform very essential aspect in general topology. The stronger and weaker forms of continuity have been brought out and analyzed by many topologists. In 1963, Levine[12] studied the weak forms of continuity called semi-continuity. Mashhour et al[15, 16], Rajamani et al[24] have introduced pre-continuity, α-continuity and αgs-continuity respectively, which are weaker forms of continuous functions.

Recently, the authors [21] have brought out and analyzed the notion of θ-sgp-closed sets. In this paper we will extend the study of θ-sgp-closed sets by all of introducing and characterizing θ-sgp-continuous and θ-sgp-irresolute functions.

2. PRELIMINARIES

In this paper, no separation axioms are taken up on the notations (X, τ) and (Y, σ) (or simply X and Y) which stands for topological spaces except clearly stated. If A ⊂ X, then Cl(A) and Int(A) stands for closure of A and the interior of A in X respectively.

The consequently definitions are useful in the continuation.

Definition 2.1: Let A ⊂ X. Then A is called

(i) a semi-open set [12] if A ⊆ Cl(Int(A)).

(ii) a pre-open set [15] if A ⊆ Int(Cl(A)).

Definition 2.2: A subset A of X is said to be pre-generalized closed(briefly pg-closed)[14] if pCl(A) ⊆ U whenever A ⊆ U and U is pre-open in X.

Definition 2.3: A space X is said to be:

(i) a pre-$T_{1/2}$ space[14] if every pg-closed set is pre-closed.

(ii) a pre-T_1 space[11] if, for x, y ∈ X such that x ≠ y, there exists a pair of preopen sets, one containing *x* but not *y*, and their other containing *y* but not *x*.

Definition 2.4: The union of all pre-θ-open sets of X contained in A is called pre-θ-interior [1] and is denoted by $pInt_\theta(A)$.

Definition 2.5: Let A ⊂ X. Then A is called

(i) g-closed set[13] if Cl(A) ⊆ U and U is open in X.

(ii) sg-closed set[4] if sCl(A) ⊆ U and U is semi-open in X. The complement of a sg-closed set is called a sg-open set.

(iii) sgp-closed[17](resp. gpr-closed[10], gp-closed[14]) sets if pCl(A) ⊆ U whenever A ⊆ U and U is semi-open(resp. regular open, open) in X.

(iv) αgs-closed set[23] if αCl(A) ⊂ U whenever A ⊂ U and U is semi-open in X.

(v) gsp-closed set[8] if spCl(A) ⊆ U whenever A ⊆ U and U is open in X.

(vi) θ-g-closed set[9] if $Cl_\theta(A)$ ⊆ U whenever A ⊆ U and U is open in X.

(vii) θ-gs-closed [19](resp. θ-sg-closed[6]) sets if $sCl_\theta(A)$ ⊂ U whenever A ⊂ U and U is open(resp. semi-open) in X.

Definition2.6: The pre-θ-closure denoted by $pCl_\theta(A)$, is the set of all pre-θ-cluster points of A. A subset A is called pre-θ-closed set [1] if A = $pCl_\theta(A)$. The complement of pre-θ-closed set is pre-θ-open set.

Definition 2.7: A function f : X → Y is called

(i) pre-semi-closed[25] if f(V) is semi-closed in Y for every semi-closed set V of X.

(ii) semi-continuous[12](resp. pg-continuous[20], α-continuous[16], gsp-continuous[8], gp-continuous[3], gpr-continuous[10], θ-g-continuous[9], αgs-continuous[24], sgp-continuous[17], θ-sg-continuous[6], θ-gs-continuous[18]) if $f^{-1}(V)$ is semi-closed(resp. pg-closed, α-closed, gsp-closed, gp-closed, gpr-closed, θ-g-closed, αgs-closed, sgp-closed, θ-sg-closed, θ-gs-closed) in X for every closed set V of Y.

(iii) pre-θ-continuous [2] if $f^{-1}(V)$ is a pre-θ-open set of X for every open set V of Y.

Definition 2.8: A function f: X → Y is said to be irresolute [7] if $f^{-1}(v) \in SO(X)$ for every $V \in SO(Y)$.

Lemma 2.9[5]: For any subset A of a topological space X, $pCl(A) \subset pCl_\theta(A)$.

Lemma 2.10[21]: Let A be a θ-sgp-closed subset of X. Then,

(i) $pCl_\theta(A) \setminus A$ does not contain a nonempty pre-closed set.

(ii) $pCl_\theta(A) \setminus A$ is θ-sgp-open.

Theorem 2.11[22]: For any $x \in X$, $x \in$ θ-sgpCl(A) if and only if $A \cap V \neq Ø$ for every θ-sgp-open set V containing x.

3. SOME MORE PROPERTIES OF θ-SEMIGENERALIZED PRE CLOSED SETS

We, recall the following definition.

Definition 3.1: A subset A of a topological space X is called θ-Semigeneralized pre-closed set[21] (briefly, θ-sgp-closed) if $pCl_\theta(A) \subset U$ whenever $A \subset U$ and U is semi-open in X.

The complement of θ-Semigeneralized pre-closed set is called θ-Semigeneralized pre-open (briefly, θ-sgp-open).

Theorem 3.2: Every θ-sgp-closed set is pg-closed set but conversely it is not true.

Proof: It is true that $pCl(A) \subset pCl_\theta(A)$ for every subset A of X.

Example 3.3: Let X = {a, b, c} and τ = {X, Ø, {a}, {c}, {a, c}}. Then a subset A = {a, b} is pg-closed set but not θ-sgp-closed set.

Theorem 3.4: A topological space X is a pre-$T_{1/2}$ space if and only if every θ-sgp-closed set is pre-closed.

Proof: Necessity. Let $A \subset X$ be a θ-sgp-closed set. By Lemma 3.2, A is pg-closed. Since X is a pre-$T_{1/2}$ space , A is pre-closed.

Sufficiency. Let $x \in X$. If $\{x\}$ is not pre-closed, then $\{x\}^c$ is not pre-open and thus the only superset of $\{x\}^c$ is X. Trivially $\{x\}^c$ is θ-sgp-closed. By hypothesis $\{x\}^c$ is pre-closed or equivalently $\{x\}$ is pre-open. Hence X is a pre-$T_{1/2}$ space.

Theorem 3.5: Let A be a pre-open subset of a space X. The set A is θ-sgp-closed set if and only if A is pg-closed set.

Proof: It is true that $pCl(A) \subset pCl_\theta(A)$ for every $A \in PO(X, \tau)$.

Theorem 3.6: A space X is a pre-T_1 if and only if every θ-sgp-closed set is pre-θ-closed.

Proof: Necessity. Let $A \subset X$ be a θ-sgp-closed set and $x \in pCl_\theta(A)$. Since X is pre-T_1, $\{x\}$ is pre-closed and thus by Lemma 2.10, $x \notin pCl_\theta(A) \setminus A$. Since $x \in pCl_\theta(A)$, then $x \in A$. This shows that $pCl_\theta(A) \subset A$ or equivalently A is pre-θ-closed.

Sufficiency. Let $x \in X$. Assume that $\{x\}$ is not pre-closed. Then $\{x\}^c$ is not pre-open but is θ-sgp-closed since the only pre-open superset of $\{x\}^c$ is X. By hypothesis $\{x\}^c$ is pre-θ-closed and thus $\{x\}$ is pre-θ-open. Since a singleton is pre-θ-open if and only if it pre-regular, $\{x\}$ is pre-regular.

Definition 3.7: A space X is called $T_{\theta sgp}$-space if all θ-sgp-closed set is semi-closed set.

Definition 3.8: A space X is called ${}_{\theta sgp}T_{\theta\text{-}g}$-space if all θ-sgp-closed set is θ-g-closed set.

Theorem 3.9: A space X is a $T_{\theta sgp}$-space if and only if every $\{x\}$ is either θ-g-closed or semi-open.

Proof: Necessity. Let $x \in X$ and assume that $\{x\}$ is not θ-g-closed. Then clearly $X\setminus\{x\}$ is not θ-g-open and $X\setminus\{x\}$ is trivially θ-g-closed. Since X is a $T_{\theta sgp}$-space, it is semi-closed. Therefore, $\{x\}$ is semi-open.

Sufficiency. Let $A \subset X$ be θ-sgp-closed. Let $x \in pCl_\theta(A)$. We will show that $x \in A$. Consider the subsequent two cases:

Case-(i): the set $\{x\}$ is θ-g-closed. Then if $x \notin A$, then $A \subset X - \{x\}$. As A is θ-sgp-closed and $X - \{x\}$ is θ-g-open, $pCl_\theta(A) \subset X - \{x\}$ and hence $x \notin pCl_\theta(A)$. This is a contradiction. Therefore, $x \in A$.

Case-(ii): the set $\{x\}$ is semi-open. Since $x \in pCl_\theta(A)$, then $\{x\} \cap A \neq \emptyset$. Thus $x \in A$. So, in both cases, $x \in A$. This shows that A is semi-closed.

4. θ-sgp-CONTINUOUS FUNCTIONS

In this section we introduced the conception of θ-sgp-continuous functions in topological spaces by utilizing θ-sgp-closed set and inspect some of their properties.

Definition 4.1: A function f: X → Y is called θ-Semigeneralized pre continuous function (briefly θ-sgp-continuous) if $f^{-1}(F)$ is θ-sgp-closed in X for every semi-closed set F of Y.

Remark 4.2: θ-sgp-continuous and continuous functions are independent of each other as examine from the consequently examples.

Example 4.3: Let X = Y = {a, b, c} and τ = {X, Ø, {a,b}}, σ = {Y, Ø, {a}, {b}, {a.b}, {a,c}} on X and Y respectively. Define f : X → Y by f(a) = b, f(b) = a, f(c) = c is θ-sgp-continuous function, but not a continuous function since $f^{-1}(\{a\}) = \{b\}$ is not an open set in X.

Example 4.4: Let X = Y = {a, b, c} and τ = {X, Ø, {a}, {c}, {a,c}, {b,c}}, σ = {Y, Ø, {a}, {c}, {a,c}} on X and Y respectively. Define identity function f : X → Y is continuous function but not θ-sgp-continuous function since $f^{-1}(\{c\}) = \{c\}$ is not θ-sgp-closed set in X.

Theorem 4.5: If f: X → Y is pre-θ-continuous, then f is θ-sgp-continuous but converse is not true.

Proof: Let F be a semi-closed set in Y. Then $f^{-1}(F)$ is pre-θ-closed set in X as f is pre-θ-continuous. Since every pre-θ-closed set is θ-sgp-closed set, $f^{-1}(F)$ is θ-sgp-closed set in X. Therefore f is a θ-sgp-continuous.

Example 4.6: Let X = Y = {a, b, c} and τ = {X, Ø, {a}, {b}, {a,b}, {a,c}}, σ = {Y, Ø, {a}, {c}, {a,c}} on X and Y respectively. Define f: X → Y by f(a) = a, f(b) = c, f(c) = b is θ-sgp-continuous but not pre-θ-continuous since $f^{-1}(\{b\}) = \{c\}$ is not pre-θ-closed set in X.

Theorem 4.7: If f: X → Y is θ-g-continuous, then f is θ-sgp-continuous but not conversely.

Proof: Let f: X → Y be a function. Let F be a closed set in Y then F be a semi-closed set in Y. Then $f^{-1}(F)$ is θ-g-closed set in X as f is a θ-g-continuous. Since every θ-g-closed set is θ-sgp-closed set, $f^{-1}(F)$ is θ-sgp-closed set in X. Therefore f is a θ-sgp-continuous.

Example 4.8: Let X = Y = {a, b, c} and τ = {X, Ø, {a,b}}, σ = {Y, Ø, {a}, {b}, {a.b}, {a,c}} on X and Y respectively. Define identity function f: X → Y is θ-sgp-continuous, but not θ-g-continuous since $f^{-1}(\{b\}) = \{b\}$ is not θ-g-closed set in X.

Remark 4.9: θ-sgp-continuous and θ-gs-continuous are independent of each other as examine from the following examples.

Example 4.10: Let X = Y = {a, b, c} and τ = {X, Ø, {a}, {b,c}}, σ = {Y, Ø, {a,b}} on X and Y respectively. Define f : X → Y by f(a) = c, f(b) = b, f(c) = a is θ-sgp-continuous, but not θ-gs-continuous since f^{-1}({c}) = {a} is not θ-gs-closed set in X.

Example 4.11: Let X = Y = {a, b, c} and τ = {X, Ø, {a}, {b}, {a,b}, {a,c}}, σ = {Y, Ø, {a,b}} on X and Y respectively. Define f : X → Y by f(a) = c, f(b) = b, f(c) = a is θ-gs-continuous, but not θ-sgp-continuous since f^{-1}({c}) = {a} is not θ-sgp-closed set in X.

Remark 4.12: The examples show that the conceptions of θ-sgp-continuous, α-continuous and αgs-continuous functions are independent.

Example 4.13: Let X = Y = {a, b, c} and τ = {X, Ø, {a,b}}, σ = {Y, Ø, {a}, {b}, {a.b}, {a,c}} on X and Y respectively. Define f : X → Y by f(a) = b, f(b) = a, f(c) = c is θ-sgp-continuous, but not α-continuous since f^{-1}({b}) = {a} is not α-closed set in X.

Example 4.14: Let X = Y = {a, b, c} and τ = {X, Ø, {a}, {c}, {a,c}, {b,c}}, σ = {Y, Ø, {a}, {c}, {a,c}} on X and Y respectively. Define identity function f : X → Y is α-continuous, but not θ-sgp-continuous since f^{-1}({c}) = {c} is not θ-sgp-closed set in X.

Example 4.15: Let X = Y = {a, b, c} and τ = {X, Ø, {a,b}}, σ = {Y, Ø, {a}, {b}, {a.b}, {a,c}} on X and Y respectively. Define f : X → Y by f(a) = b, f(b) = a, f(c) = c is θ-sgp-continuous but not αgs-continuous since f^{-1}({b}) = {a} is not αgs-closed set in X.

Example 4.16: Let X = Y = {a, b, c} and τ = {X, Ø, {a}, {c}, {a,c}, {b,c}}, σ = {Y, Ø, {a}, {c}, {a,c}} on X and Y respectively. Define identity function f : X → Y is αgs-continuous, but not θ-sgp-continuous since f^{-1}({c}) = {c} is not θ-sgp-closed set in X.

Remark 4.17: The concepts of θ-sgp-continuous and θ-sg-continuous are independent of each other as seen from the following examples.

Example 4.18: Let X = Y = {a, b, c} and τ = {X, Ø, {a}, {b,c}}, σ = {Y, Ø, {a.b}} on X and Y respectively. Define f : X → Y by f(a) = c, f(b) = b, f(c) = a is θ-sgp-continuous but not θ-sg-continuous since f^{-1}({c}) = {a} is not θ-sg-closed set in X.

Example 4.19: Let X = Y = {a, b, c} and τ = {X, Ø, {a}, {b}, {a,b}, {a,c}}, σ = {Y, Ø, {a,b}} on X and Y respectively. Define f : X → Y by f(a) = c, f(b) = b, f(c) = a is θ-sg-continuous but not θ-sgp-continuous since f^{-1}({c}) = {a} is not θ-sgp-closed set in X.

Theorem 4.20: If f: X → Y is θ-sgp-continuous, then f is gsp-continuous.

Proof: Let f: X → Y be a function. Let F be a closed set in Y then F be a semi-closed set in Y. Then $f^{-1}(F)$ is θ-sgp-closed set in X as f is a θ-sgp-continuous. Since every θ-sgp-closed set is gsp-closed set, $f^{-1}(F)$ is gsp-closed set in X. Therefore f is a gsp-continuous.

However, the converse need not be true as examine from the example.

Example 4.21: Let X = Y = {a, b, c} and τ = {X, Ø, {a}}, σ = {Y, Ø, {a,b}} on X and Y respectively. Define f : X → Y by f(a) = b, f(b) = a, f(c) = c is gsp-continuous but not θ-sgp-continuous since f^{-1}({c}) = {c} is not θ-sgp-closed set in X.

Theorem 4.22: If f: X → Y is θ-sgp-continuous, then f is sgp-continuous but not conversely.

Proof: Let f: X → Y be a function. Let F be a closed set in Y then F be a semi-closed set in Y. Then f^{-1}(F) is θ-sgp-closed set in X as f is a θ-sgp-continuous. Since every θ-sgp-closed set is sgp-closed set, f^{-1}(F) is sgp-closed set in X. Therefore f is a sgp-continuous.

Example 4.23: Let X = Y = {a, b, c} and τ = {X, Ø, {a}, {a,b}}, σ = {Y, Ø, {a,b}} on X and Y respectively. Define identity function f : X → Y is sgp-continuous but not θ-sgp-continuous since f^{-1}({c}) = {c} is not θ-sgp-closed set in X.

Theorem 4.24: If f : X → Y is θ-sgp-continuous, then f is gp-continuous but not conversely.

Proof: Let f : X → Y be a function. Let F be a closed set in Y then F be a semi-closed set in Y. Then f^{-1}(F) is θ-sgp-closed set in X as f is a θ-sgp-continuous. Since every θ-sgp-closed set is gp-closed set, f^{-1}(F) is gp-closed set in X. Therefore f is a gp-continuous.

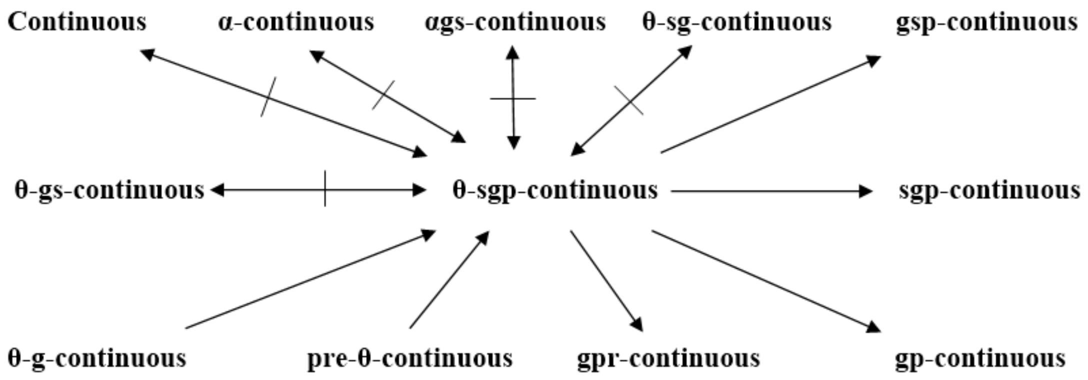
Example 4.25: Let X = Y = {a, b, c} and τ = {X, Ø, {a}, {c}, {a,c}}, σ = {Y, Ø, {a,b}} on X and Y respectively. Define f : X → Y by f(a) = a, f(b) = c, f(c) = b is gp-continuous but not θ-sgp-continuous since f^{-1}({c}) = {b} is not θ-sgp-closed set in X.

Theorem 4.26: If f : X → Y is θ-sgp-continuous, then f is gpr-continuous but not conversely.

Proof: Let f : X → Y be a function. Let F be a closed set in Y then F be a semi-closed set in Y. Then f^{-1}(F) is θ-sgp-closed set in X as f is a θ-sgp-continuous. Since every θ-sgp-closed set is gpr-closed set, f^{-1}(F) is gpr-closed set in X. Therefore f is a gpr-continuous.

Example 4.27: Let X = Y = {a, b, c} and τ = {X, Ø, {a}, {b}, {a,b}, {a,c}}, σ = {Y, Ø, {a}, {c}, {a,c}} on X and Y respectively. Define identity function f: X → Y is gpr-continuous but not θ-sgp-continuous since f^{-1}({a}) = {a} is not θ-sgp-closed set in X.

Remark 4.28: The **"Implication Diagram"** about θ-sgp-continuous function.



where A $\longrightarrow$ B (resp. A $\longleftrightarrow$ B) shows A implies B but not conversely (resp. A and B are independent).

Theorem 4.29: If f: X → Y be θ-sgp-continuous and X is $T_{\theta sgp}$-space, then f is semi-continuous.

Proof: Consider A be a closed subset of Y then A be a semi-closed set in Y. Then $f^{-1}(A)$ is θ-sgp-closed set in X as f is a θ-sgp-continuous. Since X is $T_{\theta sgp}$-space, so $f^{-1}(A)$ is semi- closed. Therefore f is semi-continuous.

Theorem 4.30: If f: X → Y be θ-sgp-continuous and X is ${}_{\theta sgp}T_{\theta\text{-}g}$-space, then f is θ-g-continuous.

Proof: Let A be a closed subset of Y then A be a semi-closed set in Y. Then $f^{-1}(A)$ is θ-sgp-closed set in X as f is a θ-sgp-continuous. Since X is ${}_{\theta sgp}T_{\theta\text{-}g}$-space, so $f^{-1}(A)$ is θ-g-closed. Therefore f is θ-g-continuous.

Theorem 4.31: A function f: X → Y be θ-sgp-continuous if and only if, for all semi-open set A of Y, $f^{-1}(A)$ is θ-sgp-open in X.

Proof: Necessity. Let f: X → Y be θ-sgp-continuous and A be an semi-open set in Y. Then Y – A is semi-closed in Y and since f is θ-sgp-continuous, $f^{-1}(Y - A)$ is θ-sgp-closed in X. But $f^{-1}(Y - A) = X - f^{-1}(A)$ and so that $f^{-1}(A)$ is θ-sgp-open in X.

Sufficiency. Assume that $f^{-1}(A)$ is θ-sgp-open in X for each semi-open set A in Y. Let F be a semi-closed set in Y. Then Y – F is semi-open in Y and by assumption $f^{-1}(Y - F)$ is θ-sgp-open in X, since $f^{-1}(Y – F) = X - f^{-1}(F)$, we have $f^{-1}(F)$ is θ-sgp-closed in X and so f is θ-sgp-continuous.

Theorem 4.32: If f: X → Y is θ-sgp-continuous and g: Y → Z is semi-continuous, then their composition gof : X → Z is θ-sgp-continuous.

Proof: Consider F be any closed set in Z. Since g is semi-continuous, $g^{-1}(F)$ is semi-closed in Y. Since f is θ-sgp-continuous and $g^{-1}(F)$ is semi-closed in Y. But $f^{-1}(g^{-1}(F)) = (gof)^{-1}(F)$ is θ-sgp-closed in X and gof is θ-sgp-continuous.

Theorem 4.33: Let f : X → Y be a function.

(a) The subsequent statements are identical:

(i) f is θ-sgp-continuous.

(ii) The inverse image of every semi-open set in Y is θ-sgp-open in X.

(b) If f: X → Y is θ-sgp-continuous, then f(θ-sgp-Cl(A)) ⊂ Cl(f(A)) for every subset A of X.

(c) The following statements are equivalent:

(i) For each x ∈ X and each semi-open set V containing f(x) there exists a θ-sgp-open set U
containing x such that f(U) ⊂ V.

(ii) For every subset A of X, f(θ-sgp-Cl(A)) ⊂ Cl(f(A)).

Proof: (a) Straightforward.

(b) Let A ⊂ X. Then Cl(f(A)) is closed in Y. Since f is θ-sgp-continuous therefore $f^{-1}(Cl(A))$ is θ-sgp-closed in X. Now A ⊂ $f^{-1}(f(A))$ ⊂ $f^{-1}(Cl(f(A)))$, we obtain θ-sgp-Cl(A) ⊂ $f^{-1}(Cl(f(A)))$ and then f(θ-sgp-Cl(A)) ⊂ f ($f^{-1}(Cl(f(A)))$) ⊂ Cl(f(A)).

(c) (i) ⇒ (ii): Let y ∈ f(θ-sgp-Cl(A)) and let V be any semi-open neighbourhood of y. Then there exist x ∈ X and θ-sgp-open set U such that f(x) = y, x ∈ U, x ∈ θ-sgp-Cl(A) and f(U) ⊂ V. By Theorem 2.12[12], U ∩ A ≠ Ø and hence f(A) ∩ V ≠ Ø. Hence y = f(x) ∈ Cl(f(A)).

(ii) ⇒ (i): Let x ∈ X and V be any semi-open set containing f(x). Let A = $f^{-1}(Y \setminus V)$. Since f(θ-sgp-Cl(A)) ⊂ Cl(f(A)) ⊂ Y \ V, then θ-sgp-Cl(A) = A. Since x ∉ θ-sgp-Cl(A) , there exists a θ-sgp-open set U containing x such that U ∩ A = Ø and hence f(U) ⊂ f(X \ A) ⊂ V.

Theorem 4.34: Every restriction of θ-sgp-continuous is θ-sgp-continuous.

Proof: Let f : X → Y be θ-sgp-continuous and A be any subset of X. For any semi-open subset S of Y, $(f/A)^{-1}(S)$ = A ∩ $f^{-1}(S)$. But f being θ-sgp-continuous, $f^{-1}(S)$ is θ-sgp-open in X and hence A ∩ $f^{-1}(S)$ is relatively θ-sgp-open subset of A. That is, $(f/A)^{-1}(S)$ is θ-sgp-open subset of A. Hence (f/A) is θ-sgp-continuous.

5. θ-sgp-IRRESOLUTE FUNCTIONS

Definition 5.1: A function f : X → Y is called θ-Semigeneralized pre irresolute (briefly θ-sgp-irresolute) if $f^{-1}(F)$ is θ-sgp-closed in X for every θ-sgp-closed set F of Y.

Remark 5.2: The irresolute and θ-sgp-irresolute functions are independent of each other as seen from the following examples.

Example 5.3: Let X = Y = {a, b, c} and τ = {X, Ø, {a}, {b}, {a,b}, {a,c}}, σ = {Y, Ø, {a,b}} on X and Y respectively. Define identity function f : X → Y is irresolute function but not θ-sgp-irresolute function since $f^{-1}(\{a\})$ = a is not θ-sgp-closed set in X.

Example 5.4: Let X = Y = {a, b, c} and τ = {X, Ø, {a}, {b,c}}, σ = {Y, Ø, {a}, {b}, {a,b}, {a,c}} on X and Y respectively. Define f : X → Y by f(a) = a, f(b) = c, f(c) = b is θ-sgp-irresolute function but not irresolute function since $f^{-1}(\{b\})$ = c is not semi-open in X.

Theorem 5.5: A function f : X → Y θ-sgp-irresolute if and only if, for every θ-sgp-open A of Y, $f^{-1}(A)$ is θ-sgp-open in X.

Proof: Necessity. If f : X → Y be θ-sgp-irresolute, then for every θ-sgp-closed B of Y, $f^{-1}(B)$ is θ-sgp-closed in X. If A is any θ-sgp-open subset of Y, then Y – A is θ-sgp-closed. Thus $f^{-1}(Y – A)$ is θ-sgp-closed, but $f^{-1}(Y – A) = X - f^{-1}(A)$ so that $f^{-1}(A)$ is θ-sgp-open in X.

Sufficiency. If for all θ-sgp-open subsets A of Y, $f^{-1}(A)$ is θ-sgp-open in X and if B is any θ-sgp-closed subset of Y, then Y – B is θ-sgp-open. Also $f^{-1}(Y – B) = X - f^{-1}(B)$ is θ-sgp-open. Thus $f^{-1}(B)$ is θ-sgp-closed.

Remark 5.6: The θ-sgp-irresolute function and θ-sgp-continuous function are independent of each other as seen from the following examples.

Example 5.7: Let X=Y={a, b, c} and τ = {X, Ø, {a}, {b}, {a,b},{a,c}}, σ = {Y, Ø, {a,b}} on X and Y respectively. Define identity function f : X → Y is θ-sgp-continuous but not θ-sgp-irresolute since $f^{-1}(\{a\})$ = a is not θ-sgp-closed set in X.

Example 5.8: Let X=Y={a, b, c} and τ = {X, Ø, {a}, {b}, {a,b},{a,c}}, σ = {Y, Ø, {a}, {c}, {a,c}} on X and Y respectively. Define f : X → Y by f(a) = b, f(b) = a, f(c) = c is θ-sgp-irresolute but not θ-sgp-continuous since $f^{-1}(\{b\})$ = {a} is not θ-sgp-closed set in X.

Remark 5.9: If f: X → Y is θ-sgp-irresolute and X is $T_{\theta sgp}$-space then it is θ-sgp-continuous.

Theorem 5.10: Let X, Y and Z be any topological spaces and f: X → Y and g : Y → Z be two functions. Then the following are true:

(i) If f and g are θ-sgp-irresolute, then their composition gof : $X \to Z$ is θ-sgp-irresolute.

(ii) If f is θ-sgp-irresolute and g is θ-sgp-continuous, then their composition gof : $X \to Z$ is θ-sgp-continuous.

(iii) If f is irresolute and g is θ-sgp-continuous and Y is a $T_{\theta sgp}$-space, then their composition gof : $X \to Z$ is semi-continuous.

Proof: (i) Let $A \subset Z$ is θ-sgp-open set then $g^{-1}(F)$ is θ-sgp-open and $f^{-1}(g^{-1}(F))$ is θ-sgp-open since g and f are θ-sgp-irresolute functions. But $f^{-1}(g^{-1}(F)) = (gof)^{-1}(F)$ is θ-sgp-open. Therefore gof is θ-sgp-irresolute.

(ii) Let V be an semi-open set in Z. Then $g^{-1}(V)$ is θ-sgp-open in Y as g is θ-sgp-continuous. Since f is θ-sgp-irresolute, $f^{-1}(g^{-1}(V))$ is θ-sgp-open in X. But $f^{-1}(g^{-1}(F)) = (gof)^{-1}(A)$. Therefore gof is θ-sgp-continuous.

(iii) Let V be closed set in Z. Therefore it is semi-closed set in Z. Then $g^{-1}(V)$ is θ-sgp-closed in Y, since g is θ-sgp-continuous. As Y is a $T_{\theta sgp}$-space, $g^{-1}(V)$ is semi-closed in Y. Irresoluteness of f implies that $f^{-1}(g^{-1}(V))$ is semi-closed in X. Hence gof is semi-continuous.

Theorem 5.11: If f: $X \to Y$ is bijective, open and θ-sgp-continuous, then f is θ-sgp-irresolute.

Proof: Let $V \in \theta SGPC(Y)$ and let $f^{-1}(V) \subset U$ where $U \in \tau$. Clearly $V \subset f(U)$. Since $f(U) \in \sigma$ and since $V \in \theta SGPC(Y)$, then $\theta sgpCl(V) \subset f(U)$ and thus $f^{-1}(\theta sgpCl(V)) \subset U$. Since f is θ-sgp-continuous and since $\theta sgpCl(V)$ is closed in Y, then $\theta sgpCl(V) \subset f^{-1}(\theta sgpCl(V)) \subset U$ and hence $\theta sgpCl(f^{-1}(V)) \subset U$. Therefore, $f^{-1}(V) \in \theta SGPC(X)$. Hence f is θ-sgp-irresolute.

Theorem 5.12: Let f : $X \to Y$ be a pre-semi-closed and θ-sgp-irresolute surjection. If X is a $T_{\theta sgp}$-space, then Y is also a $T_{\theta sgp}$-space.

Proof: Let F be any θ-sgp-closed set in Y. Since f is θ-sgp-irresolute, $f^{-1}(F)$ is θ-sgp-closed set in X. Since X is $T_{\theta sgp}$-space, $f^{-1}(F)$ is semi-closed in X and hence $f(f^{-1}(F)) = F$ is semi-closed in Y. This shows that Y is also a $T_{\theta sgp}$-space.

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