Global Journal of Pure and Applied Mathematics.
ISSN 0973-1768 Volume 13, Number 9 (2017), pp. 5805-5811

http://www.ripublication.com

A Statistical Approach for Estimating the Fuzzy Reliability of the Web Server

R. Neelambari [1]

[1] *Department of Mathematics, Saranathan College of Engineering, Trichy, Tamil Nadu, India.*

Abstract

The fuzzy reliability and performance of webserver, based on the reliability of the intermediate systems, between the server and ultimate end user is studied. A Statistical approach has been employed for finding the triangular fuzzy number.

1. INTRODUCTION

Various kind of data's are being transferred from the server to ultimate user by means of Internet using a long chain of systems and services . Subsequent delays or failure at each node of this link will leads to the total failure and latency in web services, that is the failure of any one system or service in the path between server and user will in turn affect the failure of the data transfer as far as the user is concerned.

The Intermediate systems called subsystems are inherently distributed and requires the cooperation between a server and a client system in order to accomplish their tasks and it offer a platform for universal access to web resources for immense user population.

In the refered papers by B. Prabha [5] and Razzak[1], the authors modified probability assumption as the fuzzy-state assumption: At any given time, the system has only two states, one is the fuzzy success state and another is the fuzzy failure state. They followed the same methodology as in Cai and Zhang [2] Cheng and Mon [3] all the authors mentioned above did not initially apply a statistical methodology. They used the α-cut of a triangular fuzzy number to get the interval and find the fuzzy reliability of the serial system and the parallel system. Chen likewise omitted the statistical approach when he used fuzzy numbers to find the fuzzy reliability of both the serial system and the parallel system. Jing-Shing Yao [4], used statistical data method for estimating the reliability of the parallel and serial systems using t distribution. In this paper we use a similar method to find the fuzzy reliability of the system using statistical point estimator.

2. FUZZY SYSTEM RELIABILITY MODEL

In most server architectures, the unreliability of any one sub-system or failure of their service in the path between server and user cause a serious effect in the reliability of the entire system as far as the user is concerned. Hence study of the reliability of the system plays a vital role in managing this whole complex system to provide the desired level of service to an end user.

Suppose if there be infinite number of user seeking for some information in a web server say W_i, with different arrival rate. Let there be subsystems S_1, S_2, …, S_n between the web server and the end users. Suppose the reliabilities of each subsystem R_1, R_2, …, R_n ($0 \leq R_j \leq 1$, $j = 1, 2, \ldots, n$) are unknown. Then the sample reliability of the subsystems *Sj are considered from for k days from the past datas*. Let the statistical data be denoted by $R_{jk} \in [0, 1]$, Let *Rj* (unknown) be the population reliability of the webserver *Sj. Then .Rj* is given by

$$\bar{R}_j = (1/n_j) \sum_{k=1}^{n_j} R_{jk}. \tag{1}$$

The reliability of the of a series system is denoted by

$$1 - \prod_{j=1}^{n} (1 - R_j) \tag{2}$$

From the statistical point of view, the point estimate of *Rj is* $\bar{R}_j$. Therefore, $\bar{R}_j$ can be used instead of the reliability of S_j. Hence the reliability of the system can be given by

$$1 - \prod_{j=1}^{n} (1 - \bar{R}_j) \tag{3}$$

3. ESTIMATION OF FUZZY RELIABILITY OF THE WEB SERVERS

The interval estimate for *Rj* is expressed as: $\bar{R}_j - Z(\eta_1)\left({s_j}/{\sqrt{n_j}}\right), \bar{R}_j, \bar{R}_j + Z(\eta_1)\left({s_j}/{\sqrt{n_j}}\right)$. Assuming η_1=.05, we can say that, in repeated sampling, 95% of the intervals constructed this way will include *Rj* . This is based on the probability of occurrence of different values of $\bar{R}_j$.With repeated sampling from a normally distributed population with a known standard deviation, 100(1-α) percent of all intervals in the form $\bar{R}_j \pm Z(\eta)\left({s_j}/{\sqrt{n_j}}\right)$ will, in the long run, include the population reliability, *Rj*. The quantity 1-α is called the *confidence coefficient* or *confidence level* and the interval, $\bar{R}_j \pm Z(\eta)\left({s_j}/{\sqrt{n_j}}\right)$, is called the confidence interval for *Rj*, j=1,2,3,…n and $S_j^{\,2} = {1}/{(n_j-1)}\sum_{m=1}^{n_j}\left(R_{jm}-\bar{R}_j\right)^2$. Let Z be a normal random variable. $z(\eta)$ satisfies the following conditions:

i) $P\left(Z \geq z(\eta_k)\right) = \eta_k, k = 1,2.$ (4)

ii) $0<\bar{R}_j - Z(\eta_1)\left({s_j}/{\sqrt{n_j}}\right) < 1$ and $0 < \bar{R}_j + Z(\eta_2)\left({s_j}/{\sqrt{n_j}}\right) < 1$,j=1,2,..n (5)

Equation (5) gives the interval for the point estimator of Rj. We use this confidence limit to characterize the triangular fuzzy number as follows

$$\tilde{R}_j = \left[\bar{R}_j - Z(\eta_1)\left({s_j}/{\sqrt{n_j}}\right), \bar{R}_j, \bar{R}_j + Z(\eta_2)\left({s_j}/{\sqrt{n_j}}\right)\right] \quad (6)$$

From the fig.1 the membership grade of $\bar{R}_j$ in $\tilde{R}_j$ is 1. The farther the point in intervals from both sides of $\bar{R}_j$ given has follows

$$\left[\bar{R}_j - Z(\eta_1)\left({s_j}/{\sqrt{n_j}}\right), \bar{R}_j, \bar{R}_j + Z(\eta_2)\left({s_j}/{\sqrt{n_j}}\right)\right]$$

has the lower membership grade. By this method the confidence limit and the fuzzy membership grade both have same properties. Thus if there exists a correspondence between membership grade and confidence level, it is reasonable to set up a triangular fuzzy number as in equation (7).Using the definition of signed distance we get

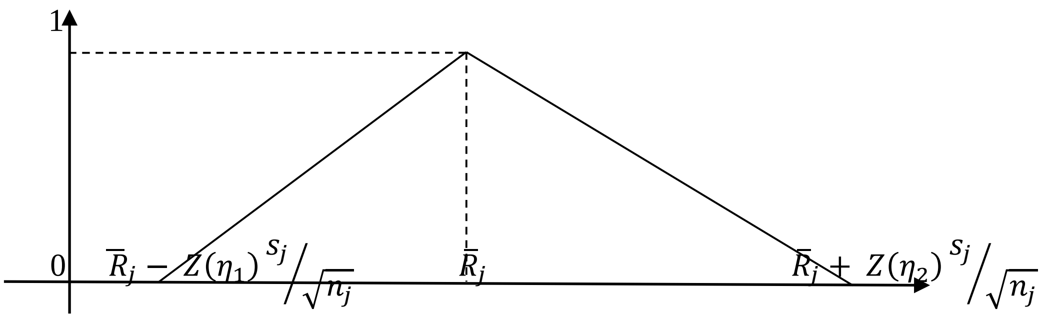

fig1: Triangular fuzzy number $\bar{R}_j$

$$\dot{R}_j = d(\tilde{R}_j, \tilde{0}) = \bar{R}_j + ((1/4)(Z(\eta_2) - Z(\eta_1))\left({s_j}/{\sqrt{n_j}}\right) \in (5)$$

This is the estimate value of R_j from a fuzzy point of view. When $\eta_1 = \eta_2 = {\eta}/{2}$, $\dot{R}_j = \bar{R}_j$ for each j ∈ { 1,2,3,…..n}. The α - level set of $\tilde{R}_j$, 0 ≤ α ≤ 1 is $[R_{jl}(\alpha), R_{ju}(\alpha)]$, where

$$R_{jl}(\alpha) = \bar{R}_j - (1-\alpha)Z(\eta_1)\left({s_j}/{\sqrt{n_j}}\right) \text{ and } \bar{R}_j + (1-\alpha)Z(\eta_2)\left({s_j}/{\sqrt{n_j}}\right) \quad (7)$$

$$\tilde{R}_j = \bigcup_{0\le\alpha\le1}[R_{jl}(\alpha), R_{ju}(\alpha); \alpha], \text{ j= 1,2,…n.} \quad (8)$$

Thus the fuzzy web system reliability using the triangular fuzzy number (8) is given as follows:

$$\tilde{1} - \tilde{R}_j = \bigcup_{0\le\alpha\le1}(1,1,1) - [R_{jl}(\alpha), R_{ju}(\alpha); \alpha]$$

$$= \bigcup_{0\le\alpha\le1}(1,1,1) - \begin{bmatrix} \bar{R}_j - (1-\alpha)Z(\eta_1)\left({s_j}/{\sqrt{n_j}}\right), \\ \bar{R}_j + (1-\alpha)Z(\eta_2)\left({s_j}/{\sqrt{n_j}}\right); \alpha \end{bmatrix}$$

$$= \bigcup_{0\le\alpha\le1}\begin{bmatrix} 1 - \bar{R}_j - (1-\alpha)Z(\eta_1)\left({s_j}/{\sqrt{n_j}}\right), \\ 1 - \bar{R}_j + (1-\alpha)Z(\eta_2)\left({s_j}/{\sqrt{n_j}}\right), \\ 1-\alpha \end{bmatrix}$$

$$= \bigcup_{0\le\alpha\le1}[Q_{jl}(\alpha), Q_{ju}(\alpha); \alpha]$$

where the upper and lower values are given as

$$Q_{jl}(\alpha) = 1 - \bar{R}_j - (1-\alpha)Z(\eta_1)\left({s_j}/{\sqrt{n_j}}\right) \text{ and}$$

$$Q_{ju}(\alpha) = 1 - \bar{R}_j + (1-\alpha)Z(\eta_2)\left({s_j}/{\sqrt{n_j}}\right), 0 \le \alpha \le 1. \quad (9)$$

From equation (6)

$0 \le Q_{jl}(\alpha) \le Q_{ju}(\alpha)$ for all α∈[0,1] and j = 1,2,….,n.

In general we have

$$[(\tilde{1}\Theta\tilde{R}_1)\otimes(\tilde{1}\Theta\tilde{R}_1)\otimes \dots \otimes(\tilde{1}\Theta\tilde{R}_1)] = \bigcup_{0\le\alpha\le1}\left[\prod_{j=1}^{n} Q_{jl}(\alpha), \prod_{j=1}^{n} Q_{ju}(\alpha); \alpha\right]$$

hence we have

$$\tilde{1}\Theta\left[\left(\tilde{1}\Theta\tilde{R}_1\right)\otimes\left(\tilde{1}\Theta\tilde{R}_1\right)\otimes\ldots\otimes\left(\tilde{1}\Theta\tilde{R}_1\right)\right]$$

$$= \bigcup_{0\le\alpha\le1}\left[1-\prod_{j=1}^{n} Q_{jl}(\alpha)\,,1-\prod_{j=1}^{n} Q_{ju}(\alpha)\,;\alpha\right] \quad (10)$$

Note:

To get the estimate reliability of the web system in the fuzzy sense as follows.

$$d\left[\tilde{1}\Theta\left(\tilde{1}\Theta\tilde{R}_1\right)\otimes\left(\tilde{1}\Theta\tilde{R}_1\right)\otimes\ldots\otimes\left(\tilde{1}\Theta\tilde{R}_1\right),\tilde{0}\right]$$

$$= \left({}^{1}/{}_{2}\right)\int_0^1\left(1-\prod_{j=1}^{n} Q_{jl}(\alpha)+1-\prod_{j=1}^{n} Q_{ju}(\alpha)\right)d\alpha \quad (11)$$

4. ILLUSTRATION:

Let us consider a web system with 3 web server. The reliability of these servers considered for a period of 10 days. From the data's collected the point estimate of their reliabilities are found and using the statistical confidence interval the fuzzy reliability of these subsystems are expressed as triangular fuzzy number the fuzzy reliability of the system as well as the estimated reliability of the web system is determined. Let γ= 0.02, γ1 = 0.01, γ=0.01. As mentioned above

$\bar{R}_j = \left({}^{1}/{}_{n}\right)\sum_{m=1}^{n_j} R_{qj}$ and $s_j^2 = \left({}^{1}/{}_{n}\right)\sum_{m=1}^{n_j}\left(R_{qj}-\bar{R}_j\right)^2$.

From the table of the Z- distribution,j = 1,2,3, we get the following data,

$$Z(\eta_1) = -2.58, Z(\eta_2) = 2.58$$

From table below the triangular fuzzy number is tabulated

Table 1. Statistical data

Web server1	Sample size	Sample mean	Sample standard deviation
W_1	N_1= 35	$\bar{R}_1$=0.78	s_1= 0.03
W_2	N_2=30	$\bar{R}_2$=0.8	s_2= 0.02
W_3	N_3=50	$\bar{R}_3$=0.85	s_3= 0.01

Using equation (8) the statistical confidence intervals are estimated as follows:

The statistical confidence intervals:

Table 2. Two endpoints in fig.1

j	$\bar{R}_j - Z(\eta_1)\left(s_j/\sqrt{n_j}\right)$	$\bar{R}_j + Z(\eta_2)\left(s_j/\sqrt{n_j}\right)$
1	0.77778	0.782211
2	0.79828	0.80172
3	0.84948	0.850516

The fuzzy reliability are given as follows:

$$\widetilde{R}_1 = (0.77778, 0.78, 0.782211), \quad \tilde{R}_2 = (0.79828, 0.80, 0.80172),$$

$$\tilde{R}_3 = (0.84948, 0.85, 0.850516),$$

From (8)

$$R_{1l}(\alpha) = 0.77778 + 0.00222\alpha; \; R_{1u}(\alpha) = 0.782211 - 0.00221\alpha$$

$$R_{2l}(\alpha) = 0.79828 + 0.00172\alpha; \; R_{2u}(\alpha) = 0.80172 - 0.00172\alpha$$

$$R_{3l}(\alpha) = 0.84948 + 0.00052\alpha; \; R_{3u}(\alpha) = 0.850516 - .000516\alpha$$

From equation (10)

$Q_{1l}(\alpha) = 0.217789 + 0.00222\alpha; Q_{1u}(\alpha) = 0.22222 - 0.00222\alpha$

$Q_{2l}(\alpha) = 0.19828 + 0.00172\alpha; \; Q_{2u}(\alpha) = 0.20172 - 0.00172\alpha$

$Q_{3l}(\alpha) = 0.149484 + 0.000516\alpha; Q_{3u}(\alpha) = 0.150514 - 0.00052\alpha$

Since we have ,

$0 \leq Q_{jl}(\alpha) \leq Q_{ju}(\alpha)$ for all $\alpha \in [0,1]$ and j = 1,2,….,n.

The fuzzy reliability of the web system is given by

$\tilde{1}\Theta[(\tilde{1}\Theta\tilde{R}_1)\otimes(\tilde{1}\Theta\tilde{R}_1)\otimes\ldots\otimes(\tilde{1}\Theta\tilde{R}_1)]=$

$$\bigcup_{0\leq\alpha\leq1}\left[1-\prod_{j=1}^{n} Q_{jl}(\alpha)\,,1-\prod_{j=1}^{n} Q_{ju}(\alpha)\,;\alpha\right]$$

The estimated reliability of the web system in the fuzzy sense is given by

$$d[\tilde{1}\Theta(\tilde{1}\Theta\tilde{R}_1)\otimes(\tilde{1}\Theta\tilde{R}_1)\otimes\ldots\otimes(\tilde{1}\Theta\tilde{R}_1),\tilde{0}]$$

$$=\left({}^{1}/_{2}\right)\int_0^1\left(1-\prod_{j=1}^{n}Q_{jl}(\alpha)+1-\prod_{j=1}^{n}Q_{ju}(\alpha)\right)d\alpha$$

$$=0.954$$

5. CONCLUSION

Hence using triangular fuzzy number obtained by statistical confidence limit the fuzzy reliability of the system is estimated and using signed distance method we have also defuzzified and found the fuzzy estimate of reliability of our web system.

ACKNOWLEDGEMENT

The author thank the anonymous referees

REFERENCES

[1] K. Abdul Razak, K. Rajakumar, A Study on Fuzzy Reliability Measures, Applied Mathematical Sciences, Vol. 7, 2013, no. 67, 3335 – 3343 HIKARI Ltd.

[2] Cai. K. Y., Wen. C. Y. and Zhang. M. L., Fuzzy Variables as a basis for a theory of fuzzy reliability in the possibility context, Fuzzy sets and systems, Vol. 42, pp. 145-172, 1991.

[3] Chen, S. M., Fuzzy system reliability analysis using fuzzy number arithmetic operations, Fuzzy Sets and Systems,Vol. 64, pp. 31-38, 1994.

[4] Jing-Shing Yao, Jin-Shieh Su And Teng-San Shih, Fuzzy System Reliability Analysis Using Triangular Fuzzy Numbers Based on Statistical Data, Journal Of Information Science and Engineering 24, 1521-1535 (2008)

[5] B. Praba, R. Sujatha and S. Srikrishna, Fuzzy Reliability Measures of Fuzzy Probabilistic Semi-Markov Model, International Journal of Recent Trends in Engineering, Vol 2, No. 2, Nov 2009.