Global Journal of Pure and Applied Mathematics.
ISSN 0973-1768 Volume 13, Number 7 (2017), pp. 3909-3918

http://www.ripublication.com

Quotient -3 Cordial Labeling for Star Related Graphs

P. Sumathi
*Department of Mathematics, C. Kandaswami Naidu College for Men,
Anna Nagar, Chennai 600 102, India.*

A. Mahalakshmi
*Department of Mathematics, Srimuthukumaran Institute of Technology,
Mangadu Chennai-600122, India.*

A. Rathi
*Department of Mathematics, Dr. M.G.R. Educational and Research Institute,
University, Maduravoyal, Chennai 600 095, India.*

Abstract

In this paper, a new type of labeling namely quotient-3 cordial labeling f is introduced. Let G (V, E) be a simple graph of order p and size q. Let f : V(G)→ Z_4 –{0} be a function. For each edge set E(G) define the labeling f*: E(G) → Z_3 by f*(uv) = $\left\lceil \frac{f(u)}{f(v)} \right\rceil$ (mod 3) where f(u) ≥ f(v). The function f is called quotient-3 cordial labeling of G if the number of vertices labeled with i and the number of vertices labeled with j differ by at most 1, the number of edges labeled with k and the number of edges labeled with l differ by at most 1 where $1 \leq i, j \leq 3$, $i \neq j$ and $0 \leq k, l \leq 2$, $k \neq l$. Here it is proved that S_n and some star related graphs like $S(S_n)$, $sp(P_n, K_{1,n})$, $B_{n,n}$, $S(B_{n,n})$ are quotient-3 cordial graphs and also proved that a graph obtained by attaching a star S_n with any vertex of C_3 are quotient-3 cordial graphs.

Key words: Star, bistar, subdivision graph, quotient-3 cordial graph.
AMS Subject Classification: 05C78

1. INTRODUCTION

Here the graphs considered are finite, simple, undirected and non trivial. Graph theory has a good development in the graph labeling and has a broad range of applications. Refer Gallian [5] for more information. The cordial labeling concept was first introduced by cahit [2]. We have introduced quotient-3 cordial labeling. A graph G is said to be quotient-3 cordial graph if it receives quotient-3 cordial labeling. Let $v_f(i)$ denotes the number of vertices labeled with i and $e_f(k)$ denotes the number of edges labeled with k ,$1 \leq i \leq 3$, $0 \leq k \leq 2$.

2. DEFINITIONS

Definition: 2.1 Let G (V, E) be a simple graph of order p and size q. Let f : $V(G) \to Z_4 - \{0\}$ be a function. For each edge set E(G) define the labeling $f^* : E(G) \to Z_3$ by $f^*(uv) = \left\lceil \frac{f(u)}{f(v)} \right\rceil \pmod 3$ where $f(u) \geq f(v)$. The function f is called quotient-3 cordial labeling of G if the number of vertices labeled with i and the number of vertices labeled with j differ by at most 1, the number of edges labeled with k and the number of edges labeled with l differ by at most 1 where $1 \leq i, j \leq 3$, $i \neq j$ and $0 \leq k, l \leq 2$, $k \neq l$.

Definition: 2.2 A complete bipartite graph $K_{1,n}$ is said to be a star graph and it is denoted by S_n.

Definition: 2.3 sp (P_n, $k_{1,n}$) is obtained from a path P_n by attaching the centre of the star $K_{1,n}$ at one end of the path P_n.

Definition: 2.4 Two copies of $k_{1,n}$ are joined by the centre vertices with an edge is called a bistar and is denoted by $B_{n,n}$.

Definition: 2.5 From the graph G, a new graph is obtained by subdividing each edge of G with a new vertex is called subdivision of G and it is denoted by S(G).

3. MAIN RESULT

Theorem: 3.1 Any star S_n is quotient-3 cordial.

Proof: Let $V(S_n) = \{ u, v_i : 1 \leq i \leq n\}$ and $E(S_n) = \{(uv_i) : 1 \leq i \leq n\}$.

Here $|V(G)| = n + 1$, $|E(G)| = n$.

Define f : V(G) →{1,2,3} by f(u) = 1
For i , $1 \leq i \leq n$.
$f(v_i) = 3$, if $i \equiv 1 \pmod 3$
$f(v_i) = 2$, if $i \equiv 2 \pmod 3$
$f(v_i) = 1$, if $i \equiv 0 \pmod 3$

Table 1 shows that the star S_n is quotient-3 cordial.

Nature of n	**$v_f(1)$**	**$v_f(2)$**	**$v_f(3)$**
$n \equiv 0 \pmod 3$	$\frac{n}{3}+1$	$\frac{n}{3}$	$\frac{n}{3}$
$n \equiv 1 \pmod 3$	$\frac{n+2}{3}$	$\frac{n+2}{3}-1$	$\frac{n+2}{3}$
$n \equiv 2 \pmod 3$	$\frac{n+1}{3}$	$\frac{n+1}{3}$	$\frac{n+1}{3}$

Nature of n	**$e_f(0)$**	**$e_f(1)$**	**$e_f(2)$**
$n \equiv 0 \pmod 3$	$\frac{n}{3}$	$\frac{n}{3}$	$\frac{n}{3}$
$n \equiv 1 \pmod 3$	$\frac{n+2}{3}$	$\frac{n+2}{3}-1$	$\frac{n+2}{3}-1$
$n \equiv 2 \pmod 3$	$\frac{n+1}{3}$	$\frac{n+1}{3}-1$	$\frac{n+1}{3}$

Table 1

Illustration: Figure 1 gives the quotient-3 cordial labeling for S_8.

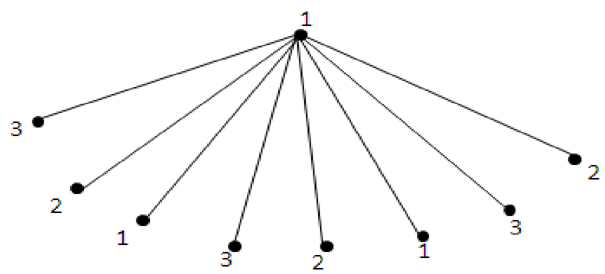

Figure 1

Theorem: 3.2 The graph $S(S_n)$ is quotient-3 cordial.

Proof: Let $V(G) = \{ u, u_i, v_i : 1 \leq i \leq n\}$ and $E(G) = \{(uu_i), (u_iv_i) : 1 \leq i \leq n\}$.

Here $|V(G)| = 2n + 1$, $|E(G)| = 2n$.

Define $f : V(G) \rightarrow \{1,2,3\}$ by $f(u)=1$

Labeling of the vertices u_i and v_i, $1 \leq i \leq n$ is given below.

Case (i): When $n \equiv 0, 1 \pmod 3$

For i , $1 \leq i \leq n$

$f(u_i) = 3$, if $i \equiv 1,2 \pmod 3$

$f(u_i) = 1$, if $i \equiv 0 \pmod 3$

$f(v_i) = 2$, if $i \equiv 1,2 \pmod 3$

$f(v_i) = 1$, if $i \equiv 0 \pmod 3$

Case (ii): When $n \equiv 2 \pmod 3$

Labeling of u_i and v_i, $1 \leq i \leq n-1$ are same as in case (i)

Assign $f(u_n) = 1$, $f(v_n) = 3$.

Table 2 gives that the graph $S(S_n)$ is quotient-3 cordial

Nature of n	$v_f(1)$	$v_f(2)$	$v_f(3)$
$n \equiv 0 \pmod 3$	$\frac{2n}{3}+1$	$\frac{2n}{3}$	$\frac{2n}{3}$
$n \equiv 1 \pmod 3$	$\frac{2n+1}{3}$	$\frac{2n+1}{3}$	$\frac{2n+1}{3}$
$n \equiv 2 \pmod 3$	$\frac{2n-1}{3}+1$	$\frac{2n-1}{3}$	$\frac{2n-1}{3}+1$

Nature of n	$e_f(0)$	$e_f(1)$	$e_f(2)$
$n \equiv 0 \pmod 3$	$\frac{2n}{3}$	$\frac{2n}{3}$	$\frac{2n}{3}$
$n \equiv 1 \pmod 3$	$\frac{2n+1}{3}$	$\frac{2n+1}{3}-1$	$\frac{2n+1}{3}$
$n \equiv 2 \pmod 3$	$\frac{2n-1}{3}+1$	$\frac{2n-1}{3}$	$\frac{2n-1}{3}$

Table 2

Illustration: Figure 2 shows that the quotient -3 cordial labeling S (S_7).

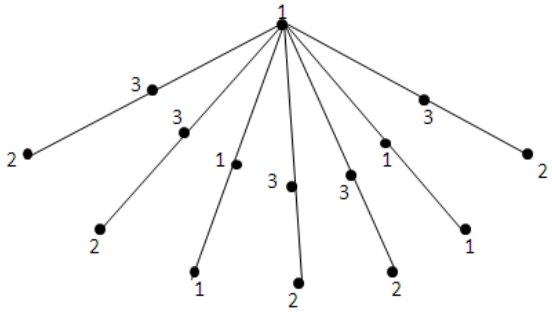

Figure 2

Theorem: 3.3 The graph sp(P_n, $K_{1,n}$) is quotient-3 cordial.

Proof: Let $V(G)=\{u_i, v_i : 1 \le i \le n\}$ and $E(G)=\{[(u_i u_{i+1}) : 1 \le i \le n-1] \cup [(u_1 v_i) : 1 \le i \le n]\}$

Here $|V(G)| = 2n$, $|E(G)| = 2n-1$.

Define $f : V(G) \to \{1,2,3\}$

Label the vertices u_i, $1 \le i \le n$ as follows.

$f(u_i) = 1$, if $i \equiv 1,2 \pmod 6$

$f(u_i) = 3$, if $i \equiv 0,3 \pmod 6$

$f(u_i) = 2$, if $i \equiv 4,5 \pmod 6$

Labeling of v_i, $1 \le i \le n$ is given below.

Case (i): When $n \equiv 0, 1, 2, 4, 5 \pmod 6$

For i, $1 \le i \le n$

$f(v_i) = 3$, if $i \equiv 1 \pmod 3$

$f(v_i) = 2$, if $i \equiv 2 \pmod 3$

$f(v_i) = 1$, if $i \equiv 0 \pmod 3$

Case (ii): When $n \equiv 3 \pmod 6$

Label the vertices v_i, $1 \le i \le n-1$ as in case (i)

Assign $f(v_n) = 2$.

Table 3 gives the quotient-3 cordial for sp(P_n, $K_{1,n}$)

Nature of n	$v_f(1)$	$v_f(2)$	$v_f(3)$
n ≡ 0,3 (mod 6)	$\frac{2n}{3}$	$\frac{2n}{3}$	$\frac{2n}{3}$
n ≡ 1,4 (mod 6)	$(\frac{2n+1}{3})$	$(\frac{2n+1}{3})-1$	$(\frac{2n+1}{3})$
n ≡ 2(mod 6)	$(\frac{2n-1}{3})+1$	$(\frac{2n-1}{3})$	$(\frac{2n-1}{3})$
n ≡ 5 (mod 6)	$(\frac{2n-1}{3})$	$(\frac{2n-1}{3})+1$	$(\frac{2n-1}{3})$

Nature of n	$e_f(0)$	$e_f(1)$	$e_f(2)$
n ≡ 0 (mod 6)	$\frac{2n}{3}-1$	$\frac{2n}{3}$	$\frac{2n}{3}$
n ≡ 1,4 (mod 6)	$(\frac{2n+1}{3})$	$(\frac{2n+1}{3})-1$	$(\frac{2n+1}{3})-1$
n ≡ 2,5(mod 6)	$(\frac{2n-1}{3})$	$(\frac{2n-1}{3})$	$(\frac{2n-1}{3})$
n ≡ 3 (mod 6)	$\frac{2n}{3}$	$\frac{2n}{3}-1$	$\frac{2n}{3}$

Table 3

Theorem: 3.4 The Bistar $B_{n,n}$ is quotient-3 cordial.

Proof: Let V(G)={u,v, u_i, v_i : $1 \le i \le n$} and E(G)={ (uv),(uu_i),(vv_i) : $1 \le i \le n$}

Here $|V(G)| = 2n + 2$, $|E(G)|$ = 2n+1.

Define f : V(G)→{1,2,3}

Assign f(u) = 1 and f(v) = 1 for all n

When n = 1, $f(u_1) = 2$ and $f(v_1) = 3$

When n > 1, labeling of u_i, $1 \le i \le n$ is given below

$f(u_i) = 3$, if $i \equiv 1 \pmod 3$

$f(u_i) = 2$, if $i \equiv 2 \pmod 3$

$f(u_i) = 1$, if $i \equiv 0 \pmod 3$

Labeling of v_i, $1 \le i \le n$ is given below

Case (i): When $n \equiv 2 \pmod 3$

$f(v_i) = 3$, if $i \equiv 1 \pmod 3$

$f(v_i) = 2$, if $i \equiv 2 \pmod 3$

$f(v_i) = 1$, if $i \equiv 0 \pmod 3$

Case (ii): When n ≡ 0, 1 (mod 3)

Labeling the vertices v_i, $1 \le i \le n-1$ are same as in case (i)

Assign $f(v_n) = 2$.

Table 4 shows that the graph bistar is quotient-3 cordial.

Nature of n	$v_f(1)$	$v_f(2)$	$v_f(3)$
n ≡ 0 (mod 3)	$\frac{2n}{3}+1$	$\frac{2n}{3}+1$	$\frac{2n}{3}$
n ≡ 1 (mod 3)	$(\frac{2n+1}{3})+1$	$(\frac{2n+1}{3})$	$(\frac{2n+1}{3})$
n ≡ 2(mod 3)	$(\frac{2n+2}{3})$	$(\frac{2n+2}{3})$	$(\frac{2n+2}{3})$

Nature of n	$e_f(0)$	$e_f(1)$	$e_f(2)$
n ≡ 0 (mod 3)	$\frac{2n}{3}$	$\frac{2n}{3}$	$\frac{2n}{3}+1$
n ≡ 1 (mod 3)	$(\frac{2n+1}{3})$	$(\frac{2n+1}{3})$	$(\frac{2n+1}{3})$
n ≡ 2(mod 3)	$(\frac{2n+2}{3})$	$(\frac{2n+2}{3})-1$	$(\frac{2n+2}{3})$

Table 4

Theorem : 3.5 The graph $S(B_{n,n})$ is quotient-3 cordial.

Proof: : Let V(G) = {u, v, w, u_i, v_i, x_i, y_i : $1 \le i \le n$} and E(G) = {(uv),(vw),(uu_i),(u_iv_i), (wx_i),(x_iy_i) : $1 \le i \le n$}

Here $|V(G)| = 4n + 3$, $|E(G)|$ = 4n+2.

Define f : V(G) →{1,2,3} by f(u) = f(v) = f(w) = 1

$f(u_i) = 3$, $1 \le i \le n$

$f(v_i) = 2$, $1 \le i \le n$

Labeling of x_i and y_i , $1 \le i \le n$ are given below

Case (i): When n ≡ 0 (mod 3)

$f(x_i) = 3$, $1 \le i \le \frac{n+3}{3}$

$f(x_i) = 1$, $\frac{n}{3}+2 \le i \le n$

$f(y_i) = 2$, $1 \le i \le \frac{n+3}{3}$

$f(y_i) = 1$, $\frac{n}{3} + 2 \le i \le n$

Case (ii): When $n \equiv 1 \pmod 3$

$f(x_i) = 3$, $1 \le i \le \frac{n+2}{3}$

$f(x_i) = 1$, $\frac{n+2}{3} + 1 \le i \le n$

$f(y_i) = 2$, $1 \le i \le \frac{n+2}{3}$

$f(y_i) = 1$, $\frac{n+2}{3} + 1 \le i \le n$

Case (iii): When $n \equiv 2 \pmod 3$

$f(x_i) = 3$, $1 \le i \le \frac{n+1}{3}$

$f(x_i) = 1$, $\frac{n+1}{3} + 1 \le i \le n$

$f(y_i) = 2$, $1 \le i \le \frac{n+4}{3}$

$f(y_i) = 1$, $\frac{n+4}{3} + 1 \le i \le n$

Table 5 gives the quotient-3 cordial for $S(B_{n,n})$

Nature of n	$v_f(1)$	$v_f(2)$	$v_f(3)$
$n \equiv 0 \pmod 3$	$\frac{4n}{3}+1$	$\frac{4n}{3}+1$	$\frac{4n}{3}+1$
$n \equiv 1 \pmod 3$	$(\frac{4n+2}{3})+1$	$(\frac{4n+2}{3})$	$(\frac{4n+2}{3})$
$n \equiv 2 \pmod 3$	$(\frac{4n+1}{3})+1$	$(\frac{4n+1}{3})+1$	$(\frac{4n+1}{3})$

Nature of n	$e_f(0)$	$e_f(1)$	$e_f(2)$
$n \equiv 0 \pmod 3$	$\frac{4n}{3}+1$	$\frac{4n}{3}$	$\frac{4n}{3}+1$
$n \equiv 1 \pmod 3$	$(\frac{4n+2}{3})$	$(\frac{4n+2}{3})$	$(\frac{4n+2}{3})$
$n \equiv 2 \pmod 3$	$(\frac{4n+1}{3})$	$(\frac{4n+1}{3})$	$(\frac{4n+1}{3})+1$

Table 5

Theorem: 3.6 A graph obtained by attaching a star S_n with any vertex of C_3 are quotient-3 cordial.

Proof: Let $V(G) = \{u_i, v_j : 1 \le i \le 3 , 1 \le j \le n \}$ and $E(G) = \{ [(u_iu_{i+1}) ; 1 \le i \le 2] \cup [(u_3v_j) : 1 \le j \le n] \}$

Here $|V(G)| = n + 3$, $|E(G)|$ = n+3.

Define f : V(G) →{1,2,3} by $f(u_1) = f(u_2) = 1$, $f(u_3) = 3$

Labeling of v_j , $1 \le j \le n$ is given below

Assign the label 2, 2 and 3 for the first three vertices v_1, v_2 and v_3 respectively.

For j, $4 \le j \le n$

$f(v_j) = 1$, if $j \equiv 1 \pmod 3$

$f(v_j) = 2$, if $j \equiv 2 \pmod 3$

$f(v_j) = 3$, if $j \equiv 0 \pmod 3$

The following table 6 gives the quotient-3 cordial.

Nature of n	$v_f(1)$	$v_f(2)$	$v_f(3)$
$n \equiv 0 \pmod 3$	$\frac{n+3}{3}$	$\frac{n+3}{3}$	$\frac{n+3}{3}$
$n \equiv 1 \pmod 3$	$(\frac{n+2}{3})+1$	$(\frac{n+2}{3})$	$(\frac{n+2}{3})$
$n \equiv 2 \pmod 3$	$(\frac{n+4}{3})$	$(\frac{n+4}{3})$	$(\frac{n+4}{3})-1$

Nature of n	$e_f(0)$	$e_f(1)$	$e_f(2)$
$n \equiv 0 \pmod 3$	$\frac{n+3}{3}$	$\frac{n+3}{3}$	$\frac{n+3}{3}$
$n \equiv 1 \pmod 3$	$(\frac{n+2}{3})+1$	$(\frac{n+2}{3})$	$(\frac{n+2}{3})$
$n \equiv 2 \pmod 3$	$(\frac{n+4}{3})$	$(\frac{n+4}{3})-1$	$(\frac{n+4}{3})$

Table 6

Illustration: Figure 3 gives the quotient-3 cordial labeling.

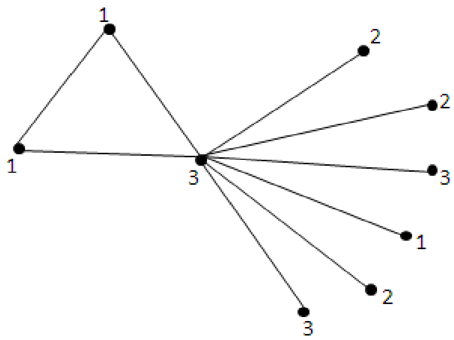

Figure 3

4. CONCLUSION

In this paper, it is proved that some star related graphs which admits quotient-3 cordial. The existence of quotient-3 cordial labeling of different families of graphs will be the future work.

5. ACKNOWLEDGMENT

Sincerely register our thanks for the valuable suggestions and feedback offered by the referees.

REFERENCES

[1] Albert William, Indra Rajasingh and S Roy, Mean Cordial Labeling of Certain graphs, J.Comp.& Math. Sci. Vol.4 (4),274-281 (2013)

[2] I. Cahit and R. Yilmaz, E3-cordial graphs, Ars Combin., 54 (2000) 119-127

[3] S. Freeda and R. S. Chellathurai, H- and H2-cordial labeling of some graphs, Open J. Discrete Math., 2 (2012) 149-155.

[4] P. Jeyanthi and A. Maheswari, 3-product cordial labeling, SUT J. Math., 48 (2) (2012) 231-240.

[5] Joseph A. Gallian, A Dynamic survey of Graph Labeling , Nineteenth edition, December 23, 2016

[6] A.Nellai Murugan and S.Heerajohn, Special Class of Mean square cordial graphs, International Journal of Applied Research 2015; 1(11): 128-131.

[7] A.Nellai Murugan and S.Heerajohn, Path related mean square cordial graphs, International Journal of Emerging Technologies in Engineering Research (IJETER),Volume 2,Issue 3, October-2015.

[8] S K Vaidya and N H Shah, on square Divisor Cordial Graphs, Journal of Scientific Research, J.Sci.Res. 6(3), 445-455 (2014)