Global Journal of Pure and Applied Mathematics.
ISSN 0973-1768 Volume 13, Number 9 (2017), pp. 6061-6080

http://www.ripublication.com

Heat transfer in a Viscoelastic Fluid over a Stretching Sheet with Frictional Heating and Work due to Deformation

Dr. V. Dhanalaxmi

Associate Professor of Mathematics
University College of Technology
Osmania University, Hyderabad
Telangana State, India 500 007

Abstract

This paper presents a study on viscoelastic boundary layer flow and heat transfer over a stretching sheet in the presence of viscous dissipation and work due to deformation. Non linear boundary layer equations are solved using Quasilinearization technique for two cases, namely, (i) the sheet with Constant Surface Temperature(CST) (ii) the sheet with Prescribed Surface Temperature (PST).The effects of various parameters on flow and temperature profiles are depicted in graphs and discussed.

I. INTRODUCTION

Boundary-layer behavior over a moving continuous solid surface is an important type of flow occurring in several engineering processes. Such processes include heat-treated materials travelling between a feed roll and a wind-uproll or materials manufactured by extrusion and many others. Since the pioneering work of Sakiadis [1], various aspects of the problem have been investigated by many authors. Crane [2], Gupta and Gupta [3] have analyzed the stretching problem with constant surface temperature, while Soundalgekar [4] investigated the Stokes problem for a viscoelastic fluid. This flow was examined by Siddappa and Khapate [5] for a special class of non-Newtonian fluids known as second-order fluids, which are viscoelastic in nature. Danberg and Fansler [6] studied the solution for the boundary layer flow past a wall that is stretched with a speed proportional to the distance along the wall.

Rajagopal et al. [7] independently examined the same flow as in [5] and obtained similarity solutions of the boundary-layer equations numerically for the case of small viscoelastic parameter k_1. It is shown that skin-friction decreases with increase in k_1. Dandapat and Gupta [8] examined the same problem with heat transfer. In [8], an exact analytical solution of the non-linear equation governing this self-similar flow which is consistent with the numerical results in [7] is given and the solutions for the temperature for various values of k_1 are presented. Later, Cortell [9]extended the work of Dandapat and Gupta [8] to study the heat transfer in an incompressible second-order fluid caused by a stretching sheet with a view to examining the influence of the viscoelastic parameter on that flow. It is found that the temperature distribution depends on k_1, in accordance with the results in [8].

In the case of fluids of differential type (see Ref. [10]), the equations of motion are in general one order higher than the Navier–Stokes equations and, they need additional boundary conditions to determine the solution completely. These important issues were studied in detail by Rajagopal [10],[11] and Rajagopal and Gupta [12]. On the other hand, Abel and Veena [13] investigated a viscoelastic fluid flow and heat transfer in a porous medium over a stretching sheet and observed that the dimensionless surface temperature profiles increases with an increase in viscoelastic parameter k_1; however, later, Abel et al. [14] studied the effect of heat transfer on MHD viscoelastic fluid over a stretching surface and an important finding was that the effect of visco-elasticity is to decrease the dimensionless surface temperature profiles in that flow. Furthermore, Char [15] studied MHD flow of a viscoelastic fluid over a stretching sheet; however, only the thermal diffusion is considered in the energy equation. Vajravelu and Rollins [16] obtained analytical solution for heat transfer characteristics in viscoelastic second order fluid over a stretching sheet with frictional heating and internal heat generation. Later, Sarma and Rao [17]extended the work of Vajravelu and Rollins [16] and studied the effect of work due to deformation in the energy equation. Vajravelu and Roper [18] and Cortell [19] analyzed the effects of work due to deformation in viscoelastic second grade fluid over a stretching sheet. Another effect which bears great importance on heat transfer is the viscous dissipation. When the viscosity of the fluid and/or velocity gradient is high, the dissipation term becomes important. Consequently, the effects of viscous dissipation are also included in the energy equation

In the present paper, the flow and heat transfer of an incompressible second order fluid past stretching sheet with viscous dissipation and work due to deformation terms in the energy equation are considered. Non linear boundary layer equations are solved using quasilinearization technique with two thermal boundary conditions, namely, (i) the sheet with Constant Surface Temperature (CST case) (ii) the sheet with Prescribed Surface Temperature (PST case). Results are in good agreement with available studies. This paper highlights the effectof work due to deformation on heat transfer characteristics of the fluid.

II. MATHEMATICAL FORMULATION

Following the postulates of gradually fading memory, Coleman and Noll [20] derived the constitutive equation of second-order fluid flow in the form

$$T = -pI + \mu A_1 + \alpha_1 A_2 + \alpha_2 A_1^2 \tag{1}$$

where T is the Cauchy stress tensor, $-pI$ is the spherical stress due to constraint of incompressibility, μ is the viscosity, α_1, α_2 are the material constants and A_1 and A_2 are the first two Rivlin–Ericken tensors [21] defined as

$$A_1 = (grad\ v) + (grad\ v)^T \tag{2}$$

$$A_2 = \frac{dA_1}{dt} + A_1 (grad\ v) + (grad\ v)^T A_1 \tag{3}$$

Here, v denotes the velocity field and d/dt is the material time derivative. If the fluid of second grade modeled by (1) is to be compatible with thermodynamics and is to satisfy the Clausius-Duhem inequality for all motions and the assumption that the specific Helmholtz free energy of the fluid is a minimum when it is locally at rest, Dunn and Fosdick [22] found that the material modulimust satisfy

$$\mu \ge 0, \alpha_1 \ge 0, \alpha_1 + \alpha_2 = 0 \tag{4}$$

But later on Fosdick and Rajagopal [23] have reported, by using the data reduction from experiments, that in the case of a second order fluid the material constants μ, α_1, α_2 should satisfy the relation

$$\mu \ge 0, \alpha_1 \le 0, \alpha_1 + \alpha_2 \ne 0 \tag{5}$$

They also reported that that the fluids modeled by (1)with the relationship (5) exhibit some anomalous behavior. A critical review on this controversial issue can be found in the work of Dunn and Rajagopal [24].It was mentioned that second-order fluid ,obeying model equation (1) with $\alpha_1 < \alpha_2, \alpha_1 < 0$ although exhibits some undesirable instability characteristics, thesecond order approximationsare valid at low shear rate. Nowin literature the fluid satisfying the model equation (1) with $\alpha_1 < 0$ is termed as second-order fluid and with $\alpha_1 > 0$ is termed as second grade fluid.

In present study, it is considered a laminar steady flow of an incompressible viscoelastic (Walters' liquid B model) fluid over a wall coinciding with the plane y = 0, the flow being confined to y > 0.Two equal and opposite forces are applied along the x-axis, so that a sheet is stretched with a velocity proportional to the distance from the origin. The resulting motion of the quiescent fluid is thus caused solely by the moving surface. The flow satisfies the rheological equation of state derived by Beard and Walters [25].

The governing boundary layer equations for momentum, in the usual form, are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{6}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}-k_0\left\{\frac{\partial}{\partial x}\left(u\frac{\partial^2 u}{\partial y^2}\right)+\frac{\partial u}{\partial y}\frac{\partial^2 v}{\partial y^2}+v\frac{\partial^3 u}{\partial y^3}\right\} \tag{7}$$

$$where\ \ \upsilon=\frac{\mu}{\rho},\ k_0>0$$

where u and v are the velocity components along the x and y directions respectively, υ are the kinematic viscosity, $k_0=-\alpha_1/\rho$ is the co-efficient of elasticity, and ρ is the density. Hence, in the case second order fluid flow k_0 takes positive value as α_1 takes negative value and other quantities have their usual meanings. In deriving (7) it is assumed that the normal stress is of the same order of magnitude as that of the shear stress, in addition to usual boundary layer approximations.

The boundary conditions for the velocity field are:

$$\begin{aligned} &u=u_w=bx,\ v=0 \quad at\ y=0, b>0\\ &u\to 0, \frac{\partial u}{\partial x}\to 0\ as\ y\to\infty \end{aligned} \tag{8}$$

The condition $\partial u/\partial y\to 0\ \ as\ \ y\to\infty$ is the augmented condition since the flow is in an unbounded domain, which has been discussed by Rajgopal [10]. In this case, the flow is caused solely by the stretching of the sheet, since the free stream velocity is zero.

Defining new variables:

$$u=bxf_\eta(\eta), v=-\sqrt{b\upsilon}\,f(\eta), \eta=\sqrt{b/\upsilon}\,y \tag{9}$$

where $f_\eta(\eta)$ denotes differentiation with respect to η. Clearly u and v defined above satisfy the continuity equation (6), and equation (7) is transformed as

$$f_\eta^2 - ff_{\eta\eta} = f_{\eta\eta\eta} - k_1\{2f_\eta f_{\eta\eta\eta} - ff_{\eta\eta\eta\eta} - f_{\eta\eta}^2\} \tag{10}$$

where $k_1 = k_0 b/\upsilon$

The boundary conditions (8) become

$$f(0)=0,\ f_\eta(0)=1 \tag{11a}$$

$$f_\eta(\eta)\to 0,\ f_{\eta\eta}(\eta)\to 0\ \ as\ \eta\to\infty \tag{11b}$$

III. HEAT TRANSFER ANALYSIS

By using boundary layer approximations, and taken into account both viscous dissipation and work due to deformation, the equation of energy for temperature T is given by

$$\rho C_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k\frac{\partial^2 T}{\partial y^2}+\mu\left(\frac{\partial u}{\partial y}\right)^2-\rho k_0\frac{\partial u}{\partial y}\left[\frac{\partial}{\partial y}\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)\right] \tag{12}$$

where k is the thermal diffusivity and C_p the specific heat of a fluid at constant pressure.Two thermal boundary conditions are considered, namely, (i) Constant Surface Temperature (CST case)

(ii). Prescribed Surface Temperature (PST case)

The heat transfer analysis for these two processes is carried out in the following sections.

Case I:Constant Surface Temperature (CST case)

In this circumstance, the boundary conditions are

$$T = T_w \text{ at y = 0, } T\to T_\infty \text{ as } y\to y_\infty \tag{13}$$

Where T_w and T_∞ are constants.

Defining non-dimensional temperature and Prandtl number (Pr) as

$$\theta(\eta)=\frac{(T-T_\infty)}{(T_w-T_\infty)},\ \Pr=\frac{\upsilon}{\alpha} \tag{14}$$

Using (9), equation (12) reduces to

$$\theta'' + \Pr f\theta' = -\Pr Ec\left[(f'')^2 - k_1 f''(f'f'' - ff''')\right] \tag{15}$$

With boundary conditions

$$\theta(0) = 1, \theta(\infty) \to 0 \tag{16}$$

Here $Ec = (b^2x^2)/c_p(T_w - T_\infty)$ represents local Eckert number for this problem. It is worth mentioning that the x-coordinate cannot be eliminated from (5.15), whereby, the temperature profiles always depend on x.

CaseII:Prescribed Surface Temperature (PST case)

For this circumstance, the boundary conditions are

$$T = T_w[= T_\infty + A\left(\frac{x}{l}\right)^2 \text{ at y=0} \tag{17a}$$

$$T \to T_\infty \qquad as \quad y \to \infty \tag{17b}$$

where l is the characteristic length.

Using (9), equation (12) reduces to

$$\theta'' + \Pr f\theta' - 2\Pr f'\theta = -\Pr Ec\left[(f'')^2 - k_1 f''(f'f'' - ff''')\right]$$
(18)

with boundary conditions

$$\theta(0) = 1, \theta(\infty) \to 0$$

IV. NUMERICAL SOLUTION OF THE PROBLEM

The flow equation (10) coupled with energy equation (15) or (18)become set of nonlinear differential equations.A numerical method, quasilinearization technique [26], isin most cases directly applicable to computer aided solutions of non-linear two-point boundary value problems. So this method is used to solve this system.

For convenience equations (10), (15) and (18) are rearranged as

$$f^{iv} = \frac{1}{k_1 f}\left[(f')^2 - ff'' - f''' + 2k_1 f'f''' - k_1(f'')^2\right] \tag{19}$$

$$\theta'' + \Pr f\theta' = -\Pr Ec\left[(f'')^2 - k_1 f''(f'f'' - ff''')\right] \qquad \text{(CST case)} \tag{20a}$$

(OR)

$$\theta'' + \Pr f\theta' - 2\Pr f'\theta = -\Pr Ec\left[(f'')^2 - k_1 f''\left(f'f'' - ff'''\right)\right] \qquad \text{(PST case)} \qquad (20b)$$

In order to implement the quasilinearization method, the equations (19) and (20) are converted to a system of first order differential equations by substituting

$$\left(f, f', f'', f''', \theta, \theta'\right) = \left(x_1, x_2, x_3, x_4, x_5, x_6\right)$$

Then equations (19) and (20) give

$$\frac{dx_1}{d\eta} = x_2$$

$$\frac{dx_2}{d\eta} = x_3$$

$$\frac{dx_3}{d\eta} = x_4$$

$$\frac{dx_4}{d\eta} = \frac{1}{k_1 x_1}\left\{x_2^2 - x_1x_3 - x_4 + 2k_1x_2x_4 - k_1x_3^2\right\} \qquad (21)$$

$$\frac{dx_5}{d\eta} = x_6$$

$$\frac{dx_6}{d\eta} = -\Pr x_1x_6 - \Pr Ec\left[x_3^2 - k_1x_3\left(x_2x_3 - x_1x_4\right)\right] \qquad \text{(CST case)}$$

(OR)

$$\frac{dx_6}{d\eta} = -\Pr x_1x_6 + 2\Pr x_2x_5 - \Pr Ec\left[x_3^2 - k_1x_3\left(x_2x_3 - x_1x_4\right)\right] \text{(PST case)}$$

Using Quasilinearization technique, the system (21) can be linearized as

$$\frac{dx_1^{r+1}}{d\eta} = x_2^{r+1}$$

$$\frac{dx_2^{r+1}}{d\eta} = x^{r+1}$$

$$\frac{dx_3^{r+1}}{d\eta} = x_4^{r+1}$$

$$\frac{dx_4^{r+1}}{d\eta} = \left(\frac{-1}{k_1\left(x_1^r\right)^2} \left(\left(x_2^r\right)^2 - x_4^r + 2k_1 x_2^r x_4^r - k_1\left(x_3^r\right)^2 \right) \right) x_1^{r+1}$$
$$+ \left(\frac{1}{k_1 x_1^r} \left(2x_2^r + 2k_1 x_4^r\right) \right) x_2^{r+1}$$
$$+ \left(\frac{1}{k_1 x_1^r} \left(- x_1^r - 2k_1 x_3^r\right) \right) x_3^{r+1}$$
$$+ \left(\frac{1}{k_1 x_1^r} \left(-1 + 2k_1 x_2^r\right) \right) x_4^{r+1} + \left(\frac{-x_4^r}{k_1 x_1^r} \right) \qquad (22)$$

$$\frac{dx_5^{r+1}}{d\eta} = x_6^{r+1}$$

$$\frac{dx_6^{r+1}}{d\eta} = \left(-\Pr x_6^r - \Pr Eck_1 x_3^r x_4^r\right) x_1^{r+1} + \left(\Pr Eck_1 \left(x_3^r\right)^2\right) x_2^{r+1}$$
$$- Ec \Pr\left(2x_3^r - k_1\left(2x_2^r x_3^r - x_1^r x_4^r\right)\right) x_3^{r+1} - \left(\Pr Eck_1 x_1^r x_3^r\right) x_4^{r+1} - \Pr x_1^r x_6^{r+1}$$
$$+ \left(Ec \Pr\left(x_3^r\right)^2 + \Pr x_1^r x_6^r - 2\Pr Eck_1 x_3^r \left(x_2^r x_3^r - x_1^r x_4^r\right)\right)$$

(OR)

$$\frac{dx_6^{r+1}}{d\eta} = \left(-\Pr x_6^r - \Pr Eck_1 x_3^r x_4^r\right) x_1^{r+1} + (2\Pr x_5^r + \Pr Eck_1 \left(x_3^r\right)^2) x_2^{r+1}$$
$$+ \left(-2Ec \Pr x_3^r + \Pr Eck_1 \left(2x_2^r x_3^r - x_1^r x_4^r\right)\right) x_3^{r+1} - \left(\Pr Eck_1 x_1^r x_3^r\right) x_4^{r+1} + (2\Pr x_2^r) x_5^{r+1}$$
$$+ (-\Pr x_1^r) x_6^{r+1} + \left(Ec \Pr\left(x_3^r\right)^2 + \Pr x_1^r x_6^r - 2\Pr x_2^r x_5^r - 2\Pr Eck_1 x_3^r \left(x_2^r x_3^r - x_1^r x_4^r\right)\right)$$

The above system of equations (22) is linear in $x_i^{r+1}(i = 1,2,...6$ and general solution can be obtained by using the principle of superposition.

The boundary conditions reduce to

$$x_1^{r+1}(\eta) = 0,\ x_2^{r+1}(\eta) = 1,\ x_5^{r+1}(\eta) = 1 \qquad at\ \eta = 0$$

$$x_2^{r+1}(\eta) \to 0, x_3^{r+1}(\eta) \to 0, x_5^{r+1}(\eta) \to 0 \qquad as\ \eta \to \infty$$

The initial values are chosen as follows:

For the homogeneous solution:

$$x_i^{h_1}(\eta) = [0\ 0\ 1\ 0\ 0\ 0]$$
$$x_i^{h_2}(\eta) = [0\ 0\ 0\ 1\ 0\ 0] \qquad (23)$$
$$x_i^{h_3}(\eta) = [0\ 0\ 0\ 0\ 0\ 1]$$

For particular solution:

$$x_i^p(\eta) = [0\,1\,0\,0\,1\,0] \quad (24)$$

The general solution of system of equations is given by

$$x_i^{r+1}(\eta) = c_1 x_i^{h_1}(\eta) + c_2 x_i^{h_2}(\eta) + c_3 x_i^{h_3}(\eta) + x_i^p(\eta) \quad (25)$$

where C_1, C_2, C_3 are the unknown constants and are determined by considering the boundary conditions as η → ∞. This solution $(x_i^{r+1}, i = 1,2,...6)$ is then compared with solution at the previous step $x_i^r, i = 1,2,...6$ and next iteration is performed if the convergence has not been achieved or greater accuracy is desired.

V. RESULTS AND DISCUSSIONS

Here, a study is presentedon flow and heat transfer of an incompressible second order fluid past a stretching sheet. The non linear differential equations of flow and heat transfer were solved by quasilinearization technique. The energy equation includes both the viscous dissipation and the work due to deformation. This later effect doesn't appear, if a viscous flow (i.e., k_1=0) takes place.

Case 1.Constant Surface Temperature (CST case): It can be seen from Table1, for fixed Prandtl number (Pr) and a given position (η**)**, $\theta(\eta)$increases as the viscoelastic parameter $k_1$increases. Dimensionless heat transfer coefficient $|-\theta'(0)|$ decreases with increase ink_1, which results an increase in temperature of the fluid.

In Fig1, values of dimensionless temperature $\theta(\eta)$ and temperature gradient $\theta'(\eta)$ for selected values of Prandtl number (Pr) with k_1=0.3, Ec=0 are given. Since thermal boundary layer thickness decreases with increasing Prandtl number, temperature $\theta(\eta)$ at a point decreases with an increase in the Prandtl number.

In Fig2, temperature$\theta(\eta)$and temperature gradient $\theta'(\eta)$ are drawn for various values of Eckert number (Ec). Temperature $\theta(\eta)$ increases with increase in viscous dissipation, because heat energy is stored in the fluid due to frictional heating. The values$|-\theta'(0)|$ decrease with increase in viscous dissipation, which yields augment in fluid's temperature.

Case 2. Prescribed Surface Temperature (PST case):

The effect of work due to deformation term in the energy equation can be analyzed from Table 2 and Table 3.It can be seen from both the tables that, at a given point, temperature $\theta(\eta)$ increases with increase in viscoelastic parameter (k_1).

From Table3, it is observed that when work due to deformation is taken into account, for given k_1, temperature $\theta(\eta)$ decreases, which is in contrast to the second grade fluids [19]. And values of $|-\theta'(0)|$ in Table3 are larger than in Table2. Physically it means that heat transfer rate is more from the sheet, which results in decrease in temperature $\theta(\eta)$.

In Fig3, Values of non-dimensional temperature $\theta(\eta)$ for various values of Prandtl number (Pr) are shown. In Fig 4, effect of viscous dissipation on temperature $\theta(\eta)$ and temperature gradient $\theta'(\eta)$ is depicted. The reasons are same as in CST case.

In Fig 5, the effect of work due to deformation term in the energy equation on temperature profiles for various values of Eckert number (Ec) is shown.It can be seen that, the presence of work due to deformation term in the energy equation reduces the temperature, which is in contrast to the case $\alpha_1>0$ [19] (i.e., second grade fluids). This effect is more significant for moderate and higher values of Eckert number (Ec).

Finally, Heat transfer characteristics at the wall in both CST and PST cases are given in Table 4. It shows that the value of $|-\theta'(0)|$ decreases with increasing values of viscoelastic parameter (k1) and Eckert number (Ec). And $|-\theta'(0)|$ increases with increasing values of Prandtl number (Pr). Therefore fluid's elasticity (k_1) and Eckert number (Ec) increases the temperature $\theta(\eta)$, while opposite trend can be seen with the Prandtl number.

CST case only:

Table 1: Values of θ (η), θ’ (η) in CST case with Pr=3, Ec=0.0

	H	θ(η)	θ’(η)
k1=0.3	0.0	1.00000	-1.29274
	0.2	0.74691	-1.21174
	0.4	0.52384	-1.00445
	1.0	0.13615	-0.34041
	1.5	0.02793	-0.07654
	2.0	0.00919	-0.0258
	3.0	0.00054	-0.00154
	4.0	0.00003	-0.00009
	5.0	$2.5\text{x}10^{-7}$	-0.00001
k1=0.5	0	1.00000	-1.21466
	0.2	0.76177	-1.14548
	0.4	0.54898	-0.96962
	1.0	0.15863	-0.36653
	1.5	0.03630	-0.09366
	2.0	0.01251	-0.03460
	3.0	0.00069	-0.00448
	4.0	0.00013	-0.00102
	5.0	$3.7\text{x}10^{-7}$	-0.00035
k1=0.7	0	1.00000	-1.07311
	0.2	0.78934	-1.01560
	0.4	0.59920	-0.87663
	1.0	0.22263	-0.39990
	1.5	0.07128	-0.14252
	2.0	0.03086	-0.06828
	3.0	0.00252	-0.01525
	4.0	0.00105	-0.00015
	5.0	$4.1\text{x}10^{-7}$	-0.00040

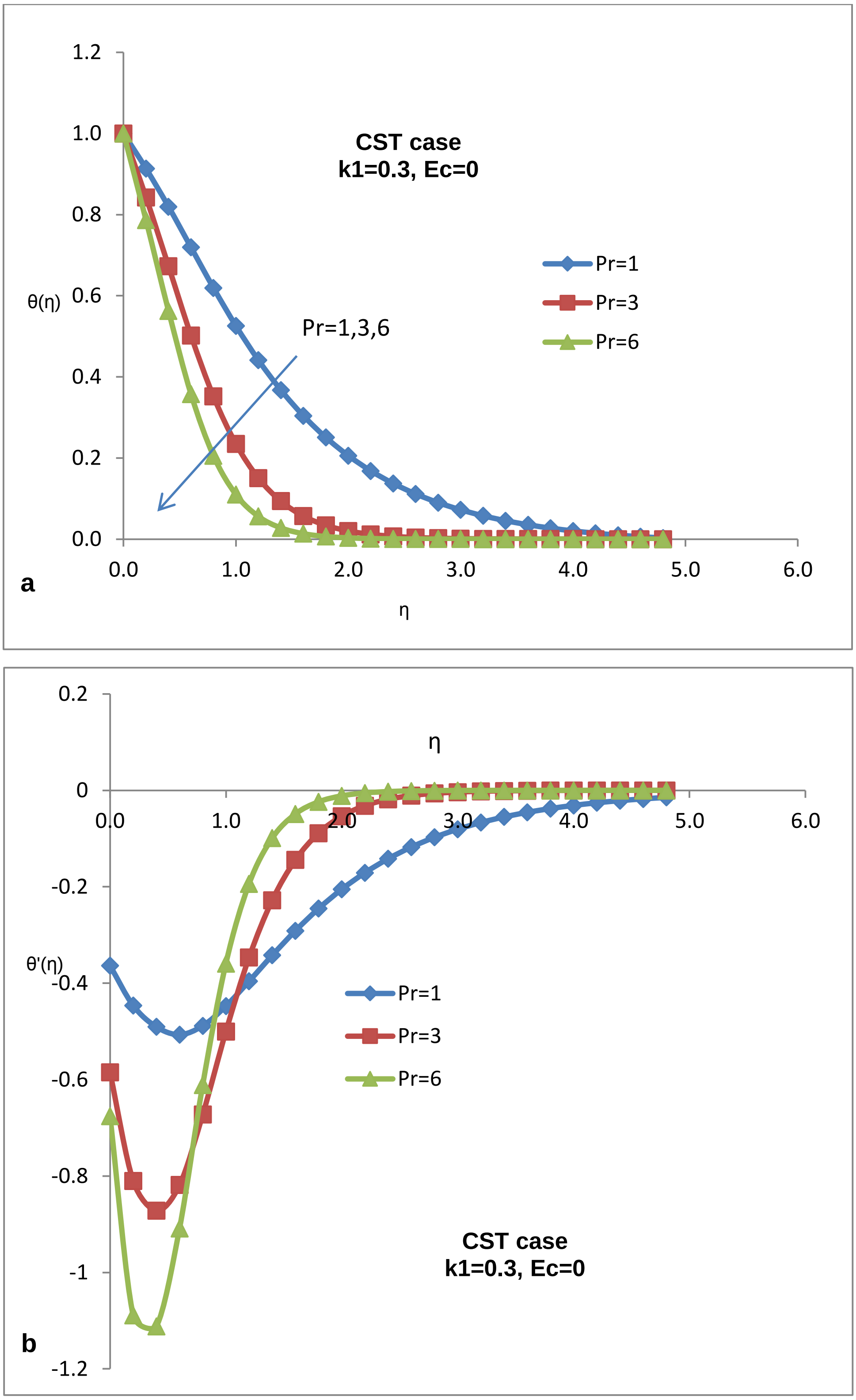


Fig.1. Effect of Prandtl number (Pr) on **(a)** Temperature θ(η) **(b)** Temperature gradient θ'(η) in CST case with k_1=0.3, Ec=0

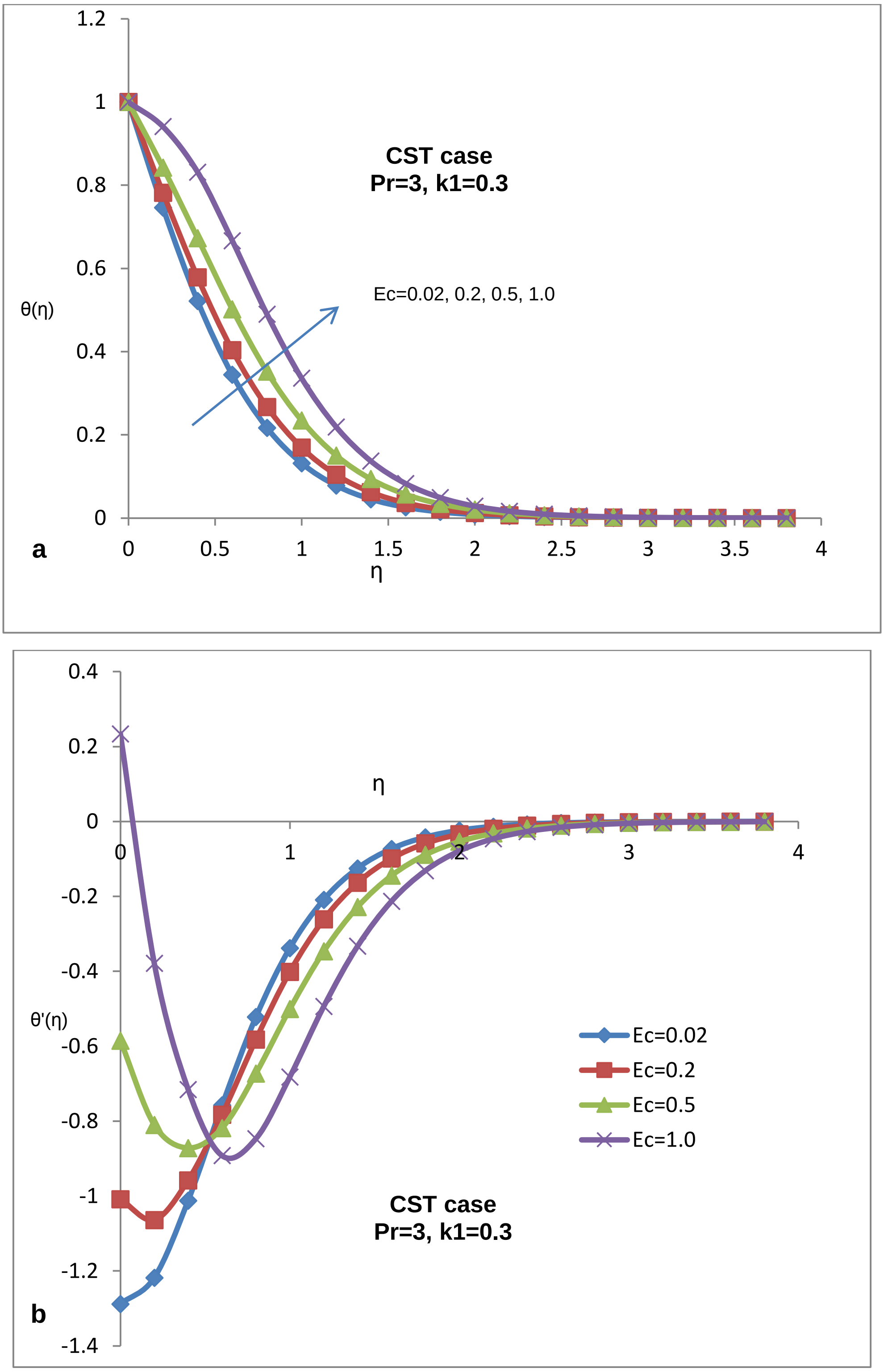


Fig 2. Effect of Eckert number (Ec) on **(a)** Temperature θ (η) **(b)** Temperature gradient θ' (η) in CST case with Pr=3, k_1=0.3

Table 2: Effect Values of viscoelastic parameter k_1 on $\theta(\eta)$, $\theta'(\eta)$ in PST case with Pr=3, Ec=0.5, When work due to deformation is not taken into account

	η	θ (η)	θ'(η)
k1=0.3	0.0	1.00000	-1.97008
	0.2	0.64297	-1.30429
	0.4	0.44045	-0.79197
	1.0	0.14617	-0.29034
	1.6	0.03976	-0.09295
	2.0	0.01505	-0.03778
	3.0	0.00106	-0.00293
	4.0	0.00006	-0.00018
	5.0	0.00000	-0.00002
K1=0.5	0.0	1.00000	-1.96787
	0.2	0.68299	-1.27100
	0.4	0.47299	-0.86144
	1.0	0.15705	-0.29549
	1.6	0.04887	-0.09824
	2.0	0.02152	-0.04495
	3.0	0.00243	-0.00556
	4.0	0.00016	-0.00062
	5.0	0.00001	-0.00026
k1=0.7	0.0	1.00000	-1.52267
	0.2	0.73524	-1.14405
	0.4	0.53644	-0.85628
	1.0	0.19950	-0.33807
	1.6	0.07044	-0.12525
	2.0	0.03405	-0.06352
	3.0	0.00367	-0.01140
	4.0	0.00037	-0.00572
	5.0	0.00002	-0.00450

Table 3: Effect Values of viscoelastic parameter k_1 on $\theta(\eta)$, $\theta'(\eta)$ in PST case with Pr=3, Ec=0.5, When work due to deformation is taken into account

	η	$\theta(\eta)$	$\theta'(\eta)$
k1=0.3	0.0	1	-2.27114
	0.2	0.62464	-1.40343
	0.4	0.40396	-0.86193
	1.0	0.11113	-0.24745
	1.6	0.02739	-0.06674
	2.0	0.0101	-0.02571
	3.0	0.00075	-0.00193
	4.0	0.00009	-0.00012
	5.0	0.00005	-0.00001
	0.0	1	-2.20301
	0.2	0.64857	-1.3841
	0.4	0.42509	-0.89027
	1.0	0.1224	-0.25404
	1.6	0.03474	-0.07391
	2.0	0.01471	-0.03193
	3.0	0.00156	-0.00367
	4.0	0.00008	-0.0004
	4.5	0.00003	-0.00016
	0.0	1	-2.08753
	0.2	0.66495	-1.32853
	0.4	0.44899	-0.86767
	1.0	0.14608	-0.26731
	1.6	0.04902	-0.08943
	2.0	0.02342	-0.04402
	3.0	0.00263	-0.00771
	4.0	0.00041	-0.00386
	4.5	0.00001	-0.00010

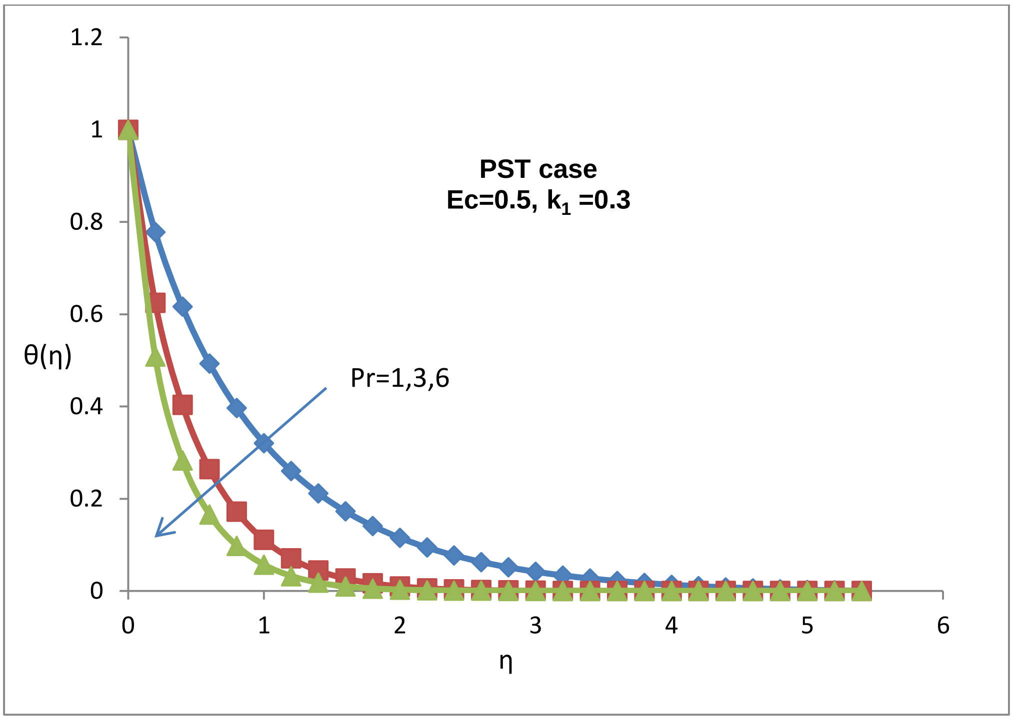
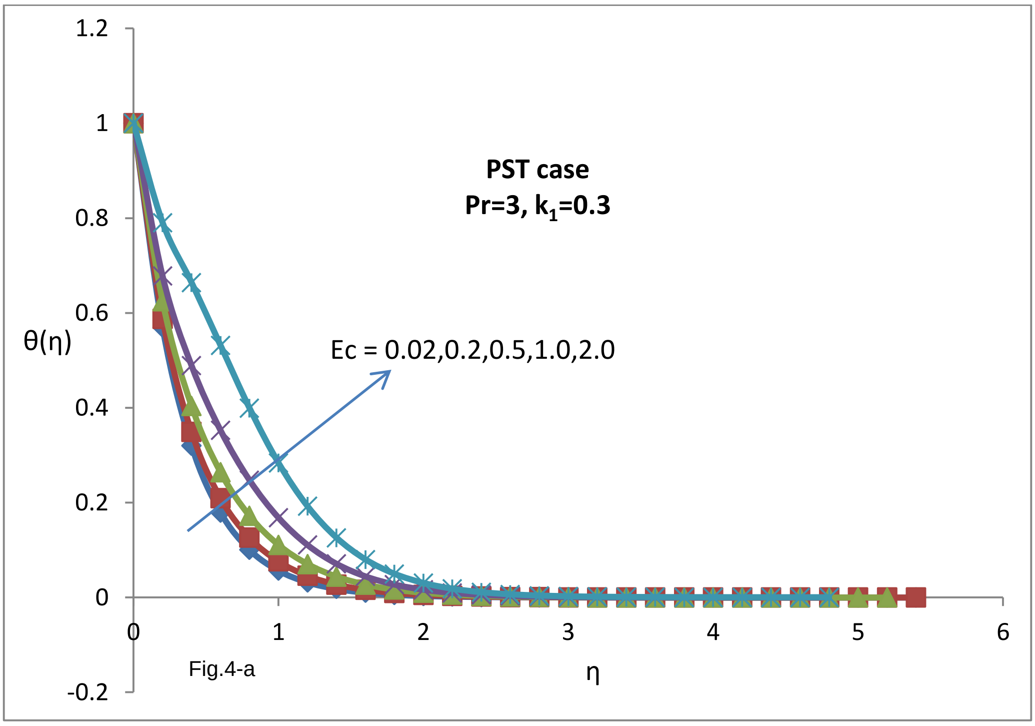


Fig 3.Effect of Prandtl number on Temperature profiles .



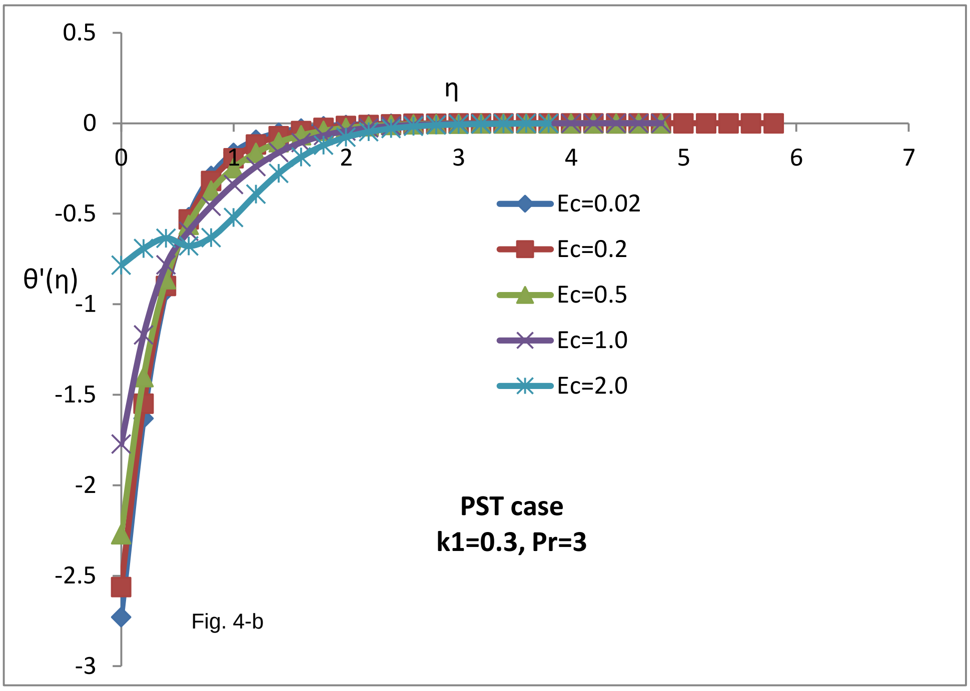


Fig 4. Effect of Eckert number (Ec) on **(a)** Temperature θ (η) **(b)** Temperature gradient θ' (η) in PST case with Pr=3, k_1=0.3

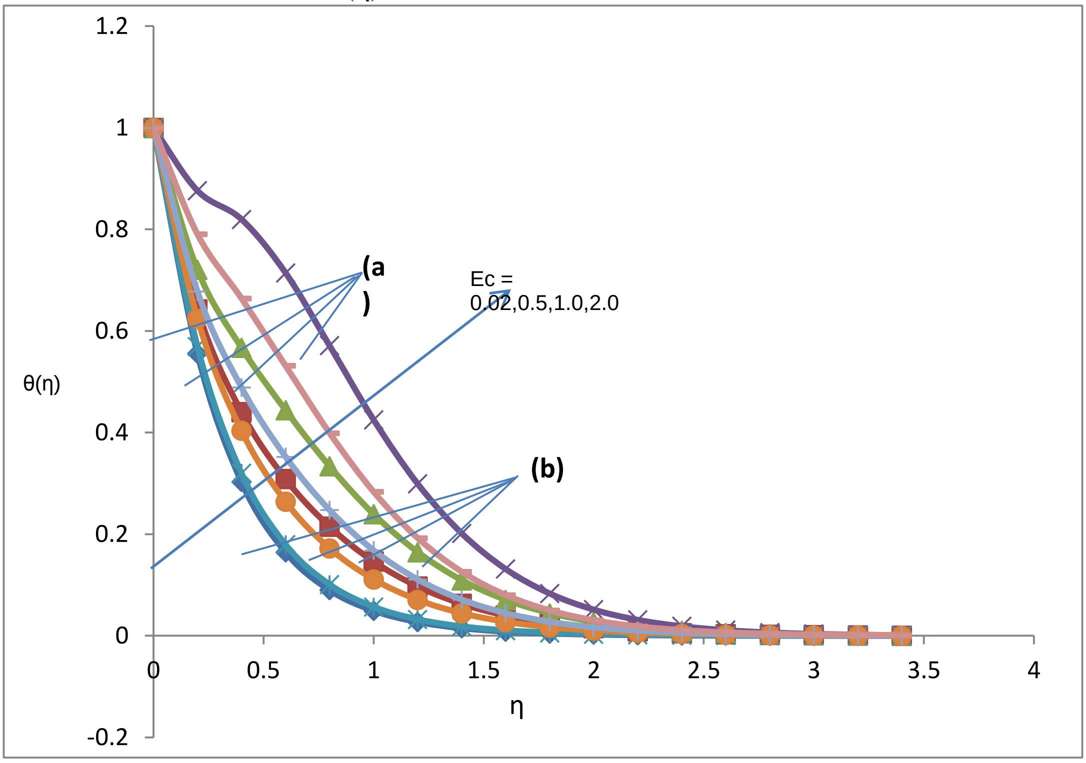


Fig 5. Temperature profiles for various values of Eckert number in PST case **(a)** without work due to deformation **(b)** with work due to deformation**.**

Table 4: The wall heat transfer θ’ (0) in CST and PST cases:

k_1	Pr	Ec	CST case - θ’ (0)	PST case - θ’ (0)
0.3	3.0	0.5	1.29274	2.27114
0.5			1.21466	2.20301
0.7			1.07311	2.08753
0.3	1.0	0.5	0.36325	1.24676
	3.0		0.58478	2.27114
	6.0		0.67562	3.20128
0.3	3.0	0.02		2.72905
		0.2		2.56293
		0.5		2.27114
		1.0		1.77267

REFERENCES:

[1] B.C. Sakiadis, “Boundary-layer behavior on continuous solid surfaces,” *Amer. Inst. Chem. Eng. J.*, vol 7, pp. 26–28, 1961..

[2] L.J. Crane, “Flow past a stretching plate,” *Z. Angew. Math. Phys.*, vol 21, pp.645–647, 1970.

[3] P.S. Gupta, A.S. Gupta, “Heat and mass transfer on a stretching sheet with suction or blowing,” *Canad.J. Chem. Eng.*, vol 55, pp. 744–746, 1977.

[4] V.M. Soundalgekar, “Stokes problem for elastic-viscous fluid,” *Rheol. Acta.*vol13, pp. 177–179, 1974.

[5] B. Siddappa, B.S. Khapate, “Rivlin–Ericksen fluid flow past a stretching sheet,” Rev. Roumaine Sci. Tech.Mech. Appl. 2 (1976) 497–505.

[6] J.E. Danberg, K.S. Fansler, A non similar moving wall boundary-layer problem, Quart. Appl. Math. 34 (1976) 305–309.

[7] K.R. Rajagopal, T.Y. Na, A.S. Gupta, Flow of a viscoelastic fluid over a

stretching sheet, Rheol. Acta 23(1984) 213–215.

[8] B.S. Dandapat, A.S. Gupta, Flow and heat transfer in a viscoelastic fluid over a stretching sheet, Internat. J.Non-Linear Mech. 24 (1989) 215–219.

[9] R. Cortell, Similarity solutions for flow and heat transfer of a viscoelastic fluid over a stretching sheet, Internat. J. Non-Linear Mech. 29(1984) 155–161.

[10] K.R. Rajagopal, On boundary conditions for fluids of the differential type, in: A. Sequeira (Ed.), Navier Stokes Equations and Related non-linear Problems, Plenum Press, New York, 1995, pp. 273–278.

[11] K.R. Rajagopal, On the creeping flow of second order fluid, J. Non-Newtonian Fluid Mech. 15 (1984) 239–246.

[12] K.R. Rajagopal, A.S. Gupta, An exact solution for the flow of a non-Newtonian fluid past an infinite porous plate, Meccanica 19 (1984)158–160.

[13] S. Abel, P.H. Veena, Viscoelastic fluid flow and heat transfer in a porous medium over a stretching sheet, Internat. J. Non-Linear Mech. 33(1998) 531–540.

[14] S. Abel, M. Joshi, R.M. Sonth, Heat transfer in MHD visco-elastic fluid over a stretching surface, Z. Angew. Math. Mech. 81 (2001) 691–698.

[15] M.I. Char, Heat and mass transfer in a hydromagnetic flow of the viscoelastic fluid over a stretchingsheet, J. Math. Anal. Appl. 186 (1994)674–689.

[16] K. Vajravelu , D.Rollins, Heat transfer in a viscoelastic fluid over a stretching sheet, J. Math. Anal.Appl. 158 (1991) 241–255.

[17] M.S. Sarma, B.N. Rao, Heat transfer in a viscoelastic fluid over a stretching sheet, J. Math. Anal. Appl. 222 (1998) 268–275.

[18] K. Vajravelu, T. Roper, Flow and heat transfer in a second grade fluid over a stretching sheet,Internat. J. Non-Linear Mech. 34 (1999)1031–1036.

[19] R. Cortell, A note on flow and heat transfer of a viscoelastic fluid over a stretching sheet, Internat. J.Non-Linear Mech. 41 (2005) 78–85.

[20] B.D.Coleman, W. Noll, An approximation theorem for functionals with applications in continuous mechanics, Arch. Ration. Mech. Anal. 6 (1960)355–370.

[21] R.S. Rivlin, J.L. Ericksen, Stress deformation relations for isotropic materials, J.Ration. Mech. Anal. 4 (1955) 323–425.

[22] J.E. Dunn, R.L. Fosdick, Thermodynamics, stability and boundedness of fluids of complexity 2 and fluids of second grade, Arch. Ration. Mech. Anal. 56 (1974)191–252.

[23] R.L. Fosdick, K.R. Rajagopal, Anomalous features in the model of second orderfluids, Arch. Ration. Mech. Anal. 70 (1979) 145–152.

[24] J.E. Dunn, K.R. Rajagopal, Fluids of differential type – critical review and thermodynamic analysis, Int. J. Eng. Sci. 33 (1995) 689–729.

[25] D.W. Beard, K. Walters, Elastico-viscous boundary layer flows: part I. Twodimensional flow near a stagnation point, in: Proceedings of Cambridge Philos.Soc., 1964, pp. 667–674.

[26] R.E. Bellman, R.E. Kalaba., Quasilinearisation and nonlinear boundary value problems, American Elsevier, Newyark, 1965.