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Common Random Fixed Point Theorems under Contraction of rational Type in Multiplicative Metric Space

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Abstract

This is the paper inspired and yeasted by [1], we obtain common random fixed point theorems satisfying contractive mapping in the setup of multiplicative metric space, and additionally we are going to prove compatible and weak compatible mappings again under rational contractive maps to prove the common random fixed point theorems in multiplicative metric space.

Keywords. Random fixed point, compatible mapping, weak compatible mapping, implicit function, common fixed point, multiplicative metric space.
***AMS Subject Classification**:* 46S40, 47H10, 54H25

I. INTRODUCTION

The initiation and research in the direction of random approximation and random fixed point theorems was introduced by the Prague School of probabilistic in 1950s. since then, a lot of efforts have been devoted to random fixed point theory and applications see(5-14], [18]).Whereas In 1976 , some sufficient conditions have been given by Bharucha Reid[14] for a stochastic analogue of Schauder's fixed point theorem which was defined in a separable metric space. One of the powerful tools in current mathematical applications that we have are fixed point theorems. Wide class of problems of mechanics, physics *etc*. have been solved by the extended and generalized fixed point theorems. Many fixed point theorems have been proved in [references].

In 2008, it was found that the set of all positive real numbers is not complete with respect to usual metric by Bashirov et al. [4], which was overcome by the introduction of the concept of multiplicative metric space. Many authors Abbas [2], kang et al. [15], Sarvar and Badshah-e [17], have been proved many fixed point theorems also they tried to give more results on multiplicative metric space. Beg et al. [5, 8, 10] studied the interrelation of common random fixed points and random coincidence point of compatible random operator and proved the random fixed points for random operators.

II. PRELIMINARIES

Definition 1.1. ([3]) Let X be a nonempty set. A multiplicative metric is a mapping $\Omega: X \times X \to \mathbb{R}_+$ satisfying the following conditions:

(1.1) $\Omega(x,y) \geq 1 \ \forall\, x,y \in X$ and $\Omega(x,y)=1$ if and only if $x=y$;

(1.2) $\Omega(x,y) = \Omega(y,x) \ \forall\, x,y \in X$;

(1.3) $\Omega(x,y) \leq \Omega(x,z).\, \Omega(z,y) \ \forall\, x,y \in X$ (multiplicative triangle inequality).

Then the mapping Ω together with X, *i.e.* (X, Ω) is a multiplicative metric space.

Definition 1.2. ([15]) Let (X,d) be a multiplicative metric space and $F: \mathbb{R} \times X \to X$ be a function, where X is a non empty set. Then function $g: \mathbb{R} \to X$ is said to be a *random common fixed point* of the function F is $F(t,g(t))=g(t)$ for all $t \in R$.

Definition 1.3. ([15]) Let (X, Ω) be a multiplicative metric space. Then a sequence $\{x_n\}$ in X is said to be

(1.4) a *multiplicative convergent* to x if for every multiplicative open ball $B_\varepsilon(x)=\{ y \mid \Omega(x,y) \leq \varepsilon, \varepsilon > 1\}$, there exists a natural number N such that $n \geq N$, *then* $x_n \in B_\varepsilon(x)$, *i.e.* $\Omega(x_n, x) \to 1$ as $n \to \infty$.

(1.5) *a multiplicative Cauchy sequence* if $\forall \varepsilon > 1$ for all m, n >N, that is, $\Omega(x_n, x_m) \to 1$ as $n \to \infty$.

(1.6) we call a multiplicative metric space *complete* if every multiplicative Cauchy sequence in it is multiplicative convergent to $x \in X$.

(1.7) Let (X,d) be a multiplicative metric space. Two mappings $\alpha, \gamma: \mathbb{R} \times X \to X$ are said to be

a. *Compatible* for each t$\in \mathbb{R}$ if $\lim_{n\to\infty} \Omega(\alpha(t,\gamma(t, \vartheta_n(t))),\gamma(t,\alpha(t, \vartheta_n(t)))=1$,
Whenever $\lim_{n\to\infty} \alpha\left(t,\gamma(t, \vartheta_n(t))\right)$, $\lim_{n\to\infty} \gamma\left(t,\alpha(t, \vartheta_n(t))\right)$ exist in *X and* $\lim_{n\to\infty} \alpha\left(t,\gamma(t, \vartheta_n(t))\right)=\lim_{n\to\infty} \gamma\left(t,\alpha(t, \vartheta_n(t))\right)$, $t \in \mathbb{R}$, where $\{\vartheta_n\}$ is a sequence of mappings.

b. *Weakly compatible* for each t$\in \mathbb{R}$ if $\gamma(t,\vartheta(t)) = \alpha(t,\vartheta(t))$ for some mapping ϑ implies $\gamma\left(t,\alpha(t,\vartheta(t))\right) = \alpha\left(t,\gamma(t,\vartheta(t))\right)$ for every t$\in \mathbb{R}$.

III. MAIN RESULTS

Now we prove the following theorems for compatible and weakly compatible mappings satisfying implicit function in a multiplicative metric space.

Theorem 2.1. *Let us consider complete multiplicative metric space (X, Ω) and a function Φ:* $\mathbb{R}$ x *X→X be a function satisfying the following conditions*

$$(E1)\quad \Omega(\Phi(t,x),\Phi(t,y)) \le \left\{ \begin{array}{c} \frac{\Omega(y,\Phi(t,x))}{\Omega(\Phi(t,x),\Phi(t,y))}\cdot\left[\frac{\Omega(y,\Phi(t,y))+\Omega(x,\Phi(t,y))}{1+\Omega(x,y)}\right]. \\ \left[\frac{\Omega(x,y)\Omega(y,\Phi(t,x))+\Omega(\Phi(t,y),x)\Omega(y,\Phi(t,x))}{\Omega(y,\Phi(t,x))+\Omega(\Phi(t,x),\Phi(t,y))}\right]. \\ \left[\frac{\Omega(x,y)+\Omega(\Phi(t,x),\Phi(t,y))}{\Omega(x,\Phi(t,x))+\Omega(y,\Phi(t,y))}\right]\left[\frac{\Omega(x,\Phi(t,y))+\Omega(x,\Phi(t,x))}{1+\Omega(\Phi(t,x),\Phi(t,y))}\right] \end{array} \right\}^{\frac{\lambda}{2}}$$

For all x, y ∈ X, where $t \in \mathbb{R}$ *and* $\lambda \in [0,1)$.
Then Φ have a unique random fixed point.

Proof. A sequence $\{\vartheta_n\}$can be defined as $\vartheta_n(t) = \Phi(t, \vartheta_{n-1}(t))$, where ϑ_n is a arbitrary function defined as $\vartheta_n : \mathbb{R} \to X$ for $t \in \mathbb{R}$ *and n=0,1,2,3…*

If we define $\vartheta_{n+1}(t) = \vartheta_n(t)$ for some n and $t \in \mathbb{R}$, *then* $\vartheta_n(t)$ is a random fixed point

Let $\vartheta_{n+1}(t) \neq \vartheta_n(t)$ for each n and $t \in \mathbb{R}$ then for all $t \in \mathbb{R}$,substituting x = $\vartheta_{n-1}(t)$ and y = $\vartheta_n(t)$ in *(E1)* we have

$\Omega(\vartheta_n(t), \vartheta_{n+1}(t))= \Omega\left(\Phi(t, \vartheta_{n-1}(t)),\Phi(t, \vartheta_n(t))\right)$

$$\leq \left\{ \begin{array}{c} \frac{\Omega(\vartheta_n(t), \Phi(t, \vartheta_{n-1}(t)))}{\Omega(\Phi(t, \vartheta_{n-1}(t)), \Phi(t, \vartheta_n(t)))} \cdot \\ \left[\frac{\Omega(\vartheta_n(t), \Phi(t, \vartheta_n(t))) + \Omega(\vartheta_{n-1}(t), \Phi(t, \vartheta_n(t)))}{1 + \Omega(\vartheta_{n-1}(t), \vartheta_n(t))}\right] \cdot \\ \left[\frac{\Omega(\vartheta_{n-1}(t), \vartheta_n(t)) . \Omega(\vartheta_n(t), \Phi(t, \vartheta_{n-1}(t))) + \Omega(\Phi(t, \vartheta_n(t)), \vartheta_{n-1}(t)) . \Omega(\vartheta_n(t), \Phi(t, \vartheta_{n-1}(t)))}{\Omega(\vartheta_n(t), \Phi(t, \vartheta_{n-1}(t))) + \Omega(\Phi(t, \vartheta_{n-1}(t)), \Phi(t, \vartheta_n(t)))}\right] \cdot \\ \left[\frac{\Omega(\vartheta_{n-1}(t), \vartheta_n(t)) + \Omega(\Phi(t, \vartheta_{n-1}(t)), \Phi(t, \vartheta_n(t)))}{\Omega(\vartheta_{n-1}(t), \Phi(t, \vartheta_{n-1}(t))) + \Omega(\vartheta_n(t), \Phi(t, \vartheta_n(t)))}\right] \cdot \\ \left[\frac{\Omega(\vartheta_{n-1}(t), \Phi(t, \vartheta_n(t))) + \Omega(\vartheta_{n-1}(t), \Phi(t, \vartheta_{n-1}(t)))}{1 + \Omega(\Phi(t, \vartheta_{n-1}(t)), \Phi(t, \vartheta_n(t)))}\right] \end{array} \right\}^{\frac{\lambda}{2}}$$

$$\leq$$

$$\left\{ \begin{array}{c} \frac{\Omega(\vartheta_n(t), \vartheta_n(t))}{\Omega(\vartheta_n(t), \vartheta_{n+1}(t))} \cdot \\ \left[\frac{\Omega(\vartheta_n(t), \vartheta_{n+1}(t)) + \Omega(\vartheta_{n-1}(t), \vartheta_{n+1}(t))}{1 + \Omega(\vartheta_{n-1}(t), \vartheta_n(t))} \cdot\right] \\ \left[\frac{\Omega(\vartheta_{n-1}(t), \vartheta_n(t)) . \Omega(\vartheta_n(t), \vartheta_n(t)) + \Omega(\vartheta_{n-1}(t), \vartheta_{n+1}(t)) . \Omega(\vartheta_n(t), \vartheta_n(t))}{\Omega(\vartheta_n(t), \vartheta_n(t)) + \Omega(\vartheta_{n+1}(t), \vartheta_n(t))}\right] \cdot \\ \left[\frac{\Omega(\vartheta_{n-1}(t), \vartheta_n(t)) + \Omega(\vartheta_n(t), \vartheta_{n+1}(t))}{\Omega(\vartheta_{n-1}(t), \vartheta_n(t)) + \Omega(\vartheta_n(t), \vartheta_{n+1}(t))}\right] \cdot \\ \left[\frac{\Omega(\vartheta_{n-1}(t), \vartheta_{n+1}(t)) + \Omega(\vartheta_{n-1}(t), \vartheta_n(t))}{1 + \Omega(\vartheta_n(t), \vartheta_{n+1}(t))}\right] \end{array} \right\}^{\frac{\lambda}{2}}$$

$$\leq \left\{ \begin{array}{c} \frac{1}{\Omega(\vartheta_n(t), \vartheta_{n+1}(t))} \cdot \\ \left[\frac{\Omega(\vartheta_n(t), \vartheta_{n+1}(t)) + \Omega(\vartheta_{n-1}(t), \vartheta_n(t)) \Omega(\vartheta_n(t), \vartheta_{n+1}(t))}{1 + \Omega(\vartheta_{n-1}(t), \vartheta_n(t))}\right] \\ \left[\frac{\Omega(\vartheta_{n-1}(t), \vartheta_n(t)) + \Omega(\vartheta_{n-1}(t), \vartheta_n(t)) \Omega(\vartheta_n(t), \vartheta_{n+1}(t))}{1 + \Omega(\vartheta_{n+1}(t), \vartheta_n(t))}\right] \cdot \\ \left[\frac{\Omega(\vartheta_{n-1}(t), \vartheta_n(t)) + \Omega(\vartheta_n(t), \vartheta_{n+1}(t))}{\Omega(\vartheta_{n-1}(t), \vartheta_n(t)) + \Omega(\vartheta_n(t), \vartheta_{n+1}(t))}\right] \cdot \\ \left[\frac{\Omega(\vartheta_{n-1}(t), \vartheta_n(t)) . \Omega(\vartheta_n(t), \vartheta_{n+1}(t)) + \Omega(\vartheta_{n-1}(t), \vartheta_n(t))}{1 + \Omega(\vartheta_n(t), \vartheta_{n+1}(t))}\right] \end{array} \right\}^{\frac{\lambda}{2}}$$

$$\leq \{\Omega^2(\vartheta_{n-1}(t), \vartheta_n(t))\}^{\frac{\lambda}{2}}$$

hence, we have

$$\begin{aligned} \Omega(\vartheta_n(t), \vartheta_{n+1}(t)) &\leq \Omega^{\lambda}(\vartheta_{n-1}(t), \vartheta_n(t)) \\ &\leq \Omega^{\lambda^2}(\vartheta_{n-2}(t), \vartheta_{n-1}(t)) \\ &\leq \Omega^{\lambda^3}(\vartheta_{n-3}(t), \vartheta_{n-2}(t)) \leq \cdots \leq \Omega^{\lambda^n}(\vartheta_0(t), \vartheta_1(t)) \end{aligned}$$

For n, m ∈ ℕ having m<n, we have

$$\Omega(\vartheta_m(t), \vartheta_n(t)) \leq$$
$$\Omega(\vartheta_m(t), \vartheta_{m+1}(t)).\Omega(\vartheta_{m+1}(t), \vartheta_{m+2}(t)) \dots \Omega(\vartheta_{n-1}(t), \vartheta_n(t))$$
$$\leq$$
$$\Omega^{\lambda^m}(\vartheta_0(t), \vartheta_1(t)).\Omega^{\lambda^{m+1}}(\vartheta_0(t), \vartheta_1(t)) \dots \Omega^{\lambda^{n-1}}(\vartheta_0(t), \vartheta_1(t))$$
$$\leq \Omega^{\frac{\lambda^m}{1-\lambda}}(\vartheta_0(t), \vartheta_1(t))$$

Taking n approaches to infinity, $\Omega(\vartheta_m(t), \vartheta_n(t)) \to 1$. This implies that $\{\vartheta_m(t)\}$ is a multiplicative Cauchy sequence. Since the space is complete it implies that the convergent point (say $\vartheta(t)$) belongs to *X.*

now it is to be claim that $\Phi(t, \vartheta(t)) = \vartheta(t)$, substituting *x*= $\vartheta(t)$ and *y*= $\vartheta_n(t)$ in *(E1)*

$$\Omega(\Phi(t,\vartheta(t)), \vartheta_{n+1}(t)) \leq \Omega(\Phi(t,\vartheta(t)), \Phi(t, \vartheta_n(t)))$$
$$\leq$$
$$\left\{\begin{array}{c} \frac{\Omega(\vartheta_n(t),\Phi(t,\vartheta(t)))}{\Omega(\Phi(t,\vartheta(t)),\Phi(t,\vartheta_n(t)))}. \\ \left[\frac{\Omega(\vartheta_n(t),\Phi(t,\vartheta_n(t)))+\Omega(\vartheta(t),\Phi(t,\vartheta_n(t)))}{1+\Omega(\vartheta(t),\vartheta_n(t))}\right]. \\ \left[\frac{\Omega(\vartheta(t),\vartheta_n(t))\Omega(\vartheta_n(t),\Phi(t,\vartheta(t)))+\Omega(\Phi(t,\vartheta_n(t)),\vartheta(t))\Omega(\vartheta_n(t),\Phi(t,\vartheta(t)))}{\Omega(\vartheta_n(t),\Phi(t,\vartheta(t)))+\Omega(\Phi(t,\vartheta(t)),\Phi(t,\vartheta_n(t)))}\right]. \\ \left[\frac{\Omega(\vartheta(t),\vartheta_n(t))+\Omega(\Phi(t,\vartheta(t)),\Phi(t,\vartheta_n(t)))}{\Omega(\vartheta(t),\Phi(t,\vartheta(t)))+\Omega(\vartheta_n(t),\Phi(t,\vartheta_n(t)))}\right]. \\ \left[\frac{\Omega(\vartheta(t),\Phi(t,\vartheta_n(t)))+\Omega(\vartheta(t),\Phi(t,\vartheta(t)))}{1+\Omega(\Phi(t,\vartheta(t)),\Phi(t,\vartheta_n(t)))}\right] \end{array}\right\}^{\frac{\lambda}{2}}$$

$$\leq$$

$$\left\{\begin{array}{c} \frac{\Omega(\vartheta_n(t),\Phi(t,\vartheta(t)))}{\Omega(\Phi(t,\vartheta(t)),\vartheta_{n+1}(t))}. \\ \left[\frac{\Omega(\vartheta_n(t),\vartheta_{n+1}(t))+\Omega(\vartheta(t),\vartheta_{n+1}(t))}{1+\Omega(\vartheta(t),\vartheta_n(t))}\right]. \\ \left[\frac{\Omega(\vartheta(t),\vartheta_n(t))\Omega(\vartheta_n(t),\Phi(t,\vartheta(t)))+\Omega(\vartheta_{n+1}(t),\vartheta(t))\Omega(\vartheta_n(t),\Phi(t,\vartheta(t)))}{\Omega(\vartheta_n(t),\Phi(t,\vartheta(t)))+\Omega(\Phi(t,\vartheta(t)),\vartheta_{n+1}(t))}\right]. \\ \left[\frac{\Omega(\vartheta(t),\vartheta_n(t))+\Omega(\Phi(t,\vartheta(t)),\vartheta_{n+1}(t))}{\Omega(\vartheta(t),\Phi(t,\vartheta(t)))+\Omega(\vartheta_n(t),\vartheta_{n+1}(t))}\right]. \\ \left[\frac{\Omega(\vartheta(t),\vartheta_{n+1}(t))+\Omega(\vartheta(t),\Phi(t,\vartheta(t)))}{1+\Omega(\Phi(t,\vartheta(t)),\vartheta_{n+1}(t))}\right] \end{array}\right\}^{\frac{\lambda}{2}}$$

Taking n approaches to infinity, we have

$$\Omega\big(\Phi(t,\vartheta(t)),\vartheta(t)\big) \leq \left\{\begin{array}{c} \frac{\Omega(\vartheta(t),\Phi(t,\vartheta(t)))}{\Omega(\Phi(t,\vartheta(t)),\vartheta(t))}. \\ \left[\frac{\Omega(\vartheta(t),\vartheta(t))+\Omega(\vartheta(t),\vartheta(t))}{1+\Omega(\vartheta(t),\vartheta(t))}\right]. \\ \left[\frac{\Omega(\vartheta(t),\vartheta(t))\Omega(\vartheta(t),\Phi(t,\vartheta(t)))+\Omega(\vartheta(t),\vartheta(t))\Omega(\vartheta(t),\Phi(t,\vartheta(t)))}{\Omega(\vartheta(t),\Phi(t,\vartheta(t)))+\Omega(\Phi(t,\vartheta(t)),\vartheta(t))}\right]. \\ \left[\frac{\Omega(\vartheta(t),\vartheta(t))+\Omega(\Phi(t,\vartheta(t)),\vartheta(t))}{\Omega(\vartheta(t),\Phi(t,\vartheta(t)))+\Omega(\vartheta(t),\vartheta(t))}\right] \\ \left[\frac{\Omega(\vartheta(t),\vartheta(t))+\Omega(\vartheta(t),\Phi(t,\vartheta(t)))}{1+\Omega(\Phi(t,\vartheta(t)),\vartheta(t))}\right] \end{array}\right\}^{\frac{\lambda}{2}}$$

$$\leq \left\{\begin{array}{c} 1.1. \\ 1. \\ 1.1 \end{array}\right\}^{\frac{\lambda}{2}}$$

Hence, we have

$$\Omega\big(\Phi(t,\vartheta(t)),\vartheta(t)\big) = 1$$

Implies,

$$\Phi(t,\vartheta(t)) = \vartheta(t)$$

Uniqueness

Let *(t)* be another common random fixed point of Φ . Then using the inequality *(E1)*

$$\Omega\big(\vartheta(t),\theta(t)\big) = \Omega\Big(\Phi\big(t,\vartheta(t)\big),\Phi\big(t,\theta(t)\big)\Big)$$

$$\leq$$

$$\left\{\begin{array}{c} \frac{\Omega(\theta(t),\Phi(t,\vartheta(t)))}{\Omega(\Phi(t,\vartheta(t)),\Phi(t,\theta(t)))}.\left[\frac{\Omega(\theta(t),\Phi(t,\theta(t)))+\Omega(\vartheta(t),\Phi(t,\theta(t)))}{1+\Omega(\vartheta(t),\theta(t))}\right]. \\ \left[\frac{\Omega(\vartheta(t),\theta(t))\Omega(\theta(t),\Phi(t,\vartheta(t)))+\Omega(\Phi(t,\theta(t)),\vartheta(t))\Omega(\theta(t),\Phi(t,\vartheta(t)))}{\Omega(\theta(t),\Phi(t,\vartheta(t)))+\Omega(\Phi(t,\vartheta(t)),\Phi(t,\theta(t)))}\right]. \\ \left[\frac{\Omega(\vartheta(t),\theta(t))+\Omega(\Phi(t,\vartheta(t)),\Phi(t,\theta(t)))}{\Omega(\vartheta(t),\Phi(t,\vartheta(t)))+\Omega(\theta(t),\Phi(t,\theta(t)))}\right]. \\ \left[\frac{\Omega(\vartheta(t),\Phi(t,\theta(t)))+\Omega(\vartheta(t),\Phi(t,\vartheta(t)))}{1+\Omega(\Phi(t,\vartheta(t)),\Phi(t,\theta(t)))}\right] \end{array}\right\}^{\frac{\lambda}{2}}$$

$$\leq \left\{ \begin{array}{c} \frac{\Omega(\theta(t),\vartheta(t))}{\Omega(\vartheta(t),\theta(t))} \cdot \left[\frac{\Omega(\theta(t),\theta(t))+\Omega(\vartheta(t),\theta(t))}{1+\Omega(\vartheta(t),\theta(t))}\right]. \\ \left[\frac{\Omega(\vartheta(t),\theta(t))\Omega(\theta(t),\vartheta(t))+\Omega(\theta(t),\vartheta(t))\Omega(\theta(t),\vartheta(t))}{\Omega(\theta(t),\vartheta(t))+\Omega(\vartheta(t),\theta(t))}\right]. \\ \left[\frac{\Omega(\vartheta(t),\theta(t))+\Omega(\vartheta(t),\theta(t))}{\Omega(\vartheta(t),\vartheta(t))+\Omega(\theta(t),\theta(t))}\right]\left[\frac{\Omega(\vartheta(t),\theta(t))+\Omega(\vartheta(t),\vartheta(t))}{1+\Omega(\vartheta(t),\theta(t))}\right] \end{array} \right\}^{\frac{\lambda}{2}}$$

[since $\theta(t)$ and $\vartheta(t)$ are random fixed points of Φ]

$$= \left\{ \begin{array}{c} 1.1. \\ \left[\Omega(\theta(t),\vartheta(t))\right]. \\ \left[\Omega(\theta(t),\vartheta(t))\right].1 \end{array} \right\}^{\frac{\lambda}{2}}$$

$$\Omega(\vartheta(t),\theta(t)) \leq \left\{\Omega^2(\theta(t),\vartheta(t))\right\}^{\frac{\lambda}{2}}$$

$$\Omega(\vartheta(t),\theta(t)) \leq \Omega^{\lambda}(\theta(t),\vartheta(t))$$

which is a contradiction, since $\lambda \in [0,1)$.

Hence we have, $\theta(t) = \vartheta(t)$. This completes the proof.

Theorem 2.2. *Let us consider complete multiplicative metric space (X, Ω) and functions Φ, α, γ, and β:* ℝ x *X→X be four functions satisfying the following conditions:*

$(E2)$ *α(t,X) ⊂ β (t,X) and Φ(t,X) ⊂ γ(t,X)*

$(E3)$ $\Omega\left(\alpha(t,x(t)),\Phi(t,y(t))\right)$

$$\leq \left\{ max \begin{pmatrix} \frac{\Omega\left(\beta(t,y(t)),\Phi(t,y(t))\right)[\Omega(\alpha(t,x(t)),\gamma(t,x(t))+\Omega(\beta(t,y(t)),\gamma(t,x(t))]}{\Omega\left(\Phi(t,y(t)),\beta(t,y(t))\right)+\Omega\left(\Phi(t,y(t)),\alpha(t,x(t))\right)}, \\ \frac{\Omega\left(\alpha(t,x(t)),\Phi(t,y(t))\right)[\Omega(\beta(t,y(t)),\Phi(t,y(t))+\Omega(\alpha(t,x(t)),\gamma(t,x(t))]}{\Omega\left(\Phi(t,y(t)),\alpha(t,x(t))\right)+\Omega\left(\gamma(t,x(t)),\beta(t,y(t))\right)}, \\ \frac{\Omega(\beta(t,y(t)),\gamma(t,x(t)))\left[\Omega(\Phi\left(t,y(t)),\beta(t,y(t))\right)+\Omega(\gamma(t,x(t)),\alpha(t,x(t))\right]}{\Omega(\beta(t,y(t)),\gamma(t,x(t)))+\Omega\left(\alpha(t,x(t)),\Phi(t,y(t))\right)}, \\ \frac{\Omega\left(\gamma(t,x(t)),\beta(t,y(t))\right)[\Omega(\Phi(t,y(t)),\beta(t,y(t))+\Omega(\Phi(t,y(t)),\alpha(t,x(t))]}{\Omega(\alpha(t,x(t)),\gamma(t,x(t))+\Omega(\beta(t,y(t)),\gamma(t,x(t)))} \end{pmatrix} \right\}^{\lambda}$$

For all $x, y \in X$, *where* $t \in \mathbb{R}$ *and* $\lambda \in [0,1)$.

Then Φ, α, γ, *and* β *have a unique random fixed point is and only if one of the following conditions are satisfied:*

$(E4)$ *either* α *or* γ *is continuous, the pair* (α, γ) *is compatible and the pair* (β, Φ) *is weakly compatible.*

$(E5)$ *either* Φ *or* β *is continuous, the pair* (Φ, β) *is compatible and the pair* (α, γ) *is weakly compatible.*

Proof: Let the function $\vartheta_0 : \mathbb{R} \to X$ be an arbitrary function on X, by (E2) there exists a function $\vartheta_1 : \mathbb{R} \to X$ *such that for* $t \in \mathbb{R}$ can be defined as $(t, \vartheta_0(t)) = \beta(t, \vartheta_1(t))$ and for this function $\vartheta_1 : \mathbb{R} \to X$ we can choose a function $\vartheta_2 : \mathbb{R} \to X$ such that *for* $t \in \mathbb{R}$ can be defined as $\gamma(t, \vartheta_2(t)) = \Phi(t, \vartheta_1(t))$ and so on. By using the principle of mathematical induction we can define a sequence $\{y_n(t)\}$, $t \in \mathbb{R}$, of functions as follows:

$$y_{2n+1}(t) = \beta\left(t, \vartheta_{2n+1}(t)\right) = \alpha\left(t, \vartheta_{2n}(t)\right),$$
$$y_{2n}(t) = \gamma\left(t, \vartheta_{2n}(t)\right) = \Phi\left(t, \vartheta_{2n-1}(t)\right), \tag{2.1}$$

Using *(E3)* and *(2.1)*, we have

$$\Omega(y_{2n+1}(t), y_{2n}(t)) = \Omega\left(\alpha\left(t, \vartheta_{2n}(t)\right), \Phi\left(t, \vartheta_{2n-1}(t)\right)\right)$$

$$\leq$$

$$\left\{ max \left(\begin{array}{c} \frac{\Omega(\beta(t,\vartheta_{2n-1}\ (t)),\Phi(t,\vartheta_{2n-1}\ (t)))[\Omega(\alpha(t,\vartheta_{2n}\ (t)),\gamma(t,\vartheta_{2n}\ (t))+\Omega(\beta(t,\vartheta_{2n-1}\ (t)),\gamma(t,\vartheta_{2n}\ (t))]}{\Omega(\Phi(t,\vartheta_{2n-1}\ (t)),\beta(t,\vartheta_{2n-1}\ (t)))+\Omega(\Phi(t,\vartheta_{2n-1}\ (t)),\alpha(t,\vartheta_{2n}\ (t)))}, \\ \frac{\Omega(\alpha(t,\vartheta_{2n}\ (t)),\Phi(t,\vartheta_{2n-1}\ (t)))[\Omega(\beta(t,\vartheta_{2n-1}\ (t)),\Phi(t,\vartheta_{2n-1}\ (t))+\Omega(\alpha(t,\vartheta_{2n}\ (t)),\gamma(t,\vartheta_{2n}\ (t))]}{\Omega(\Phi(t,\vartheta_{2n-1}\ (t)),\alpha(t,\vartheta_{2n}\ (t)))+\Omega(\gamma(t,\vartheta_{2n}\ (t)),\beta(t,\vartheta_{2n-1}\ (t)))}, \\ \frac{\Omega(\beta(t,\vartheta_{2n-1}\ (t)),\gamma(t,\vartheta_{2n}\ (t)))[\Omega(\Phi(t,\vartheta_{2n-1}\ (t),\beta(t,\vartheta_{2n-1}\ (t)))+\Omega(\gamma(t,\vartheta_{2n}\ (t)),\alpha(t,\vartheta_{2n}\ (t))]}{\Omega(\beta(t,\vartheta_{2n-1}\ (t))\ ,\gamma(t,\vartheta_{2n}\ (t)))+\Omega(\ \alpha(t,\vartheta_{2n}\ (t)),\Phi(t,\vartheta_{2n-1}\ (t)))}, \\ \frac{\Omega(\ \gamma(t,\vartheta_{2n}\ (t)),\beta(t,\vartheta_{2n-1}\ (t)))[\Omega(\Phi(t,\vartheta_{2n-1}\ (t)),\beta(t,\vartheta_{2n-1}\ (t))\ +\Omega(\Phi(t,\vartheta_{2n-1}\ (t)),\alpha(t,\vartheta_{2n}\ (t))]}{\Omega(\alpha(t,\vartheta_{2n}\ (t)),\gamma(t,\vartheta_{2n}\ (t))+\Omega(\beta(t,\vartheta_{2n-1}\ (t))\ ,\gamma(t,\vartheta_{2n}\ (t)))} \end{array} \right) \right\}^{\lambda}$$

$$\leq$$

$$\left\{ max \left(\begin{array}{c} \frac{\Omega(y_{2n-1}(t),y_{2n}(t))[\Omega(y_{2n+1}(t),y_{2n}(t)+\Omega(y_{2n-1}(t),y_{2n}(t))]}{\Omega(y_{2n}(t),y_{2n-1}(t))+\Omega(y_{2n}(t),y_{2n+1}(t))}, \\ \frac{\Omega(y_{2n+1}(t),y_{2n}(t))[\Omega(y_{2n-1}(t),y_{2n}(t))+\Omega(y_{2n+1}(t),y_{2n}(t))]}{\Omega(y_{2n}(t),y_{2n+1}(t))+\Omega(y_{2n}(t),y_{2n-1}(t))}, \\ \frac{\Omega(y_{2n-1}(t),y_{2n}(t))[\Omega(y_{2n}(t),y_{2n-1}(t))+\Omega(y_{2n}(t),y_{2n+1}(t))]}{\Omega(y_{2n-1}(t)\ ,y_{2n}(t))+\Omega(\ y_{2n+1}(t),y_{2n}(t))}, \\ \frac{\Omega(y_{2n-1}(t),y_{2n}(t))[\Omega(y_{2n}(t),y_{2n-1}(t))\ +\Omega(y_{2n}(t),y_{2n+1}(t))]}{\Omega(y_{2n+1}(t),y_{2n}(t))+\Omega(y_{2n-1}(t),y_{2n}(t))} \end{array} \right) \right\}^{\lambda}$$

$$\leq \left\{ max \begin{pmatrix} \Omega(y_{2n-1}(t), y_{2n}(t)), \\ \Omega(y_{2n+1}(t), y_{2n}(t)), \\ \Omega(y_{2n-1}(t), y_{2n}(t)), \\ \Omega(y_{2n-1}(t), y_{2n}(t)) \end{pmatrix} \right\}^{\lambda}$$

$if\ \Omega(y_{2n+1}(t), y_{2n}(t)) \leq\ \Omega^{\lambda}(y_{2n+1}(t), y_{2n}(t))$, a contradiction since $\lambda \in$ *[0,1)*.

Implies,

$$\Omega(y_{2n+1}(t), y_{2n}(t)) \leq\ \Omega^{\lambda}(y_{2n-1}(t), y_{2n}(t)), \qquad (2.2)$$

$$\Omega(y_{2n+2}(t), y_{2n+1}(t)) =\ \Omega\left((\Phi(t,\ \vartheta_{2n+1}\ (t)), \alpha(t,\ \vartheta_{2n}\ (t))\right)$$

$$= \Omega\left((\alpha(t,\ \vartheta_{2n}\ (t)), \Phi(t,\ \vartheta_{2n+1}\ (t))\right)$$

$$\leq$$

$$\left\{ max \left(\begin{array}{c} \frac{\Omega(\beta(t,\vartheta_{2n+1}\ (t)),\Phi(t,\vartheta_{2n+1}\ (t)))[\Omega(\alpha(t,\vartheta_{2n}\ (t)),\gamma(t,\vartheta_{2n}\ (t))+\Omega(\beta(t,\vartheta_{2n+1}\ (t)),\gamma(t,\vartheta_{2n}\ (t))]}{\Omega(\Phi(t,\vartheta_{2n+1}\ (t)),\beta(t,\vartheta_{2n+1}\ (t)))+\Omega(\Phi(t,\vartheta_{2n+1}\ (t)),\alpha(t,\vartheta_{2n}\ (t)))}, \\ \frac{\Omega(\alpha(t,\vartheta_{2n}\ (t)),\Phi(t,\vartheta_{2n+1}\ (t)))[\Omega(\beta(t,\vartheta_{2n+1}\ (t)),\Phi(t,\vartheta_{2n+1}\ (t))+\Omega(\alpha(t,\vartheta_{2n}\ (t)),\gamma(t,\vartheta_{2n}\ (t))]}{\Omega(\Phi(t,\vartheta_{2n+1}\ (t)),\alpha(t,\vartheta_{2n}\ (t)))+\Omega(\gamma(t,\vartheta_{2n}\ (t)),\beta(t,\vartheta_{2n+1}\ (t)))}, \\ \frac{\Omega(\beta(t,\vartheta_{2n+1}\ (t)),\gamma(t,\vartheta_{2n}\ (t)))[\Omega(\Phi(t,\vartheta_{2n+1}\ (t)),\beta(t,\vartheta_{2n+1}\ (t)))+\Omega(\gamma(t,\vartheta_{2n}\ (t)),\alpha(t,\vartheta_{2n}\ (t))]}{\Omega(\beta(t,\vartheta_{2n+1}\ (t))\ ,\gamma(t,\vartheta_{2n}\ (t)))+\Omega(\ \alpha(t,\vartheta_{2n}\ (t)),\Phi(t,\vartheta_{2n+1}\ (t)))}, \\ \frac{\Omega(\ \gamma(t,\vartheta_{2n}\ (t)),\beta(t,\vartheta_{2n+1}\ (t)))[\Omega(\Phi(t,\vartheta_{2n+1}\ (t)),\beta(t,\vartheta_{2n+1}\ (t))\ +\Omega(\ \Phi(t,\vartheta_{2n+1}\ (t)),\alpha(t,\vartheta_{2n}\ (t))]}{\Omega(\alpha(t,\vartheta_{2n}\ (t)),\gamma(t,\vartheta_{2n}\ (t))+\Omega(\beta(t,\vartheta_{2n+1}\ (t))\ ,\gamma(t,\vartheta_{2n}\ (t)))} \end{array} \right) \right\}^{\lambda}$$

$$\leq \left\{ max \begin{pmatrix} \frac{\Omega(y_{2n+1}(t),y_{2n+2}(t))[\Omega(y_{2n+1}(t),y_{2n}(t))+\Omega(y_{2n+1}(t),y_{2n}(t))]}{\Omega(y_{2n+2}(t),y_{2n+1}(t))+\Omega(y_{2n+2}(t),y_{2n+1}(t))}, \\ \frac{\Omega(y_{2n+1}(t),y_{2n+2}(t))[\Omega(y_{2n+1}(t),y_{2n+2}(t))+\Omega(y_{2n+1}(t),y_{2n}(t))]}{\Omega(y_{2n+2}(t),y_{2n+1}(t))+\Omega(y_{2n}(t),y_{2n+1}(t))}, \\ \frac{\Omega(y_{2n+1}(t),y_{2n}(t))[\Omega(y_{2n+2}(t),y_{2n+1}(t)+\Omega(y_{2n}(t),y_{2n+1}(t))]}{\Omega(y_{2n+1}(t),y_{2n}(t))+\Omega(y_{2n+1}(t),y_{2n+2}(t))}, \\ \frac{\Omega(y_{2n}(t),y_{2n+1}(t))[\Omega(y_{2n+2}(t),y_{2n+1}(t))+\Omega(y_{2n+2}(t),y_{2n+1}(t)]}{\Omega(y_{2n+1}(t),y_{2n}(t))+\Omega(y_{2n+1}(t),y_{2n}(t))} \end{pmatrix} \right\}^{\lambda}$$

$$\leq \left\{ max \begin{pmatrix} \Omega(y_{2n+1}(t), y_{2n}(t)), \\ \Omega(y_{2n+1}(t), y_{2n+2}(t)), \\ \Omega(y_{2n+1}(t), y_{2n}(t)), \\ \Omega(y_{2n+2}(t), y_{2n+1}(t)) \end{pmatrix} \right\}^{\lambda}$$

$if\ \Omega(y_{2n+2}(t), y_{2n+1}(t)) \leq \Omega^{\lambda}(y_{2n+2}(t), y_{2n+1}(t))$, a contradiction, since $\lambda \in$ *[0,1).*

Implies,

$$\Omega(y_{2n+2}(t), y_{2n+1}(t)) \leq \Omega^{\lambda}(y_{2n+1}(t), y_{2n}(t))$$

(2.3)

Using (2.2) and (2.3), we have

$$\Omega(y_{n+1}(t), y_n(t)) \leq \Omega^{\lambda}(y_n(t), y_{n-1}(t)), \text{ for all n}$$

hence

$$\begin{aligned} \Omega(y_{n+1}(t), y_n(t)) &\leq \Omega^{\lambda}(y_n(t), y_{n-1}(t)) \\ &\leq \Omega^{\lambda^2}(y_{n-1}(t), y_{n-2}(t)) \\ &\ \ . \\ &\ \ . \\ &\ \ . \\ &\leq \Omega^{\lambda^n}(y_1(t), y_0(t)) \end{aligned}$$

(2.4)

Let *m, n* $\in \mathbb{N}$ with *m* >*n*. then we have

$$\Omega\big(y_m\ (t),\ y_n\ (t)\big)$$
$$\leq \Omega\big(y_m\ (t),\ y_{m-1}\ (t)\big).\Omega\big(y_{m-1}\ (t),\ y_{m-2}\ (t)\big) \ldots \Omega\big(y_{n+1}\ (t),\ y_n\ (t)\big)$$
$$\leq$$
$$\Omega^{\lambda^{m-1}}\big(y_1\ (t),\ y_0\ (t)\big).\Omega^{\lambda^{m-2}}\big(y_1\ (t),\ y_0\ (t)\big) \ldots \Omega^{\lambda^n}\big(y_1\ (t),\ y_0\ (t)\big)$$

$$\leq \varOmega^{\frac{\lambda^n}{1-\lambda}}\big(y_1(t), y_0(t)\big)$$

Taking $n \to \infty$, $\varOmega\big(y_m(t), y_n(t)\big) \to 0$, hence $\{y_n(t)\}$ is a multiplicative Cauchy sequence. Since X is complete so $\{y_n(t)\}$ is a multiplicative convergent to a point $y(t) \in X$ as n approaches to infinity and also subsequences of $\{y_n(t)\}$ also are multiplicative convergent to a point $y(t) \in X$ and $t \in \mathbb{R}$

$$\beta\big(t, \vartheta_{2n+1}(t)\big) \to y(t),\ \varPhi\big(t, \vartheta_{2n+1}(t)\big) \to y(t) \tag{2.5}$$
$$\alpha\big(t, \vartheta_{2n}(t)\big) \to y(t), \qquad \gamma\big(t, \vartheta_{2n}(t)\big) \to y(t)$$

Now, since the pair α, γ is compatible, then by (2.5), we obtain

$$\lim_{n\to\infty} \varOmega\Big(\alpha\Big(t, \gamma\big(t, \vartheta_{2n}(t)\big)\Big), \gamma\Big(t, \alpha\big(t, \vartheta_{2n}(t)\big)\Big) = 1. \tag{2.6}$$

Suppose that γ is continuous, it follows that

$$\gamma\Big(t, \gamma\big(t, \vartheta_{2n}(t)\big)\Big) \to \gamma\big(t, y(t)\big), \gamma\Big(t, \alpha\big(t, \vartheta_{2n}(t)\big)\Big) \to \gamma\big(t, y(t)\big) \tag{2.7}$$

Then by definition of compatibility, we have

$$\alpha\Big(t, \gamma\big(t, \vartheta_{2n}(t)\big)\Big) \to \gamma\,(t, y(t)),\ t \in \mathbb{R}\ . \tag{2.8}$$

Using condition (E3), we have

$$\varOmega\Big(\alpha\Big(t, \gamma\big(t, \vartheta_{2n}(t)\big)\Big), \varPhi\big(t, \vartheta_{2n+1}(t)\big)\Big)^{\lambda}$$
$$\leq$$
$$\left\{ max \left(\begin{array}{c} \dfrac{\varOmega\Big(\beta\big(t,\vartheta_{2n+1}(t)\big),\varPhi\big(t,\vartheta_{2n+1}(t)\big)\Big)\Big[\varOmega(\alpha\big(t,\gamma(t,\vartheta_{2n}(t))\big),\gamma\big(t,\gamma(t,\vartheta_{2n}(t))\big)+\varOmega(\beta(t,\vartheta_{2n+1}(t)),\gamma\big(t,\gamma(t,\vartheta_{2n}(t))\big)\Big]}{\varOmega\Big(\varPhi\big(t,\vartheta_{2n+1}(t)\big),\beta\big(t,\vartheta_{2n+1}(t)\big)\Big)+\varOmega\Big(\varPhi\big(t,\vartheta_{2n+1}(t)\big),\alpha\big(t,\gamma(t,\vartheta_{2n}(t))\big)\Big)}, \\ \dfrac{\varOmega\Big(\alpha\big(t,\gamma(t,\vartheta_{2n}(t))\big),\varPhi\big(t,\vartheta_{2n+1}(t)\big)\Big)\Big[\varOmega(\beta\big(t,\vartheta_{2n+1}(t)\big),\varPhi\big(t,\vartheta_{2n+1}(t)\big)+\varOmega(\alpha\big(t,\gamma(t,\vartheta_{2n}(t))\big),\gamma(t,\gamma(t,\vartheta_{2n}(t)))\Big]}{\varOmega\Big(\varPhi(t,\vartheta_{2n+1}(t)),\alpha\big(t,\gamma(t,\vartheta_{2n}(t))\big)\Big)+\varOmega\Big(\gamma(t,\gamma(t,\vartheta_{2n}(t))),\beta\big(t,\vartheta_{2n+1}(t)\big)\Big)}, \\ \dfrac{\varOmega\big(\beta(t,\vartheta_{2n+1}(t)),\gamma(t,\gamma(t,\vartheta_{2n}(t)))\big)\Big[\varOmega(\varPhi\big(t,\vartheta_{2n+1}(t)\big),\beta\big(t,\vartheta_{2n+1}(t)\big)\Big)+\varOmega(\gamma\big(t,\gamma(t,\vartheta_{2n}(t))\big),\alpha\big(t,\gamma(t,\vartheta_{2n}(t))\big)\Big]}{\varOmega\big(\beta(t,\vartheta_{2n+1}(t)),\gamma(t,\gamma(t,\vartheta_{2n}(t)))\big)+\varOmega\Big(\alpha\big(t,\gamma(t,\vartheta_{2n}(t))\big),\varPhi\big(t,\vartheta_{2n+1}(t)\big)\Big)}, \\ \dfrac{\varOmega\Big(\gamma\big(t,\gamma(t,\vartheta_{2n}(t))\big),\beta\big(t,\vartheta_{2n+1}(t)\big)\Big)\Big[\varOmega(\varPhi(t,\vartheta_{2n+1}(t)),\beta\big(t,\vartheta_{2n+1}(t)\big)+\varOmega(\varPhi\big(t,\vartheta_{2n+1}(t)\big),\alpha\big(t,\gamma(t,\vartheta_{2n}(t))\big)\Big]}{\varOmega(\alpha\big(t,\gamma(t,\vartheta_{2n}(t))\big),\gamma(t,\gamma(t,\vartheta_{2n}(t)))+\varOmega\big(\beta(t,\vartheta_{2n+1}(t)),\gamma(t,\gamma(t,\vartheta_{2n}(t)))\big)} \end{array} \right) \right\}^{\lambda}$$

Letting n approaches to infinity, we have

$$\Omega\left(\gamma(t,y(t)),y(t)\right)$$

$$\leq \left\{ max \begin{pmatrix} \frac{\Omega(y(t),y(t))[\Omega(\gamma(t,y(t)),\gamma(t,y(t))+\Omega(y(t),\gamma(t,y(t))]}{\Omega(y(t),y(t))+\Omega\left(y(t),\gamma(t,y(t))\right)}, \\ \frac{\Omega\left(\gamma(t,y(t)),y(t)\right)[\Omega(y(t),y(t))+\Omega(\gamma(t,y(t)),\gamma(t,y(t)))]}{\Omega\left(y(t),\gamma(t,y(t))\right)+\Omega\left(\gamma(t,y(t))),y(t)\right)}, \\ \frac{\Omega(y(t),\gamma(t,y(t))))[\Omega((y(t),y(t))+\Omega(\gamma(t,y(t)),\gamma(t,y(t))]}{\Omega\left(y(t)\,,\gamma(t,y(t))\right)+\Omega\left(\gamma(t,y(t)),y(t)\right)}, \\ \frac{\Omega\left(\gamma(t,y(t)),y(t)\right)[\Omega(y(t),y(t))\ +\Omega(y(t),\gamma(t,y(t))]}{\Omega\left(\gamma(t,y(t)),\gamma(t,y(t))\right)+\Omega\left(y(t),\gamma(t,y(t))\right)} \end{pmatrix} \right\}^{\lambda}$$

$$\leq \left\{ max \begin{pmatrix} 1, \\ 1, \\ 1, \\ \Omega\left(\gamma(t,y(t)),y(t)\right) \end{pmatrix} \right\}^{\lambda}$$

$\Omega\left(\gamma(t,y(t)),y(t)\right) \leq \Omega^{\lambda}\left(\gamma(t,y(t)),y(t)\right)$, a contradiction since $\lambda \in [0,1)$.

Which implies that, $\gamma\ (t,y(t))=y(t)$ *(2.9)*

Now we prove that $\alpha(t,y(t)) = y(t)$. if possible $\alpha(t,y(t)) \neq y(t)$,
*U*sing condition *(E3)*

$$\Omega\left(\alpha(t,y(t)),\Phi(t,\ \vartheta_{2n-1}\ (t))\right)$$

$$\leq$$

$$\left\{ max \begin{pmatrix} \frac{\Omega\left(\beta(t,\vartheta_{2n+1}\ (t)),\Phi(t,\vartheta_{2n+1}\ (t))\right)[\Omega(\alpha(t,y(t)),\gamma(t,y(t))+\Omega(\beta(t,\vartheta_{2n+1}\ (t)),\gamma(t,y(t))]}{\Omega\left(\Phi(t,\vartheta_{2n+1}\ (t)),\beta(t,\vartheta_{2n+1}\ (t))\right)+\Omega\left(\Phi(t,\vartheta_{2n+1}\ (t)),\alpha(t,y(t))\right)}, \\ \frac{\Omega\left(\alpha(t,y(t)),\Phi(t,\vartheta_{2n+1}\ (t))\right)[\Omega(\beta(t,\vartheta_{2n+1}\ (t)),\Phi(t,\vartheta_{2n+1}\ (t))+\Omega(\alpha(t,y(t)),\gamma(t,y(t))]}{\Omega\left(\Phi(t,\vartheta_{2n+1}\ (t)),\alpha(t,y(t))\right)+\Omega\left(\gamma(t,y(t)),\beta(t,\vartheta_{2n+1}\ (t))\right)}, \\ \frac{\Omega(\beta(t,\vartheta_{2n+1}\ (t)),\gamma(t,y(t)))\left[\Omega(\Phi\left(t,\vartheta_{2n+1}\ (t)),\beta(t,\vartheta_{2n+1}\ (t))\right)+\Omega(\gamma(\ t,y(t)),\alpha(t,y(t))\right]}{\Omega(\beta(t,\vartheta_{2n+1}\ (t))\,,\gamma(t,y(t)))+\Omega\left(\alpha(t,y(t)),\Phi(t,\vartheta_{2n+1}\ (t))\right)}, \\ \frac{\Omega\left(\gamma(t,y(t)),\beta(t,\vartheta_{2n+1}\ (t))\right)[\Omega(\Phi(t,\vartheta_{2n+1}\ (t)),\beta(t,\vartheta_{2n+1}\ (t))\ +\Omega(\Phi(t,\vartheta_{2n+1}\ (t)),\alpha(t,y(t))]}{\Omega(\alpha(t,y(t)),\gamma(t,y(t))+\Omega(\beta(t,\vartheta_{2n+1}\ (t))\,,\gamma(t,y(t)))} \end{pmatrix} \right\}^{\lambda}$$

Taking *n*→∞, we have

$$\Omega\left(\alpha\big(t,y(t)\big),y(t)\right)$$

$$\leq \left\{ max\left(\begin{array}{c} \frac{\Omega(y(t),y(t))[\Omega(\alpha(t,y(t)),y(t)+\Omega(y(t),y(t)]}{\Omega(y(t),y(t))+\Omega\left(y(t),\alpha(t,y(t))\right)}, \\ \frac{\Omega\left(\alpha(t,y(t)),y(t)\right)[\Omega(y(t),y(t))+\Omega(\alpha(t,y(t)),y(t)]}{\Omega\left(y(t),\alpha(t,y(t))\right)+\Omega(y(t),y(t))}, \\ \frac{\Omega(y(t),y(t))[\Omega(y(t),y(t))+\Omega(y(t),\alpha(t,y(t))]}{\Omega(y(t)\,,y(t))+\Omega\left(\alpha(t,y(t)),y(t)\right)}, \\ \frac{\Omega(\,y(t),y(t))[\Omega(y(t),y(t))\,+\Omega(y(t),\alpha(t,y(t))]}{\Omega\left(\alpha(t,y(t)),y(t)\right)+\Omega(y(t)\,,y(t))} \end{array}\right)\right\}^{\lambda}$$

[using (2.5),(2.9)]

$$\leq \left\{ max\left(\begin{array}{c} 1, \\ \Omega\left(\alpha\big(t,y(t)\big),y(t)\right), \\ 1, \\ 1 \end{array}\right)\right\}^{\lambda}$$

Implies,

$\Omega\left(\alpha\big(t,y(t)\big),y(t)\right) \leq \Omega^{\lambda}\left(\alpha\big(t,y(t)\big),y(t)\right)$, a contradiction since $\lambda \in [0,1)$.

And thus we have,

$$\alpha\big(t,y(t)\big) = y(t).$$

Hence we have,

$$\alpha\big(t,y(t)\big) = \gamma(t,y(t)) = y(t). \qquad (2.10)$$

Now, since $y(t) = \alpha\big(t,y(t)\big) \in \alpha(t,X)) \subset \beta(t,X)$, $\exists$ $h(t) \in X$, such that $y(t) = \beta(t,h(t))$ for

$t \in \mathbb{R}$. Now we prove that $\Phi(t,h(t))=y(t)$. if possible $\Phi\big(t,h(t)\big) \neq y(t)$,

Using condition (E3), we obtain

$$\Omega\left(y(t),\Phi\big(t,h(t)\big)\right) = \Omega\left(\alpha\big(t,y(t)\big),\Phi\big(t,h(t)\big)\right)$$

$$\leq \left\{ max \begin{pmatrix} \frac{\Omega\left(\beta(t,h(t)),\Phi(t,h(t))\right)[\Omega(\alpha(t,y(t)),\gamma(t,y(t))+\Omega(\beta(t,h(t)),\gamma(t,y(t))]}{\Omega\left(\Phi(t,h(t)),\beta(t,h(t))\right)+\Omega\left(\Phi(t,h(t)),\alpha(t,y(t))\right)}, \\ \frac{\Omega\left(\alpha(t,y(t)),\Phi(t,h(t))\right)[\Omega(\beta(t,h(t)),\Phi(t,h(t))+\Omega(\alpha(t,y(t)),\gamma(t,y(t))]}{\Omega\left(\Phi(t,h(t)),\alpha(t,y(t))\right)+\Omega\left(\gamma(t,y(t)),\beta(t,h(t))\right)}, \\ \frac{\Omega(\beta(t,h(t)),\gamma(t,y(t)))\left[\Omega(\Phi(t,h(t)),\beta(t,h(t))\right)+\Omega(\gamma(t,y(t)),\alpha(t,y(t))]}{\Omega(\beta(t,h(t)),\gamma(t,y(t)))+\Omega\left(\alpha(t,y(t)),\Phi(t,h(t))\right)}, \\ \frac{\Omega\left(\gamma(t,y(t)),\beta(t,h(t))\right)[\Omega(\Phi(t,h(t)),\beta(t,h(t))+\Omega(\Phi(t,h(t)),\alpha(t,y(t))]}{\Omega(\alpha(t,y(t)),\gamma(t,y(t))+\Omega(\beta(t,h(t)),\gamma(t,y(t)))} \end{pmatrix} \right\}^{\lambda}$$

Hence we obtain

$$\Omega\left(y(t),\Phi(t,h(t))\right) \leq \left\{ h\, max \begin{pmatrix} \frac{\Omega\left(y(t),\Phi(t,h(t))\right)[\Omega(y(t),y(t)+\Omega(y(t),y(t)]}{\Omega\left(\Phi(t,h(t)),y(t)\right)+\Omega\left(\Phi(t,h(t)),y(t)\right)}, \\ \frac{\Omega\left(y(t),\Phi(t,h(t))\right)[\Omega(y(t),\Phi(t,h(t))+\Omega(y(t),y(t)]}{\Omega(\Phi(t,h(t)),y(t))+\Omega(y(t),y(t))}, \\ \frac{\Omega(y(t),y(t))[\Omega(\Phi(t,h(t)),y(t))+\Omega(y(t),y(t)]}{\Omega(y(t),y(t))+\Omega\left(y(t),\Phi(t,h(t))\right)}, \\ \frac{\Omega(y(t),y(t))[\Omega(\Phi(t,h(t)),y(t))+\Omega(\beta(t,h(t)),y(t)]}{\Omega(y(t),y(t))+\Omega(y(t),y(t))} \end{pmatrix} \right\}^{\lambda}$$

$$\leq \left\{ h\, max \begin{pmatrix} 1, \\ \Omega\left(y(t),\Phi(t,h(t))\right), \\ 1, \\ \Omega\left(y(t),\Phi(t,h(t))\right) \end{pmatrix} \right\}^{\lambda}$$

$\Omega\left(y(t),\Phi(t,h(t))\right) \leq \Omega^{\lambda}\left(y(t),\Phi(t,h(t))\right)$, a contradiction since $\lambda \in$ *[0,1).*

And thus we have, $\Phi(t,h(t)) = y(t)\ for$ t ∈ ℝ. hence we have,

$$\Phi(t,h(t)) = \beta(t,h(t)) = y(t)\ for \text{ t} \in \mathbb{R}. \qquad (2.11)$$

Since the pair (Φ,β) is weakly compatible, we have $\Phi\left(t,\beta(t,h(t))\right) = \beta\left(t,\Phi(t,h(t))\right)$, that is,

$$\Phi(t,y(t)) = \beta(t,y(t)). \qquad (2.12)$$

From condition (E3) and (2.10), we have

$$\Omega\Big(y(t),\Phi\big(t,y(t)\big)\Big) = \Omega\Big(\alpha\big(t,y(t)\big),\Phi\big(t,y(t)\big)\Big)$$

$$\leq$$

$$\left\{ max \left(\begin{array}{c} \frac{\Omega\left(\beta(t,y(t)),\Phi(t,y(t))\right)[\Omega(\alpha(t,y(t)),\gamma(t,y(t))+\Omega(\beta(t,y(t)),\gamma(t,y(t))]}{\Omega\left(\Phi(t,y(t)),\beta(t,y(t))\right)+\Omega\left(\Phi(t,y(t)),\alpha(t,y(t))\right)}, \\ \frac{\Omega\left(\alpha(t,y(t)),\Phi(t,y(t))\right)[\Omega(\beta(t,y(t)),\Phi(t,y(t))+\Omega(\alpha(t,y(t)),\gamma(t,y(t))]}{\Omega\left(\Phi(t,y(t)),\alpha(t,y(t))\right)+\Omega\left(\gamma(t,y(t)),\beta(t,y(t))\right)}, \\ \frac{\Omega\left(\beta(t,y(t)),\gamma(t,y(t))\right)\left[\Omega(\Phi\left(t,y(t)),\beta(t,y(t))\right)+\Omega(\gamma(t,y(t)),\alpha(t,y(t))\right]}{\Omega\left(\beta(t,y(t)),\gamma(t,y(t))\right)+\Omega\left(\alpha(t,y(t)),\Phi(t,y(t))\right)}, \\ \frac{\Omega\left(\gamma(t,y(t)),\beta(t,y(t))\right)[\Omega(\Phi(t,y(t)),\beta(t,y(t))+\Omega(\Phi(t,y(t)),\alpha(t,y(t))]}{\Omega(\alpha(t,y(t)),\gamma(t,y(t))+\Omega\left(\beta(t,y(t)),\gamma(t,y(t))\right)} \end{array} \right) \right\}^{\lambda}$$

$$\leq \left\{ max \left(\begin{array}{c} \frac{\Omega\left(\beta(t,y(t)),\Phi(t,y(t))\right)[\Omega(y(t),y(t))+\Omega(\Phi(t,y(t)),y(t))]}{\Omega\left(\Phi(t,y(t)),\Phi(t,y(t))\right)+\Omega(\Phi(t,y(t)),y(t))}, \\ \frac{\Omega\left(y(t),\Phi(t,y(t))\right)[\Omega(\Phi(t,y(t)),\Phi(t,y(t))+\Omega(y(t),y(t)]}{\Omega(\Phi(t,y(t)),y(t))+\Omega\left(y(t),\Phi(t,y(t))\right)}, \\ \frac{\Omega(\Phi(t,y(t)),y(t))\left[\Omega(\Phi\left(t,y(t)),\Phi(t,y(t))\right)+\Omega(y(t),y(t))\right]}{\Omega(\Phi(t,y(t)),y(t))+\Omega\left(y(t),\Phi(t,y(t))\right)}, \\ \frac{\Omega\left(y(t),\Phi(t,y(t))\right)[\Omega(\Phi(t,y(t)),\Phi(t,y(t))+\Omega(\Phi(t,y(t)),y(t)]}{\Omega(y(t),y(t)+\Omega(\Phi(t,y(t)),y(t))} \end{array} \right) \right\}^{\lambda}$$

[using (2.10) and (2.12)]

$$\leq \left\{ max \left(\begin{array}{c} 1, \\ 1, \\ 1, \\ \Omega\Big(y(t),\Phi\big(t,y(t)\big)\Big) \end{array} \right) \right\}^{\lambda}$$

$\Omega\Big(y(t),\Phi\big(t,y(t)\big)\Big) \leq \Omega^{\lambda}\Big(y(t),\Phi\big(t,y(t)\big)\Big)$, a contradiction since $\lambda \in$ *[0,1).*

We get, y(t) = $\Phi\big(t,y(t)\big) = \beta\big(t,y(t)\big)$ for t ∈ ℝ

(2.13) Using *(2.10)* and *(2.13),* we have

y(t)= $\Phi\big(t,y(t)\big) = \beta\big(t,y(t)\big) = \alpha\big(t,y(t)\big) = \gamma(t,y(t))$, t ∈ ℝ

that is, $y(t)$ is a common random fixed point of Φ, β, α and γ.

For uniqueness, let *z(t)* $\in X$ be another common random fixed point of $\Phi, \beta, \alpha\ and\ \gamma$, using condition *(E3)*

$$\Omega\big(y(t), z(t)\big) = \Omega\Big(\alpha\big(t, y(t)\big), \Phi\big(t, z(t)\big)\Big)$$
$$\leq$$
$$\left\{ max \begin{pmatrix} \frac{\Omega\big(\beta(t,z(t)),\Phi(t,z(t))\big)[\Omega(\alpha(t,y(t)),\gamma(t,y(t))+\Omega(\beta(t,z(t)),\gamma(t,y(t))]}{\Omega\big(\Phi(t,z(t)),\beta(t,z(t))\big)+\Omega\big(\Phi(t,z(t)),\alpha(t,y(t))\big)}, \\ \frac{\Omega\big(\alpha(t,y(t)),\Phi(t,z(t))\big)[\Omega(\beta(t,z(t)),\Phi(t,z(t))+\Omega(\alpha(t,y(t)),\gamma(t,y(t))]}{\Omega\big(\Phi(t,z(t)),\alpha(t,y(t))\big)+\Omega\big(\gamma(t,y(t)),\beta(t,z(t))\big)}, \\ \frac{\Omega(\beta(t,z(t)),\gamma(t,y(t)))\big[\Omega(\Phi\big(t,z(t)),\beta(t,z(t))\big)+\Omega(\gamma(t,y(t)),\alpha(t,y(t))\big]}{\Omega(\beta(t,z(t)),\gamma(t,y(t)))+\Omega\big(\alpha(t,y(t)),\Phi(t,z(t))\big)}, \\ \frac{\Omega\big(\gamma(t,y(t)),\beta(t,z(t))\big)[\Omega(\Phi(t,z(t)),\beta(t,z(t))+\Omega(\Phi(t,z(t)),\alpha(t,y(t))]}{\Omega(\alpha(t,y(t)),\gamma(t,y(t))+\Omega(\beta(t,z(t)),\gamma(t,y(t)))} \end{pmatrix} \right\}^{\lambda}$$

$$\leq \left\{ max \begin{pmatrix} \frac{\Omega\big(z(t),z(t)\big)[\Omega(y(t),y(t)+\Omega(z(t),y(t)]}{\Omega\big(z(t),z(t)\big)+\Omega\big(z(t),y(t)\big)}, \\ \frac{\Omega\big(y(t),z(t)\big)[\Omega(z(t),z(t))+\Omega(y(t),y(t)]}{\Omega(z(t)),y(t))+\Omega\big(y(t),z(t)\big)}, \\ \frac{\Omega(z(t),y(t))\big[\Omega\big(z(t),z(t)\big)+\Omega(y(t),y(t)\big]}{\Omega(z(t),y(t))+\Omega\big(y(t),z(t)\big)}, \\ \frac{\Omega\big(y(t),z(t)\big)[\Omega(z(t),z(t))+\Omega(z(t),y(t)]}{\Omega(y(t),y(t)+\Omega(z(t),y(t))} \end{pmatrix} \right\}^{\lambda}$$

$\Omega\big(y(t), z(t)\big) \leq \Omega^{\lambda}\big(y(t), z(t)\big)$, which is a contradiction since $\lambda \in$ *[0,1)*. Which gives

$y(t) = z(t)$ for $t \in \mathbb{R}$.

Hence $\Phi, \beta, \alpha\ and\ \gamma$ have unique common fixed point. Now, suppose that α is continuous, then we have

$$\alpha\Big(t, \alpha\big(t, \vartheta_{2n}(t)\big)\Big) \to \alpha(t, y(t)), \alpha\Big(t, \gamma\big(t, \vartheta_{2n}(t)\big)\Big) \to \alpha(t, y(t)) \qquad (2.14)$$

Since the pairs α, γ is compatible, (2.6) implies that

$$\gamma\Big(t, \alpha\big(t, \vartheta_{2n}(t)\big)\Big) \to \alpha(t, y(t)), \qquad (2.15)$$

Using (E3) we have,

$$\Omega\Big(\alpha\Big(t, \alpha\big(t, \vartheta_{2n}(t)\big)\Big), \Phi\big(t, \vartheta_{2n+1}(t)\big)\Big)$$

$$\leq \left\{ max \left(\begin{array}{c} \Omega\left(\beta\left(t,\ \vartheta_{2n+1}\ (t)\right), \Phi\left(t,\ \vartheta_{2n+1}\ (t)\right)\right). \\ \frac{\left[\Omega\left(\alpha\left(t,\alpha\left(t,\vartheta_{2n}\ (t)\right)\right),\gamma\left(t,\alpha\left(t,\vartheta_{2n}\ (t)\right)\right)\right)+\Omega\left(\beta\left(t,\vartheta_{2n+1}\ (t)\right),\gamma\left(t,\alpha\left(t,\vartheta_{2n}\ (t)\right)\right)\right)\right]}{\Omega\left(\Phi\left(t,\vartheta_{2n+1}\ (t)\right),\beta\left(t,\vartheta_{2n+1}\ (t)\right)\right)+\Omega\left(\Phi\left(t,\vartheta_{2n+1}\ (t)\right),\alpha\left(t,\alpha\left(t,\vartheta_{2n}\ (t)\right)\right)\right)}, \\ \Omega\left(\alpha\left(t,\alpha\left(t,\ \vartheta_{2n}\ (t)\right)\right), \Phi\left(t,\ \vartheta_{2n+1}\ (t)\right)\right). \\ \frac{\left[\Omega\left(\beta\left(t,\vartheta_{2n+1}\ (t)\right),\Phi\left(t,\vartheta_{2n+1}\ (t)\right)\right)+\Omega\left(\alpha\left(t,\alpha\left(t,\vartheta_{2n}\ (t)\right)\right),\gamma\left(t,\alpha\left(t,\vartheta_{2n}\ (t)\right)\right)\right)\right]}{\Omega\left(\Phi\left(t,\vartheta_{2n+1}\ (t)\right),\alpha\left(t,\alpha\left(t,\vartheta_{2n}\ (t)\right)\right)\right)+\Omega\left(\gamma\left(t,\alpha\left(t,\vartheta_{2n}\ (t)\right)\right),\beta\left(t,\vartheta_{2n+1}\ (t)\right)\right)}, \\ \Omega\left(\beta\left(t,\ \vartheta_{2n+1}\ (t)\right), \gamma\left(t,\alpha\left(t,\ \vartheta_{2n}\ (t)\right)\right)\right). \\ \frac{\left[\Omega\left(\Phi\left(t,\vartheta_{2n+1}\ (t)\right),\beta\left(t,\vartheta_{2n+1}\ (t)\right)\right)+\Omega\left(\gamma\left(t,\alpha\left(t,\vartheta_{2n}\ (t)\right)\right),\alpha\left(t,\alpha\left(t,\vartheta_{2n}\ (t)\right)\right)\right)\right]}{\Omega\left(\beta\left(t,\vartheta_{2n+1}\ (t)\right),\gamma\left(t,\alpha\left(t,\vartheta_{2n}\ (t)\right)\right)\right)+\Omega\left(\alpha\left(t,\alpha\left(t,\vartheta_{2n}\ (t)\right)\right),\Phi\left(t,\vartheta_{2n+1}\ (t)\right)\right)}, \\ \Omega\left(\gamma\left(t,\alpha\left(t,\ \vartheta_{2n}\ (t)\right)\right), \beta\left(t,\ \vartheta_{2n+1}\ (t)\right)\right). \\ \frac{\left[\Omega\left(\Phi\left(t,\vartheta_{2n+1}\ (t)\right),\beta\left(t,\vartheta_{2n+1}\ (t)\right)\right) +\Omega\left(\Phi\left(t,\vartheta_{2n+1}\ (t)\right),\alpha\left(t,\alpha\left(t,\vartheta_{2n}\ (t)\right)\right)\right)\right]}{\Omega\left(\alpha\left(t,\alpha\left(t,\vartheta_{2n}\ (t)\right)\right),\gamma\left(t,\alpha\left(t,\vartheta_{2n}\ (t)\right)\right)\right)+\Omega\left(\beta\left(t,\vartheta_{2n+1}\ (t)\right),\gamma\left(t,\alpha\left(t,\vartheta_{2n}\ (t)\right)\right)\right)} \end{array} \right) \right\}^{\lambda}$$

Letting n approaches to infinity, we have

$$\Omega\left(\alpha\left(t,y(t)\right),y(t)\right)$$

$$\leq \left\{ max \left(\begin{array}{c} \frac{\Omega(y(t),y(t))\left[\Omega\left(\alpha\left(t,y(t)\right),\alpha\left(t,y(t)\right)\right)+\Omega\left(y(t),\alpha\left(t,y(t)\right)\right)\right]}{\Omega(y(t),y(t))+\Omega\left(y(t),\alpha\left(t,y(t)\right)\right)}, \\ \frac{\Omega\left(\alpha\left(t,y(t)\right),y(t)\right)\left[\Omega(y(t),y(t))+\Omega\left(\alpha\left(t,y(t)\right),\alpha\left(t,y(t)\right)\right)\right]}{\Omega\left(y(t),\alpha\left(t,y(t)\right)\right)+\Omega\left(\alpha\left(t,y(t)\right),y(t)\right)}, \\ \frac{\Omega\left(y(t),\alpha\left(t,y(t)\right)\right)\left[\Omega(y(t),y(t))+\Omega\left(\alpha\left(t,y(t)\right),\alpha\left(t,y(t)\right)\right)\right]}{\Omega\left(y(t),\alpha\left(t,y(t)\right)\right)+\Omega\left(\alpha\left(t,y(t)\right),y(t)\right)}, \\ \frac{\Omega\left(\alpha\left(t,y(t)\right),y(t)\right)\left[\Omega(y(t),y(t)) +\Omega\left(y(t),\alpha\left(t,y(t)\right)\right)\right]}{\Omega\left(\alpha\left(t,y(t)\right),\alpha\left(t,y(t)\right)\right)+\Omega\left(y(t),\alpha\left(t,y(t)\right)\right)} \end{array} \right) \right\}^{\lambda}$$

[using (2.14) and and (2.15)]

$$\Omega\left(\alpha\left(t,y(t)\right),y(t)\right) \leq \Omega^{\lambda}\left(\alpha\left(t,y(t)\right),y(t)\right)$$

which is a contradiction since $\lambda \in [0,1)$. Which gives

$$\alpha\left(t,y(t)\right) = y(t) \text{ for } t \in \mathbb{R}. \tag{2.16}$$

Since $y(t)= \alpha\left(t,y(t)\right) \in \alpha(t,X) \subset \beta(t,X)$, there exists $h(t) \in X$ such that $y(t) = \beta\left(t,h(t)\right)$ for $t \in \mathbb{R}$.

Using condition *(E3)* we obtain

$$\Omega\left(\alpha\left(t,\alpha(t,\vartheta_{2n}(t))\right),\Phi(t,h(t))\right)$$

$$\leq$$

$$\left\{max\left(\begin{array}{c}\frac{\Omega\left(\beta(t,h(t)),\Phi(t,h(t))\right)\left[\Omega(\alpha\left(t,\alpha(t,\vartheta_{2n}(t))\right),\gamma\left(t,\alpha(t,\vartheta_{2n}(t))\right)+\Omega(\beta(t,h(t)),\gamma\left(t,\alpha(t,\vartheta_{2n}(t))\right))\right]}{\Omega\left(\Phi(t,h(t)),\beta(t,h(t))\right)+\Omega\left(\Phi(t,h(t)),\alpha\left(t,\alpha(t,\vartheta_{2n}(t))\right)\right)},\\ \frac{\Omega\left(\alpha\left(t,\alpha(t,\vartheta_{2n}(t))\right),\Phi(t,h(t))\right)\left[\Omega(\beta(t,h(t)),\Phi(t,h(t))+\Omega(\alpha\left(t,\alpha(t,\vartheta_{2n}(t))\right),\gamma(t,\alpha(t,\vartheta_{2n}(t))))\right]}{\Omega\left(\Phi(t,h(t)),\alpha\left(t,\alpha(t,\vartheta_{2n}(t))\right)\right)+\Omega\left(\gamma(t,\alpha(t,\vartheta_{2n}(t))),\beta(t,h(t))\right)},\\ \frac{\Omega(\beta(t,h(t)),\gamma(t,\alpha(t,\vartheta_{2n}(t))))\left[\Omega(\Phi\left(t,h(t)),\beta(t,h(t))\right)+\Omega(\gamma\left(t,\alpha(t,\vartheta_{2n}(t))\right),\alpha\left(t,\alpha(t,\vartheta_{2n}(t))\right))\right]}{\Omega(\beta(t,h(t)),\gamma(t,\alpha(t,\vartheta_{2n}(t))))+\Omega\left(\alpha\left(t,\alpha(t,\vartheta_{2n}(t))\right),\Phi(t,h(t))\right)},\\ \frac{\Omega\left(\gamma\left(t,\alpha(t,\vartheta_{2n}(t))\right),\beta(t,h(t))\right)\left[\Omega(\Phi(t,y(t)),\beta(t,h(t))+\Omega(\Phi(t,h(t)),\alpha\left(t,\alpha(t,\vartheta_{2n}(t))\right))\right]}{\Omega(\alpha\left(t,\alpha(t,\vartheta_{2n}(t))\right),\gamma(t,\alpha(t,\vartheta_{2n}(t))))+\Omega(\beta(t,h(t)),\gamma(t,\alpha(t,\vartheta_{2n}(t))))}\end{array}\right)\right\}^{\lambda}$$

letting n approaches to infinity,

$$\Omega\left(y(t),\Phi(t,h(t))\right)\leq\left\{max\left(\begin{array}{c}\frac{\Omega\left(y(t),\Phi(t,h(t))\right)[\Omega(y(t),y(t)+\Omega(y(t),y(t))]}{\Omega\left(\Phi(t,h(t)),y(t)\right)+\Omega(\Phi(t,h(t)),y(t))},\\ \frac{\Omega\left(y(t),\Phi(t,h(t))\right)[\Omega(y(t),\Phi(t,h(t))+\Omega(y(t),y(t))]}{\Omega(\Phi(t,h(t)),y(t))+\Omega(y(t)),y(t))},\\ \frac{\Omega(y(t),y(t))[\Omega(\Phi(t,h(t)),y(t))+\Omega(y(t),y(t))]}{\Omega(y(t),y(t)))+\Omega(y(t),\Phi(t,h(t)))},\\ \frac{\Omega(y(t),y(t))[\Omega(\Phi(t,y(t)),y(t))+\Omega(\Phi(t,h(t)),y(t)]}{\Omega(y(t)y(t))+\Omega(y(t),y(t))}\end{array}\right)\right\}^{\lambda}$$

[using (2.14) and (2.15)]

$$\leq\left\{max\left(\begin{array}{c}1,\\ \Omega\left(y(t),\Phi(t,h(t))\right),\\ 1,\\ \Omega(\Phi(t,h(t)),h(t)\end{array}\right)\right\}^{\lambda}$$

$\Omega\left(y(t),\Phi(t,h(t))\right)\leq\Omega^{\lambda}\left(y(t),\Phi(t,h(t))\right)$, which is a contradiction since $\lambda\in$ *[0,1)*.

$$\Phi(t,h(t))=y(t).$$

Hence $\Phi(t,h(t))=\beta(t,h(t))=y(t)\ for\ t\in\mathbb{R}$.

Since, the pair (Φ,β) is weakly compatible, the we have

$$\Phi\left(t,\beta(t,h(t))\right)=\beta\left(t,\Phi(t,h(t))\right),\text{ i.e.}$$

$$\Phi(t,y(t))=\beta(t,y(t)) \qquad (2.17)$$

$from\ condition\ (E3)$ *and (2.17), we have*

$$\Omega\left(\alpha\left(t,\vartheta_{2n}(t)\right),\Phi\left(t,y(t)\right)\right)$$

$$\leq\left\{max\left(\begin{array}{c}\frac{\Omega\left(\beta\left(t,y(t)\right),\Phi\left(t,y(t)\right)\right)\left[\Omega\left(\alpha\left(t,\vartheta_{2n}(t)\right),\gamma\left(t,\vartheta_{2n}(t)\right)\right)+\Omega\left(\beta\left(t,y(t)\right),\gamma\left(t,\vartheta_{2n}(t)\right)\right)\right]}{\Omega\left(\Phi\left(t,y(t)\right),\beta\left(t,y(t)\right)\right)+\Omega\left(\Phi\left(t,y(t)\right),\alpha\left(t,\vartheta_{2n}(t)\right)\right)},\\ \frac{\Omega\left(\alpha\left(t,\vartheta_{2n}(t)\right),\Phi\left(t,y(t)\right)\right)\left[\Omega\left(\beta\left(t,y(t)\right),\Phi\left(t,y(t)\right)\right)+\Omega\left(\alpha\left(t,\vartheta_{2n}(t)\right),\gamma\left(t,\vartheta_{2n}(t)\right)\right)\right]}{\Omega\left(\Phi\left(t,y(t)\right),\alpha\left(t,\vartheta_{2n}(t)\right)\right)+\Omega\left(\gamma\left(t,\vartheta_{2n}(t)\right),\beta\left(t,y(t)\right)\right)},\\ \frac{\Omega\left(\beta\left(t,y(t)\right),\gamma\left(t,\vartheta_{2n}(t)\right)\right)\left[\Omega\left(\Phi\left(t,y(t)\right),\beta\left(t,y(t)\right)\right)+\Omega\left(\gamma\left(t,\vartheta_{2n}(t)\right),\alpha\left(t,\vartheta_{2n}(t)\right)\right)\right]}{\Omega\left(\beta\left(t,y(t)\right),\gamma\left(t,\vartheta_{2n}(t)\right)\right)+\Omega\left(\alpha\left(t,\vartheta_{2n}(t)\right),\Phi\left(t,y(t)\right)\right)},\\ \frac{\Omega\left(\gamma\left(t,\vartheta_{2n}(t)\right),\beta\left(t,y(t)\right)\right)\left[\Omega\left(\Phi\left(t,y(t)\right),\beta\left(t,y(t)\right)\right)+\Omega\left(\Phi\left(t,y(t)\right),\alpha\left(t,\vartheta_{2n}(t)\right)\right)\right]}{\Omega\left(\alpha\left(t,\vartheta_{2n}(t)\right),\gamma\left(t,\vartheta_{2n}(t)\right)\right)+\Omega\left(\beta\left(t,y(t)\right),\gamma\left(t,\vartheta_{2n}(t)\right)\right)}\end{array}\right)\right\}^{\lambda}$$

Taking n approaches to infinity, we have

$$\leq\left\{max\left(\begin{array}{c}\frac{\Omega\left(\Phi\left(t,y(t)\right),\Phi\left(t,y(t)\right)\right)\left[\Omega\left(y(t),y(t)\right)+\Omega\left(\Phi\left(t,y(t)\right),y(t)\right)\right]}{\Omega\left(\Phi\left(t,y(t)\right),\Phi\left(t,y(t)\right)\right)+\Omega\left(\Phi\left(t,y(t)\right),y(t)\right)},\\ \frac{\Omega\left(y(t),\Phi\left(t,y(t)\right)\right)\left[\Omega\left(\Phi\left(t,y(t)\right),\Phi\left(t,y(t)\right)\right)+\Omega\left(y(t),y(t)\right)\right]}{\Omega\left(\Phi\left(t,y(t)\right),y(t)\right)+\Omega\left(y(t),\Phi\left(t,y(t)\right)\right)},\\ \frac{\Omega\left(\Phi\left(t,y(t)\right),y(t)\right)\left[\Omega\left(\Phi\left(t,y(t)\right),\Phi\left(t,y(t)\right)\right)+\Omega\left(y(t),y(t)\right)\right]}{\Omega\left(\Phi\left(t,y(t)\right),y(t)\right)+\Omega\left(y(t),\Phi\left(t,y(t)\right)\right)},\\ \frac{\Omega\left(y(t),\Phi\left(t,y(t)\right)\right)\left[\Omega\left(\Phi\left(t,y(t)\right),\Phi\left(t,y(t)\right)\right)+\Omega\left(\Phi\left(t,y(t)\right),y(t)\right)\right]}{\Omega\left(y(t),y(t)\right)+\Omega\left(\Phi\left(t,y(t)\right),y(t)\right)}\end{array}\right)\right\}^{\lambda}$$

[using 2.17]

$\Omega\left(y(t),\Phi\left(t,y(t)\right)\right)\leq\Omega^{\lambda}\left(y(t),\Phi\left(t,y(t)\right)\right)$, which is a contradiction since $\lambda\in$ *[0,1)*. Hence we have

$y(t)=\Phi\left(t,y(t)\right)$ for $t\in\mathbb{R}$. Using (2.17), we have

$$\Phi\left(t,y(t)\right)=\beta\left(t,y(t)\right)=y(t). \tag{2.18}$$

$Since\ y(t)=\Phi\left(t,y(t)\right)\in\Phi(t,X)\subset\gamma(t,X)$, there exists $g(t)\in X$ such that

$$\gamma(t,g(t))=y(t)\text{ for } t\in\mathbb{R} \tag{2.19}$$

Again using (E3), *we have*

$$\Omega\left(\alpha\left(t,g(t)\right),y(t)\right)=\Omega\left(\alpha\left(t,g(t)\right),\Phi\left(t,y(t)\right)\right)$$

$$\leq \left\{ max \begin{pmatrix} \frac{\Omega\left(\beta(t,y(t)),\Phi(t,y(t))\right)[\Omega(\alpha(t,g(t)),\gamma(t,g(t))+\Omega(\beta(t,y(t)),\gamma(t,g(t))]}{\Omega\left(\Phi(t,y(t)),\beta(t,y(t))\right)+\Omega\left(\Phi(t,y(t)),\alpha(t,g(t))\right)}, \\ \frac{\Omega\left(\alpha(t,g(t)),\Phi(t,y(t))\right)[\Omega(\beta(t,y(t)),\Phi(t,y(t))+\Omega(\alpha(t,g(t)),\gamma(t,g(t))]}{\Omega\left(\Phi(t,y(t)),\alpha(t,g(t))\right)+\Omega\left(\gamma(t,g(t)),\beta(t,y(t))\right)}, \\ \frac{\Omega(\beta(t,y(t)),\gamma(t,g(t)))\left[\Omega\left(\Phi(t,y(t)),\beta(t,y(t))\right)+\Omega(\gamma(t,g(t)),\alpha(t,g(t))\right]}{\Omega(\beta(t,y(t)),\gamma(t,g(t)))+\Omega\left(\alpha(t,g(t)),\Phi(t,y(t))\right)}, \\ \frac{\Omega\left(\gamma(t,g(t)),\beta(t,y(t))\right)[\Omega(\Phi(t,y(t)),\beta(t,y(t))+\Omega(\Phi(t,y(t)),\alpha(t,g(t))]}{\Omega(\alpha(t,g(t)),\gamma(t,g(t))+\Omega(\beta(t,y(t)),\gamma(t,g(t)))} \end{pmatrix} \right\}^{\lambda}$$

$$\leq \left\{ max \begin{pmatrix} \frac{\Omega(y(t),y(t))[\Omega(\alpha(t,g(t)),y(t))+\Omega(y(t),y(t)]}{\Omega(y(t),y(t))+\Omega\left(y(t),\alpha(t,g(t))\right)}, \\ \frac{\Omega\left(\alpha(t,g(t)),y(t)\right)[\Omega(y(t),y(t))+\Omega(\alpha(t,g(t)),y(t))]}{\Omega\left(y(t),\alpha(t,g(t))\right)+\Omega(y(t),y(t))}, \\ \frac{\Omega(y(t),y(t))[\Omega(y(t),y(t))+\Omega(y(t),\alpha(t,g(t))]}{\Omega(y(t),y(t))+\Omega\left(\alpha(t,g(t)),y(t)\right)}, \\ \frac{\Omega(y(t),y(t))[\Omega(y(t),y(t))+\Omega(y(t),\alpha(t,g(t))]}{\Omega(\alpha(t,g(t)),y(t))+\Omega(y(t),y(t))} \end{pmatrix} \right\}^{\lambda}$$

[using (2.18) and (2.19)]

$\Omega\left(\alpha(t,g(t)),y(t)\right) \leq \Omega^{\lambda}\left(\alpha(t,g(t)),y(t)\right)$, which is a contradiction since $\lambda \in$ *[0,1)*.

Hence,

$$\alpha(t,\Phi(t)) = y(t).$$

Using (2.19), we have

$\alpha(t,g(t)) = \gamma(t,g(t))$=*y(t)* for $t \in \mathbb{R}$ *(2.20)*

Since the pair (α,γ) is compatible, then $\alpha\left(t,\gamma(t,g(t))\right) = \gamma\left(t,\alpha(t,g(t))\right)$, i.e., $(t,y(t)) = \gamma(t,y(t))$. Hence using (2.18) and (2.20) we obtain $\alpha(t,y(t)) = \gamma(t,y(t)) = \Phi(t,y(t)) = \beta(t,y(t)) = y(t)$ for t $\in \mathbb{R}$, that is, y(t) is the common random fixed point of α,γ,Φ *and* β.

Uniqueness

Let *z(t)* be another fixed point

$$\Omega(y(t),z(t)) = \Omega\left(\alpha(t,y(t)),\Phi(t,z(t))\right)$$

$$\leq$$

$$\left\{ \max \left(\begin{array}{c} \frac{\Omega(\beta(t,z(t)),\Phi(t,z(t)))[\Omega(\alpha(t,y(t)),\gamma(t,y(t))+\Omega(\beta(t,z(t)),\gamma(t,y(t))]}{\Omega(\Phi(t,z(t)),\beta(t,z(t)))+\Omega(\Phi(t,z(t)),\alpha(t,y(t)))}, \\ \frac{\Omega(\alpha(t,y(t)),\Phi(t,z(t)))[\Omega(\beta(t,z(t)),\Phi(t,z(t))+\Omega(\alpha(t,y(t)),\gamma(t,y(t))]}{\Omega(\Phi(t,z(t)),\alpha(t,y(t)))+\Omega(\gamma(t,y(t)),\beta(t,z(t)))}, \\ \frac{\Omega(\beta(t,z(t)),\gamma(t,y(t)))[\Omega(\Phi(t,z(t)),\beta(t,z(t)))+\Omega(\gamma(t,y(t)),\alpha(t,y(t))]}{\Omega(\beta(t,z(t)),\gamma(t,y(t)))+\Omega(\alpha(t,y(t)),\Phi(t,z(t)))}, \\ \frac{\Omega(\gamma(t,y(t)),\beta(t,z(t)))[\Omega(\Phi(t,z(t)),\beta(t,z(t))+\Omega(\Phi(t,z(t)),\alpha(t,y(t))]}{\Omega(\alpha(t,y(t)),\gamma(t,y(t))+\Omega(\beta(t,z(t)),\gamma(t,y(t)))} \end{array} \right) \right\}^{\lambda}$$

$$\leq \left\{ \max \left(\begin{array}{c} \frac{\Omega(z(t),z(t))[\Omega(y(t),y(t))+\Omega(z(t),y(t)]}{\Omega(z(t),z(t))+\Omega(z(t),y(t))}, \\ \frac{\Omega(y(t),z(t))[\Omega(z(t),z(t))+\Omega(y(t),y(t))]}{\Omega(z(t),y(t))+\Omega(y(t),z(t))}, \\ \frac{\Omega(z(t),y(t))[\Omega(z(t),z(t))+\Omega(y(t),y(t)]}{\Omega(z(t),y(t))+\Omega(y(t),z(t))}, \\ \frac{\Omega(y(t),z(t))[\Omega(z(t),z(t))+\Omega(z(t),y(t)]}{\Omega(y(t),y(t))+\Omega(z(t)),y(t))} \end{array} \right) \right\}^{\lambda}$$

$\Omega\big(y(t),z(t)\big) \leq \Omega^{\beta}\big(y(t),z(t)\big)$, which is a contradiction since $\lambda \in [0,1)$. $y(t) = z(t)$. This completes the proof.

Similarly we can prove the result using condition (E5).

In theorem if we put $\beta=\gamma = 1$ (the identity mapping), we obtain the corollary

Corollary 2.3. *Let us consider complete multiplicative metric space (X, Ω) and functions*

Φ, α: ℝ x *X→X be two functions satisfying the following conditions*

$(E6)$

$$\Omega\left(\alpha(t,x(t)),\Phi(t,y(t))\right) \leq$$

$$\left\{ \max \left(\begin{array}{c} \frac{\Omega(y(t),\Phi(t,y(t)))[\Omega(\alpha(t,x(t)),x(t))+\Omega(y(t),x(t)]}{\Omega(\Phi(t,y(t)),y(t))+\Omega(\Phi(t,y(t)),\alpha(t,x(t)))}, \\ \frac{\Omega(\alpha(t,x(t)),\Phi(t,y(t)))[\Omega(y(t),\Phi(t,y(t))+\Omega(\alpha(t,x(t)),x(t))]}{\Omega(\Phi(t,y(t)),\alpha(t,x(t)))+\Omega(x(t),y(t))}, \\ \frac{\Omega(y(t),x(t))[\Omega(\Phi(t,y(t)),y(t))+\Omega(x(t),\alpha(t,x(t))]}{\Omega(y(t),x(t))+\Omega(\alpha(t,x(t)),\Phi(t,y(t)))}, \\ \frac{\Omega(x(t),y(t))[\Omega(\Phi(t,y(t)),y(t))+\Omega(\Phi(t,y(t)),\alpha(t,x(t))]}{\Omega(\alpha(t,x(t)),x(t))+\Omega(y(t),x(t))} \end{array} \right) \right\}^{\lambda}$$

For all x, $y \in X$*, where* $t \in \mathbb{R}$ *and* $\lambda \in$ *[0,1). Then* α*, and* Φ *have a unique random fixed point*

The proof of corollary 2.4 is immediate by assuming $\mathbb{R}$ *to be a singleton set in Theorem 2.2.*

Corollary 2.4. *Let us consider complete multiplicative metric space (X, Ω) and a function Φ, α, γ, and β: X→X be four function satisfying the following conditions*

(E7) $\alpha(X) \subset \beta(X)$, $\Phi(X) \subset \gamma(X)$

$$(E8)\quad \Omega(\alpha(x),\Phi(y)) \leq \left\{ max \left(\begin{array}{c} \frac{\Omega(\beta(y),\Phi(y))[\Omega(\alpha(x),\gamma(x))+\Omega(\beta(y),\gamma(x))]}{\Omega(\Phi(y),\beta(y))+\Omega(\Phi(y),\alpha(x))}, \\ \frac{\Omega(\alpha(x),\Phi(y))[\Omega(\beta(y),\Phi(y))+\Omega(\alpha(x),\gamma(x))]}{\Omega(\Phi(y),\alpha(x))+\Omega(\gamma(x),\beta(y))}, \\ \frac{\Omega(\beta(y),\gamma(x))[\Omega(\Phi(y),\beta(y))+\Omega(\gamma(x),\alpha(x)]}{\Omega(\beta(y),\gamma(x)+\Omega(\alpha(x),\Phi(y))}, \\ \frac{\Omega(\gamma(x),\beta(y))[\Omega(\Phi(y),\beta(y))+\Omega(\Phi(y),\alpha(x)]}{\Omega(\alpha(x),\gamma(x))+\Omega(\beta(y),\gamma(x))} \end{array} \right) \right\}^{\lambda}$$

For all x, $y \in X$*, where and* $\lambda \in$ *[0,1).*

Then Φ, α, γ, and β have a unique random fixed point is and only if one of the following conditions are satisfied:

$(E9)$ *either α or γ is continuous, the pair α, γ, is compatible and the pair (β,Φ) is weakly compatible.*

$(E10)$ *either Φ or β is continuous, the pair Φ, β, is compatible and the pair (α, γ) is weakly compatible.*

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