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A New Form of Nano Locally Closed Sets in Nano Topological Spaces

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Abstract

In this paper, the notion of NA-set, NB-set, NαA-set and NS-set are introduced and investigated. A new form of nano locally closed sets are derived by using nano w-open sets.

Keywords: NA-set, NB-set, NαA-set, NS-set, nano w-open, Nwlc*-set and Nw_t -set.

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1. INTRODUCTION

In 1986 and in 1989, Tong [5, 6]introduced two new classes of set, namely A-set and B-set and using them obtained new decomposition of continuity . L.Thivagar et al [3,4] introduced the concept of nano topological spaces which was defined in terms of approximations and boundary region of a subset of a universe U using an equivalence relation on it. The aim of this paper is to introduce NA-set, NB-set, NαA-set and NS-set and a new form of decomposition is derived by using nano w-open set in nano topological spaces.

2. PRELIMINARIES

Definition 2.1

Let $(\mathcal{U}, \tau_R(X))$ be a Nano topological space and A ⊆ $\mathcal{U}$. Then A is said to be

(i) nano semi-open[2] if A ⊂ Ncl(Nint(A))

(ii) nano pre-open[2] if A ⊂ Nint(Ncl(A))

(iii) nano β-open[2] if A ⊂ Ncl(Nint(Ncl(A)))

(iv) nano α-open[2] if A ⊂ Nint(Ncl(Nint(A)))

(v) nano regular open [2]if A = Nint(Ncl(A))

NSO$(\mathcal{U}, X)$, NPO$(\mathcal{U}, X)$, NβO$(\mathcal{U}, X)$, τ_R^{α}(X) and NRO$(\mathcal{U}, X)$ repectively denote the families of all nano semi-open, nano pre-open, nano β-open, nano α-open and nano regular open subsets of $\mathcal{U}$.

Definition 2.2[2]

Let $(\mathcal{U}, \tau_R(X))$ be a Nano topological space and A ⊆ $\mathcal{U}$. Then A is said to be a nano locally closed if A = U∩F, where U is nano open and F is a nano closed in $\mathcal{U}$.

Definition 2.3[3,4]

A nano topological space $(\mathcal{U}, X)$ is said to be extremely disconnected if the nano closure of each nano open set is nano open.

Definition 2.4[3,4]

Let $(\mathcal{U}, \tau_R(X))$ be a Nano topological space and A ⊆ $\mathcal{U}$. A is said to be semi-closed (respectively, nano pre-closed, nano β-closed, nano α-closed and nano regular closed), if its complement is semi-open(nano pre-open, nano β-open, nano α-open and nano regular open).

Theorem 2.5[1]

The intersection of a nano α-open set and a nano β-open set is nano β-open.

Throughout this paper $(\mathcal{U}, \tau_R(X))$ is a Nano topological space with respect to X ⊆ $\mathcal{U}$, R is an equivalence relarion on $\mathcal{U}$, $\mathcal{U}$/R denotes the family of equivalence classes of $\mathcal{U}$ by R.

3. NA AND NB SETS

Definition 3.1

Let $(\mathcal{U}, \tau_R(X))$ be a nano topological space and A ⊆ $\mathcal{U}$. Then A is said to be

(i) Nt-set if Nint(A) = Nint(Ncl(A)).

(ii) NA-set if A = U∩F, where U is nano open and F is nano regular closed .

(iii)NB-set if A = U∩F , where U is nano open and F is Nt-set .

(iv)NαA-set if A = U∩F, where U is nano α-open and F is nano regular closed.

(v) NS-set if A = U∩F where U is nano semi open and F is nano closed .

Theorem 3.2

(i) Every nano closed is Nt-set.

(ii) Every Nt-se t is NB-set.

(iii) Every NA-set is NS-set.

(iv) Every NαA-set is NS-set.

(v) Every nano semi-open is NS-set.

Remark 3.3

Converse of the above theorem need not be true as shown from the following example.

Example 3.4

Let $\mathcal{U}$ = {a, b, c, d} , $\mathcal{U}$/R = {{a}, {c}, {b, d}} , X = {a, b} and τ_R(X) = { $\mathcal{U}$, φ, {a}, {b, d}, {a, b, d}}. (i) {a} is Nt-set, but not nano closed.(ii) {a, b, d} is NB-set, but not Nt-set.(iii) and (iv) {c} is NS-set, but not NA-set and NαA-set.

Theorem 3.5

(i) A is Nt-set if and only if it is nano semi closed.

(ii) If A is nano regular open if and only if A is nano pre-open and Nt-set.

(iii) If A and B are two Nt-sets, then A∩B is a Nt-set.

Proof

(i) Let A be a Nt-set. Then Nint(A) = Nint(Ncl(A)) and so Nint(Ncl((A)) = Nint(A) ⊂ A

Thus, A is nano semi-closed. Conversely, let A be nano semi-closed. Then, Nint(Ncl(A)) ⊂ A

and Nint(Ncl(A))) ⊂ Nint(A).Since Nint(A) ⊂ Nint(Ncl(A))), Nint(A) = Nint(Ncl(A)). Thus, A is Nt-set.

(ii) Let A be nano regular open, then it is nano pre-open and by the definition of nano regular open, Nint(A) = Nint(Ncl(A)). Conversely, let A be nano pre-open and Nt-set. Then A is nano open and so, Nint(Ncl(A)) = A. Thus, A is nano regular open.

(iii)Let A and B be two Nt-sets. Then Nint(A∩B)= Nint(A) ∩ Nint(B) = Nint(Ncl(A)) ∩ Nint(Ncl(B)) = Nint(Ncl(A∩B)). Hence A∩B is Nt-set.

Theorem 3.6

For a subset A of a nano topological space ($\mathcal{U}$, $\tau_R(X)$), the following are equivalent:

(i)A is nano open

(ii)A is nano pre-open and NB-set

Proof

(i)⟹(ii) Let A be nano open. Every nano open is nano pre-open and A = A∩ $\mathcal{U}$.Thus A is NB-set.

(ii)⟹(i) Let A be NB-set. Then we have A = C∩F, where C is nano open and F is nano Nt-set. Since A is nano pre-open, A ⊂ Nint(Ncl(A)).Hence A = C∩F = (C∩F) ∩C ⊂ [Nint(Ncl(C)) ∩ Nint(F)] ∩ C = C∩Nint(F). Therefore A is nano open.

Remark 3.7

The union of two Nt-sets need not be Nt-set. This has been proved from the following example.

Example 3.8

Let $\mathcal{U}$ = {a, b, c, d} , $\mathcal{U}$/R = {{a}, {c}, {b, d}} , X = {a, b} and τ_R(X) = { $\mathcal{U}$, φ, {a}, {b, d}, {a, b, d}}. Here {a} and {b, d} are Nt-sets, but {a,b,d} is not Nt-set.

Theorem 3.9

The intersection of nano semi-open set and nano α-open set is nano semi-open.

Theorem 3.10

(i) Every NA-set is NαA-set

(ii) Every NαA-set is nano semi-open .

(iii) Every NαA-set is nano β-open.

(iv) Every NA-set is NB-set.

Proof

(i) The proof follows from the fact that every nano-open is nano α-open.

(ii) Let A be NαA-set. Then A = C ∩ F, where C is nano α-open and F is nano regular closed, F = Ncl(Nint(F)). So, F is nano semi-open. By the theorem 3.9, A is nano semi-open.

(iii) Let A be NαA-set. Then A = C ∩ F, where C is nano α-open and F is nano regular closed, F = Ncl(Nint(F)). So, F is nano semi-open. Every nano semi open is nano β-open. By the theorem 2.5, A is nano β-open.

(iv) Let A be NA-set. Then A = C ∩ F, where C is nano open and F is nano regular closed. Since every nano regular closed set is nano closed, F is nano closed. Hence by the theorem 3.2, A is NB-closed.

Theorem 3.11

Let A⊂U be a nano topological space (U, $\tau_R(X)$). Then the following are equivalent.

(i) A is a NA-set

(ii) A is NαA-set and a nano locally closed

(iii) A is nano semi-open and nano locally closed

Proof

(i)⟹(2) Let A be a NA-set, A = C∩F where C is nano open and F is nano regular closed. Since every nano regular closed is nano closed and nano open is nano α-open, A is nano locally closed and NαA-set.

(ii)⟹(iii) By the theorem 3.10, it is obviously true.

(iii) ⟹(i) Let A be nano semi-open and nano locally closed. Then A ⊂ Ncl(Nint(A)) and A = C∩Ncl(A), where C is nano open , Ncl(Nint(A)) = A. Thus, A is NA-set.

Theorem 3.12

For a nano topological space $(\mathcal{U}, \tau_R(X))$, the following are equivalent:

(i) $(\mathcal{U}, \tau_R(X))$ is nano extremely disconnected.

(ii) NA$(\mathcal{U}, \tau_R(X))$ is a nano topology on $\mathcal{U}$.

(iii) NSO$(\mathcal{U}, \tau_R(X))$ is a nano topology on $\mathcal{U}$.

Proof

(i)⇒(ii) If G is an NA-set then G = C∩F where C is nano open and F is nano regular closed. Since $(\mathcal{U}, \tau_R(X))$ is extremely disconnected, F is nano open. Hence G is nano open.

(ii)⇒(iii) and (3)⇒(1) are obvious .

4. Nwlc*-sets and Nw$_t$-sets

Definition 4.1

A subset A of a Nano topological space $(\mathcal{U},\tau_R$(X)) is said to be Nw-closed if Ncl(A) ⊂ F whenever A ⊂ F and F is nano semi-open.

Definition 4.2

A subset A of a Nano topological space $(\mathcal{U},\tau_R$(X)) is said to be Nwlc*-set if A = C∩F, where C is nano w-open and F is nano closed.

Definition 4.3

A subset A of a Nano topological space $(\mathcal{U},\tau_R$(X)) is said to be Nw$_t$-set if A = C∩F, where C is nano w-open and F is Nt-set.

Theorem 4.4

Intersection of two nano w-open sets is nano w-open.

Remark 4.5

Union of two nano w-open sets is need not be a nano w-open which has been shown from the following example.

Example 4.6

Let $\mathcal{U}$ = {a, b, c, d} , $\mathcal{U}$/R = {{a}, {c}, {b, d}} , X = {a, b} and τ_R(X) = { $\mathcal{U}$, φ, {a}, {b, d}, {a, b, d}}. Here {a} and {c} are two nano w-open sets, but {a, c} is not nano w-open.

Theorem 4.7

A subset A of (U, τ_R(X)) is nano closed if and only if it is both nano w-closed and NS-set.

Proof

Every nano closed set is w-closed and NS-set.

Conversely, let A be both nano w-closed NS-set. Then A = F∩G where F is nano semi-open and G is nano closed. Therefore A ⊂ F and A ⊂ G and by hypothesis, cl(A) ⊂ F and cl(A) ⊂ G. Therefore, cl(A) ⊂ F∩G = A. Hence A is nano closed.

Theorem 4.8

Let A be a subset of a Nano topological space (U, τ_R(X)), the following are equivalent.

(1) A is nano closed

(2) A is nano locally closed and a nano w-closed

(3) A is NS-set and a nano w-closed

Theorem 4.9

(i) If A is nano w-open, then A is Nwlc*-set

(ii) If A is nano closed, then A is Nwlc*-set

(iii) If A is Nt-set, then A is Nw_t-set

Theorem 4.10

(i) If A is Nwlc*-set , then A is Nw_t-set

(ii) If A is NA-set, then A is Nwlc*-set

(iii) If A is NA-set, then A is Nw_t-set

(iv) If A is NB-set, then A is Nw_t-set

Remark 4.11

Converse of the above theorem need not be true as seen from the following example.

Example 4.12

Let $\mathcal{U}$ = {a, b, c, d} , $\mathcal{U}$/R = {{a}, {c}, {b, d}} , X = {a, b} and τ_R(X) = { $\mathcal{U}$, φ, {a}, {b, d}, {a, b, d}}. (i) The set {a, b, c} is Nw_t-set, but not Nwlc*-set.

(ii) The set {b} is Nwlc*-set, but not NA-set.

(iii) The {c} is Nw_t-set, but not NA-set.

(iv) The set {b, c} is Nw_t-set, but not NB set.

Remark 4.13

The notions of NB-set and Nwlc*-set are independent.

Example 4.14

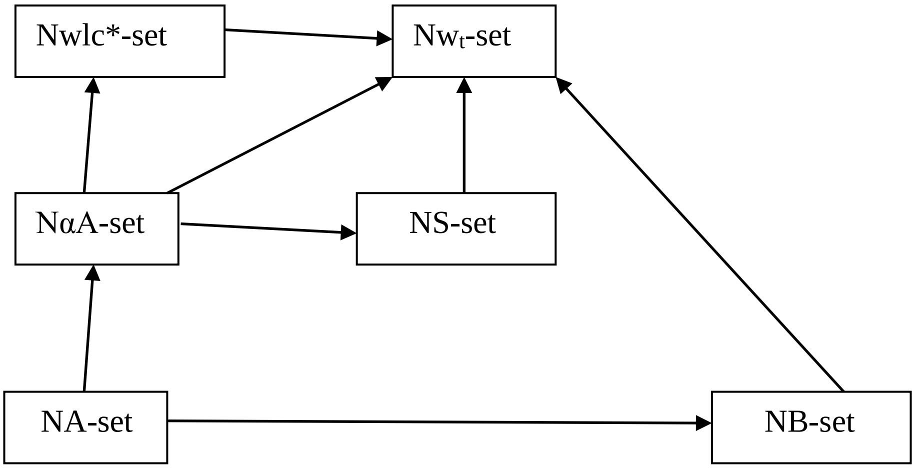
Let $\mathcal{U}$ = {a, b, c, d} , $\mathcal{U}$/R = {{a}, {c}, {b, d}} , X = {a, b} and τ_R(X) = { $\mathcal{U}$, φ, {a}, {b, d}, {a, b, d}}.Then,

(i) A = {a, b, c} is NB-set, but not Nwlc*-set.

(ii) A = {d} is Nwlc*-set, but not NB-set.

Remark 4.15

The above discussions are summarized in the following diagram



Theorem 4.16

Let A be a Nwlc*-subset of a Nano topological spaces ($\mathcal{U}$,τ_R(X)). Then the following results hold.

(1) If B is nano closed , then A∩B is an Nwlc*-set

(2) If B is nano w-open , then A∩B is an Nwlc*-set

(3) B is Nwlc*-set , then A∩B is an Nwlc*-set

Proof

(1) Let B be nano closed and A is Nwlc*-set, then A∩B = (C∩D)∩B = C∩(D∩B), where D∩B is nano closed. Thus, A∩B is an Nwlc*-set.

(2) Let B be nano w-open and A is Nwlc*-set, then A∩B = (C∩B)∩D , where C∩B is nano w-open. Thus, A∩B is an Nwlc*-set.

(3) Let A and B be two Nwlc*-sets, then A∩B = (C∩D)∩(E∩F) = (C∩E)∩(D∩F), where C∩E is nano w-open and D∩F is nano closed. Hence A∩B is Nwlc*-set.

Remark 4.17

The union of any two Nwlc*-sets need not be Nwlc*-set.

Example 4.18

Let U = {a, b, c, d}, X = {a, b} and U/R = {{a},{c},{b, d}}. τ_R(X) = {φ, U, {a}, {b, d},{a, b, d}}. Then A = {a, b}, B = {a, c} are Nwlc*-sets, but A∪B = {a, b, c} is not Nwlc*-set.

Theorem 4.19

Let A be a Nw_t-subset of a Nano topological space (U, τ_R(X)), then the following hold.

(1) If B is Nt-set, then A∩B is Nw_t-set

(2) If B is nano w-open, then A∩B is Nw_t-set

(3) If B is nano w_t-set, then A∩B is Nw_t-set

Remark 4.20

Union of two Nw_t-sets need not be Nw_t-set. This has been proved from the following example.

Example 4.21

Let U = {a, b, c, d}, X = {a, b} and U/R = {{a}, {c},{b, d}}, τ_R(X) = {φ, U, {a}, {b, d},{a, b, d}}. A = {a, c} and B = {d} are two Nw_t-sets, but A∪B = {a, c, d} is not Nw_t-set.

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