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New Approach on Regular Fuzzy Graph

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Abstract

In this paper, vertex regular fuzzy graph, total degree and totally vertex regular fuzzy graph are introduced. Vertex regular fuzzy graph and totally vertex regular fuzzy graph are compared through various examples and various properties between degree and vertex degree are provided. A necessary and sufficient condition under which they are equivalent is provided. Some properties of vertex regular fuzzy graph are studied and they are examined for totally vertex regular fuzzy graphs.

Keywords: - Degree of an edge, degree of a vertex, vertex regular fuzzy graph, totally vertex regular fuzzy graph, total degree of a vertex.

AMS Subject Classification: 05C72

I. INTRODUCTION

In 1736, the concept of graph theory was first introduced by Euler. The perception of fuzzy set was discussed by L.A. Zadeh, in 1965. In 1973, Kaufmann gave the first definition of a fuzzy graph which was based on Zadeh's fuzzy relations. A. Rosenfeld considered fuzzy relations on fuzzy sets and developed the theory of fuzzy graphs in 1975. Zadeh introduced the concept of fuzzy relations, in 1987. The theory of fuzzy graphs was developed J. N. Mordeson studied fuzzy line graphs and developed its basic properties, in 1993.

In this paper we discussed the concept of vertex regular fuzzy graphs and totally vertex regular fuzzy graphs. A comparative study between vertex regular fuzzy graphs and totally vertex regular fuzzy graph is made. Also some results on vertex regular fuzzy -

-graphs are studied and examined whether they hold for totally vertex regular fuzzy graphs. First we go through some basic definitions which can be found in [1]-[5].

II. BASIC DEFINITIONS

Throughout this paper, we shall denote the edge between two vertices x and y by xy.

Definition 2.1: A fuzzy graph G = (σ, μ) is a pair of functions $\sigma : V \to [0, 1]$ and $\mu : V \times V \to [0, 1]$ with $\mu(x, y) \leq \sigma(x) \wedge \sigma(y)$,

$\forall\, x, y \in V$, where V is a finite non- empty set and ∧ denote minimum. Where σ is a fuzzy subset of a non-empty set V and μ is a symmetric fuzzy relation on σ.

Definition 2.2: Let G = (σ, μ) be a fuzzy graph on G* = (V, E). The degree of an edge xy is $d_G(x,y) = d_G(x) + d_G(y) - 2.\mu(x,y)$.

Definition 2.3: Let G = (σ, μ) be a fuzzy graph on G* = (V, E). The total degree of an edge $xy \in E$ is defined by

$$t,d_G(x,y) = d_G(x) + d_G(y) - \mu(x,y).$$

Definition 2.4: Let G = (σ, μ) be a fuzzy graph on G* = (V, E). The degree of vertex $x \in V$ is $d_G(x) = \sum_{x \neq y} \mu(x, y)$.

The minimum degree of G is $\delta(G) = \wedge \{d_G(y),\ \forall\, y \in V\}$ and the maximum degree of G is $\Delta(G) = \vee \{d_G(y),\ \forall\, y \in V\}$.

Definition 2.5: Let G = (σ, μ) be a fuzzy graph on G* = (V, E). The total degree of vertex $x \in V$ is defined by

$$t.d_G(x) = \sum_{x \neq y} \mu(x, y) + \sigma(x) = d_G(x) + \sigma(x).$$

Example 2.6: Let G be a fuzzy graph

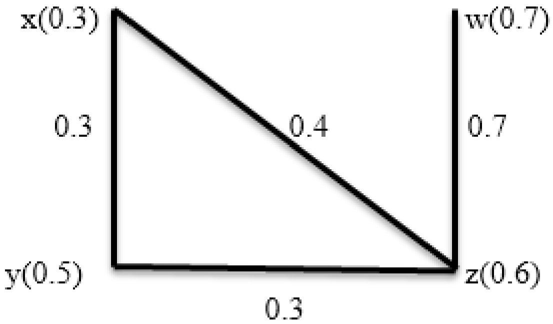


Fig.-2.1: Fuzzy Graph G = (σ, μ)

$d_G(x) = \mu(x,y) + \mu(x,z) = 0.3 + 0.4 = 0.7$; $td_G(x) = d_G(x) + \sigma(x) = 0.7 + 0.3 = 1.0$

$d_G(y) = \mu(x,y) + \mu(y,z) = 0.3 + 0.3 = 0.6$; $td_G(y) = d_G(y) + \sigma(y) = 0.6 + 0.5 = 1.1$

$d_G(z) = \mu(y,z) + \mu(x,z) + \mu(z,w) = 0.3 + 0.4 + 0.7 = 1.4$; $td_G(z) = d_G(z) + \sigma(z) = 1.4 + 0.6 = 2.0$

$d_G(w) = \mu(z,w) = 0.7$; $td_G(w) = d_G(w) + \sigma(w) = 0.7 + 0.7 = 1.4$

$\delta_v(G) = \wedge\{d_G(y),\ \forall y \in V\} = \wedge\{0.7, 0.6, 1.4, 0.7\} = 0.6 = d_G(y)$.

$\Delta_v(G) = \vee\{d_G(y),\ \forall y \in V\} = \vee\{0.7, 0.6, 1.4, 0.7\} = 1.4 = d_G(z)$.

Definition 2.7: The order and size of a fuzzy graph G are defined by $O(G) = \sum_{x \in V} \sigma(x)$ and $S(G) = \sum_{xy \in E} \mu(x,y)$

Theorem 2.8: Let G = (σ, μ) be a fuzzy graph on G* = (V, E). Then $\sum_{y \in V} d_G(y) = 2S(G)$.

Theorem 2.9: Let G = (σ, μ) be a fuzzy graph on G* = (V, E). Then $\sum_{y \in V} td_G(y) = 2S(G) + O(G)$.

Proof: The size of G is $S(G) = \sum_{xy \in E} \mu(x,y)$

∴ By definition of total degree of vertex

$$t.d_G(y) = \sum_{x \neq y} \mu(x, y) + \sigma(y) = d_G(y) + \sigma(y)$$

$$\Rightarrow \sum_{y \in V} t.d_G(y) = \sum_{y \in V} \{d_G(y) + \sigma(y)\}$$

$$\Rightarrow \sum_{y \in V} t.d_G(y) = \sum_{y \in V} d_G(y) + \sum_{y \in V} \sigma(y)$$

Hence, $\sum_{y \in V} t.d_G(y) = 2S(G) + O(G)$. (By theorem 2.8 and theorem 2.7)

III. VERTEX REGULAR FUZZY GRAPH AND TOTALLY VERTEX REGULAR FUZZY GRAPH

Definition 3.1: Let G = (σ, μ) be a fuzzy graph on G* = (V, E). If each vertex in G has same degree k, then G is said to be a vertex regular fuzzy graph or k – vertex regular fuzzy graph.

Definition 3.2: Let G = (σ, μ) be a fuzzy graph on G* = (V, E). If each vertex in G has same total degree k, then G is said to be a totally vertex regular fuzzy graph or k – totally vertex regular fuzzy graph.

Remark 3.3:

1. G is a k – vertex regular fuzzy graph if and only if $\delta_v(G) = \Delta_v(G) = k$.
2. G is a k – totally vertex regular fuzzy graph if and only if $\delta_{tv}(G) = \Delta_{tv}(G) = k$, where $\delta_{tv}(G)$ is the minimum total vertex degree of G and $\Delta_{tv}(G)$ is the maximum total vertex degree of G.

Remark 3.4:

We know that by crisp graph theory, any complete graph is vertex regular. But this result does not follow for fuzzy case.

A complete fuzzy graph need not be vertex regular. For example 3.6, G is not vertex regular, but it is a complete fuzzy graph.

Example 3.5: Consider the following fuzzy graph G = (σ, μ)

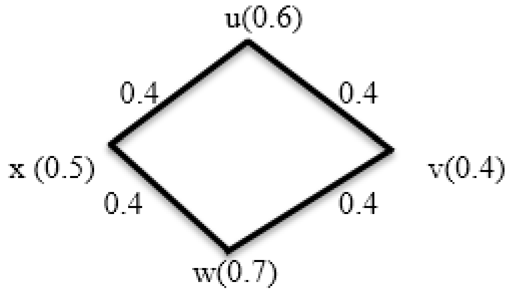


Fig.-3.1

$\delta_v(G) = \wedge\{d_G(y),\ \forall\, y \in V\} = \wedge\{0.8, 0.8, 0.8, 0.8\} = 0.8$ and $\Delta_v(G) = \vee\{d_G(y),\ \forall\, y \in V\} = \vee\{0.8, 0.8, 0.8, 0.8\} = 0.8$.

Therefore, $\delta_v(G) = \Delta_v(G) = 0.8$.

But, $\delta_{tv}(G) = \wedge\{1.4, 1.2, 1.5, 1.3\} = 1.2$ and $\Delta_{tv}(G) = \vee\{1.4, 1.2, 1.5, 1.3\} = 1.5$, since both value of $\delta_{tv}(G)$ and $\Delta_{tv}(G)$ are not equal.

So, G is 0.8 – vertex regular fuzzy graph, but G is not a totally vertex regular fuzzy graph.

Example 3.6: Consider the following fuzzy graph G = (σ, μ)

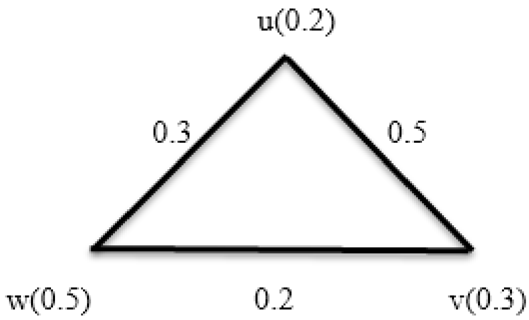


Fig.-3.2

$\delta_v(G) = \wedge\{d_G(y),\ \forall\, y \in V\} = \wedge\{0.8, 0.7, 0.5\} = 0.5$ and $\Delta_v(G) = \vee\{d_G(y),\ \forall\, y \in V\} = \vee\{0.8, 0.7, 0.5\} = 0.8$

Therefore, $\delta_v(G) \neq \Delta_v(G)$.

But, $\delta_{tv}(G) = \wedge\{1.0,\ 1.0,\ 1.0\} = 1.0$ and $\Delta_{tv}(G) = \vee\{1.0,\ 1.0,\ 1.0\} = 1.0$. Therefore, $\delta_{tv}(G) = \Delta_{tv}(G) = 1.0$.

So, G is 1.0 - totally vertex regular fuzzy graph, but G is not a vertex regular fuzzy graph.

Example 3.7: Consider the following fuzzy graph G = (σ, μ)

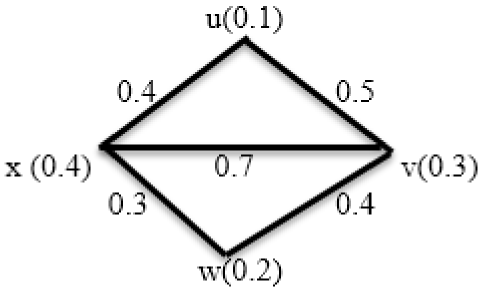


Fig.-3.3

$\delta_v(G) = \wedge\{d_G(y),\ \forall\ y \in V\} = \wedge\{0.9,\ 1.6,\ 0.7,\ 1.4\} = 0.7$ and $\Delta_v(G) = \vee\{d_G(y),\ \forall\ y \in V\} = \vee\{0.9, 1.6, 0.7, 1.4\} = 1.6$.

Therefore , $\delta_v(G) \neq \Delta_v(G)$. So, G is not a vertex regular fuzzy graph.

Also, $\delta_{tv}(G) = \wedge\{1.0, 1.9, 0.9, 1.8\} = 0.9$ and $\Delta_{tv}(G) = \vee\{1.0, 1.9, 0.9, 1.8\} = 1.9$.

Therefore, $\delta_{tv}(G) \neq \Delta_{tv}(G)$. So, G is not a totally vertex regular fuzzy graph.

But, $\delta_E(G) = \wedge\{d_G(xy), \forall\ xy \in E\} = \wedge\{1.5, 1.5, 1.5,1.5\} = 1.5$ and $\Delta_E(G) = \vee\{d_G(xy), \forall\ xy \in E\} = \vee\{1.5, 1.5, 1.5,1.5\} = 1.5$

$\therefore\ \delta_E(G) = \Delta_E(G) = 1.5$. So G is a 1.5 – edge regular fuzzy graph.

So, G is neither vertex regular fuzzy graph nor totally vertex regular fuzzy graph. But G is a 1.5 – edge regular fuzzy graph.

Example 3.8: Consider the following fuzzy graph G = (σ, μ)

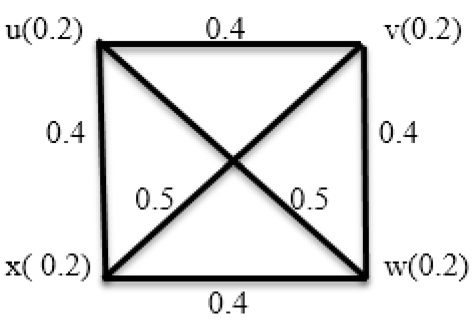


Fig.-3.4

In above figure 3.4, G is both vertex regular fuzzy graph and totally regular fuzzy graph. Also G is edge regular fuzzy graph.

Example 3.9: Consider the following fuzzy graph G = (σ, μ)

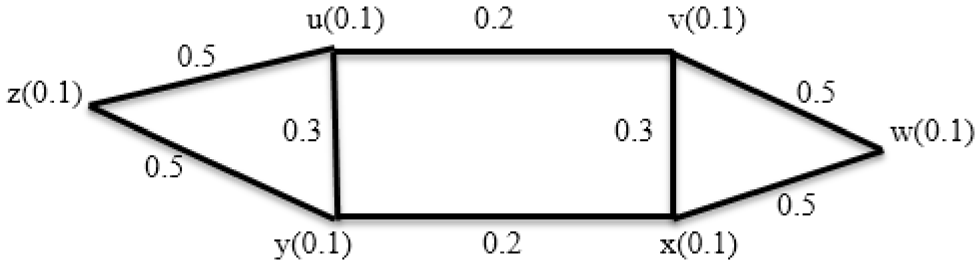


Fig.-3.5

In above figure 3.5, $\delta_v(G) = \wedge\{d_G(x), \forall x \in V\} = \wedge\{1.0, 1.0, 1.0, 1.0, 1.0, 1.0\} = 1.0$ and $\Delta_v(G) = \vee\{d_G(x), \forall x \in V\} =$

$\vee\{1.0, 1.0, 1.0, 1.0, 1.0, 1.0\} = 1.0$. Therefore $\delta_v(G) = \Delta_v(G) = 1.0$ .So, G is 1.0 – vertex regular fuzzy graph.

Also, $\delta_{tv}(G) = \wedge\{1.1, 1.1, 1.1, 1.1, 1.1, 1.1\} = 1.1$ and $\Delta_{tv}(G) = \vee\{1.1, 1.1, 1.1, 1.1, 1.1, 1.1\} = 1.1$.

Therefore, $\delta_{tv}(G) = \Delta_{tv}(G) = 1.1$.

So, G is 1.1 - totally vertex regular fuzzy graph.

But, $\delta_E(G) = \wedge\{d_G(xy), \forall xy \in E\} = \wedge\{1.6, 1.0, 1.0, 1.6, 1.0, 1.0\} = 1.0$ and $\Delta_E(G) = \vee\{d_G(xy), \forall xy \in E\} = \vee\{1.6, 1.0,$
$1.0, 1.6, 1.0, 1.0\} = 1.6$.

$\therefore$ $\delta_E(G) \neq \Delta_E(G)$. So G is a not edge regular fuzzy graph.

Therefore, G is both vertex regular fuzzy graph and totally vertex regular fuzzy graph. But G is not edge regular fuzzy graph.

Remark 3.10: From the above examples, it is clear that in general there does not exist any relationship between vertex regular fuzzy graphs, totally vertex regular fuzzy graphs, edge regular fuzzy graphs and totally edge regular fuzzy graphs. However, a necessary and sufficient condition under which two types of fuzzy graphs, vertex regular fuzzy graphs and totally vertex regular fuzzy graphs are equivalent in some particular case is provided in the following theorem.

Theorem 3.11: Let G = (σ, μ) be a fuzzy graph on G* = (V, E). Then σ is a constant function if and only if the following are
equivalent:

(1) G is a vertex regular fuzzy graph.

(2) G is a totally vertex regular fuzzy graph.

Proof: Suppose that σ is a constant function.

Let σ(x) = c, for every $x \in V$, where c is a constant.

Assume that G is a k1–vertex regular fuzzy graph.

Then d(x) = k1, for all $x \in V$.

$\therefore$ td(x) = d(x) + σ(x), for all $x \in V$.

$\Rightarrow$ td(x) = k1 + c, for all $x \in V$.

Hence G is a (k1 + c) – totally vertex regular fuzzy graph.

Thus (1) $\Rightarrow$ (2) is proved.

Now, suppose that G is a k2 –totally vertex regular fuzzy graph.

Then td(x) = k2, for all $x \in V$.

$\Rightarrow$ d(x) + σ(x) = k2, for all $x \in V$.

$\Rightarrow$ d(x) = k2 - σ(x) , for all $x \in V$.

$\Rightarrow$ d(x) = k2 - c, for all $x \in V$.

Hence G is a (k2 - c) – vertex regular fuzzy graph.

Thus (2) $\Rightarrow$ (1) is proved.

Hence (1) and (2) are equivalent.

Conversely, assume that (1) and (2) are equivalent.

i.e. G is a vertex regular if and only if G is a totally vertex regular fuzzy graph.

To prove that σ is a constant function.

Suppose that σ is not a constant function.

Then $\sigma(x) \neq \sigma(u)$ for atleast one of vertex $x,u \in V$.

Let G is a k –vertex regular fuzzy graph.

Then, $d(x) = d(u) = k$.

$\Rightarrow$ By definition of totally vertex regular fuzzy graph

$td(x) = d(x) + \sigma(x) = k + \sigma(x)$ and $td(u) = d(u) + \sigma(u) = k + \sigma(u)$

$\Rightarrow$ Since $\sigma(x) \neq \sigma(u)$, we have

$\therefore$ $td(x) \neq td(u)$.

Hence G is not a totally vertex regular, which is a contradiction to our assumption.

Now, let G is a totally vertex regular fuzzy graph.

Then, $td(x) = td(u)$.

$\therefore$ $d(x) + \sigma(x) = d(u) + \sigma(u)$

$\Rightarrow$ $d(x) - d(u) = \sigma(u) - \sigma(x)$

$\Rightarrow$ $d(x) - d(u) \neq 0$, (since $\sigma(u) \neq \sigma(x)$)

$\Rightarrow$ $d(x) \neq d(u)$.

Thus, G is not a vertex regular fuzzy graph.

This is a contradiction to our assumption.

Hence σ is a constant function.

Theorem 3.12: If a fuzzy graph G is both vertex regular and totally vertex regular, then σ is a constant function.

Proof: Let G be a k1 – vertex regular and k2 – totally vertex regular fuzzy graph.

Then $d(x) = k1$, for all $x \in V$ and $td(x) = k2$, for all $x \in V$.

Now, $td(x) = k2$, for all $x \in V$.

$\Rightarrow$ By definition of totally vertex regular fuzzy graph

$\therefore$ $d(x) + \sigma(x) = k2$, for all $x \in V$.

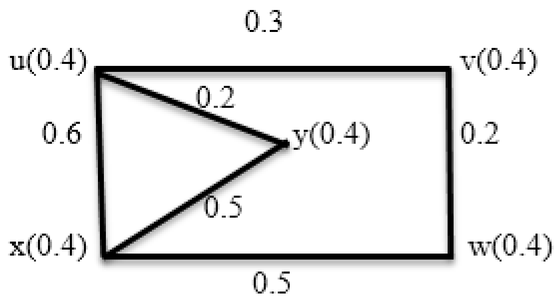
$\Rightarrow$ $k1 + \sigma(x) = k2$, for all $x \in V$. (since $d(x) = k1$)

$\Rightarrow$ $\sigma(x) = k2 - k1$, for all $x \in V$.

Hence σ is a constant function.

Remark 3.13: The converse of theorem 3.12 need not be true. It can be seen from the following example.

Consider G* = (V, E) where V = (u, v, w, x, y) and E = (uv, vw, wx, xy, xu, uy). Define G = (σ, μ) by σ(u) = σ(v)= σ(w)= σ(x)= σ(y)= 0.4 and μ(uv) = 0.3, μ(vw) = 0.2, μ(wx) = 0.5, μ(xy) = 0.5, μ(yu) = 0.2, μ(ux) = 0.6. Then σ is a constant function. But d(u) $\neq$ d(v) $\neq$ d(w) $\neq$ d(x) $\neq$ d(y). Also td(u) $\neq$ td(v) $\neq$ td(w) $\neq$ td(x) $\neq$ td(y)



So, σ is a constant function, but G is not a vertex regular fuzzy graph and also not a totally regular fuzzy graph.

Theorem 3.14: Let μ = c be a constant function in G = (σ, μ) on G* = (V, E). If G is edge regular, then G is vertex regular.

Proof: Given that μ = c be a constant function in G = (σ, μ).

Assume that G is edge regular fuzzy graph with d(xy) = k, for all xy∈ E.

To prove that G is vertex regular fuzzy graph.

$\Rightarrow$ By definition of vertex degree,

$d_G(x) = \sum_{x \neq y} \mu(xy)$, for all x∈ V.

$\Rightarrow \therefore$ $d_G(x)$ = c, for all x∈ V.

Hence G is vertex regular.

Theorem 3.15: Let σ = c’ be a constant function in G = (σ, μ) on G* = (V, E). If G is edge regular, then G is totally vertex regular. **Proof:** Given that σ = c’ be a constant function in G = (σ, μ).

Assume that G is edge regular fuzzy graph with d(xy) = k, for all xy∈ E.

To prove that G is totally vertex regular fuzzy graph.

$\Rightarrow$ By definition of totally vertex degree,

$td_G(x) = \sum_{x \neq y} \mu(x, y) + \sigma(x)$, for all x∈ V.

$= d_G(x) + c'$, for all x∈ V.

= c + c’, for all x∈ V (from above theorem 3.14 and given σ = c’).

⇒ ∴ $td_G(x)$ = constant, for all x∈ V.

Hence G is totally vertex regular.

Remark 3.16: The converse of above theorem 3.15 need not be true from example 3.9.

Definition 3.17: Let G* = (V, E) be a graph. Then G* is said to be vertex regular, if each vertex in G* has same degree.

Theorem 3.18: Let G = (σ, μ) be a fuzzy graph on G* = (V, E). If μ is a constant function, then G is vertex regular fuzzy graph if and only if G* is vertex regular graph.

Proof: Given that μ is a constant function.

Then μ(xy) = c, where c is constant.

Assume that G is vertex regular fuzzy graph.

To prove G* is vertex regular graph.

Suppose that G* is not vertex regular graph.

Then $d_{G^*}(x) \neq dG^*(y)$ for at least one vertex x,y ∈V.

⇒ By the definition of vertex degree of a fuzzy graph,

$$d_G(x) = \sum_{x\neq y} \mu(xy) \text{ , for all } x\in V.$$

$$= \sum_{x\neq y} c \text{ , for all } x\in V.$$

⇒ ∴ $d_G(x) = c(d_{G^*}(x))$, for all x∈ V (By the definition of vertex degree in a graph)

⇒ Similarly, $d_G(y) = c(d_{G^*}(y))$, for all y∈ V.

⇒ Since, $d_{G^*}(x) \neq dG^*(y)$.

⇒ ∴ $d_G(x) \neq dG(y)$.

Thus, G is not vertex regular fuzzy graph.

This is contradiction to our assumption.

Therefore, G* is vertex regular graph.

Conversely, let μ is a constant function and G* is vertex regular graph.

To prove G is vertex regular graph.

Suppose that G is not vertex regular graph.

Then, $d_G(x) \neq dG(y)$ for at least one vertex x,y ∈V.

$$\therefore \sum_{x\neq y} \mu(xy) \neq \sum_{y\neq z} \mu(yz)$$

$\Rightarrow \quad \sum_{x \neq y} c \neq \sum_{y \neq z} c$

$\Rightarrow$ $c(d_{G^*}(x)) \neq c(d_{G^*}(y))$ (By the definition of vertex degree in a graph)

$\Rightarrow \therefore$ $d_{G^*}(x) \neq dG^*(y)$.

Thus G* is not vertex regular fuzzy graph.

This is contradiction to our assumption.

Therefore, G is vertex regular fuzzy graph.

Theorem 3.19: Let G = (σ, μ) be a fuzzy graph on G* = (V, E). Then G is vertex regular if and only if μ is a constant function.

Proof: Let G is k-vertex regular fuzzy graph.

Then $d_G(x) = k$, for all x ∈ V.

To prove that μ is a constant function.

$\Rightarrow$ By definition of vertex degree,

$d_G(x) = \sum_{x \neq y} \mu(xy)$, for all x ∈ V.

$\Rightarrow \quad k = \sum_{x \neq y} \mu(xy)$

$\Rightarrow \therefore \sum_{x \neq y} \mu(xy) = k$, for all x, y∈ V.

So, μ is a constant function.

Conversely, let μ is a constant function.

Then μ = c.

To prove that G is vertex regular fuzzy graph.

$\Rightarrow$ By definition of vertex degree,

$\therefore$ $d_G(x) = \sum_{x \neq y} \mu(xy)$, for all x ∈ V.

$\Rightarrow \quad d_G(x) = c$, for all x ∈ V.

So, G is vertex regular fuzzy graph.

IV. PROPERTIES OF VERTEX REGULAR FUZZY GRAPHS

Theorem 4.1: The size of a k-vertex regular fuzzy graph G = (σ, μ) on G* = (V, E) is $\frac{\alpha k}{2}$ where α = | V |.

Proof: The size of G is $S(G) = \sum_{xy \in E} \mu(x, y)$.

Since G is a k-vertex regular fuzzy graph, then $d_G(x) = k$, for all x ∈ V.

By definition of vertex degree,

$$d_G(x) = \sum_{xy\in E} \mu(xy) \text{, for all } x\in V.$$

$$\therefore \quad \sum_{x\in V} d_G(x) = 2\sum_{xy\in E} \mu(x,y).$$

$$\Rightarrow \quad \sum_{x\in V} d_G(x) = 2\,S(G).$$

$$\Rightarrow \quad 2\,S(G) = \sum_{x\in V} d_G(x).$$

$$\Rightarrow \quad 2\,S(G) = \sum_{x\in V} k.$$

$$\Rightarrow \quad 2\,S(G) = \alpha k.$$

$$\Rightarrow \text{So, } \quad S(G) = \frac{\alpha k}{2}$$

Theorem 4.2: If G = (σ, μ) is a k1- totally vertex regular fuzzy graph on G* = (V, E), then 2S(G) + O(G) = αk ,where α = | V |.

Proof: Since G is a k-totally vertex regular fuzzy graph, $td_G(x)$ = k1 for all x∈ V.

$\Rightarrow$ By definition of totally vertex degree,

$$t.d_G(x) = d_G(x) + \sigma(x) \text{, for all } x\in V.$$

$$\Rightarrow \quad k1 = d_G(x) + \sigma(x) \text{, for all } x\in V.$$

$$\Rightarrow \quad \sum_{x\in V} k1 = \sum_{x\in V} d_G(x) + \sum_{x\in V} \sigma(x) \text{, for all } x\in V.$$

$$\Rightarrow \quad \alpha k1 = 2\sum_{xy\in E} \mu(x,y) + \sum_{x\in V} \sigma(x) \text{, for all } x\in V.$$

$$\Rightarrow \text{So, } \quad \alpha k1 = 2S(G) + O(G).$$

Corollary 4.3: If G is a k-vertex regular and a k1- totally vertex regular fuzzy graph, then O(G) = α(k1 –k).

Proof: From Theorem 4.1,

$$S(G) = \frac{\alpha k}{2}.$$

$$\Rightarrow \quad \therefore 2S(G) = \alpha k.$$

$\Rightarrow$ From Theorem 4.2,

$$2S(G) + O(G) = \alpha k1.$$

$$\Rightarrow \quad O(G) = \alpha k1 - 2S(G).$$

$$\Rightarrow \quad O(G) = \alpha k1 - \alpha k.$$

$$\Rightarrow \quad \text{So, } O(G) = \alpha(k1 - k).$$

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