Global Journal of Pure and Applied Mathematics.
ISSN 0973-1768 Volume 13, Number 9 (2017), pp. 6129–6132

http://www.ripublication.com/gjpam.htm

On Designs arising from Corona Product $H \cong G \circ K_m$

Dr. Sumathi M. P.
Department of Mathematics,
Mahajana First Grade College,
Mysore 570012, India.

Dr. Anwar Alwardi
University of Aden,
Aden, Yemen

Abstract

In this paper, we determine the number of minimum dominating sets of graph $H \cong G \circ K_m$ and prove that the set of all minimum dominating sets of $H \cong G \circ K_m$ forms a partially balanced incomplete block design with two association scheme. Finally we generalize and determine the partially balanced incomplete block designs and association scheme which are formed by the minimum dominating sets of the graphs $H \cong G \circ K_m$.

AMS subject classification: 05C50.
Keywords: Minimum dominating sets, association schemes, PBIB designs.

1. Introduction

By a graph, we mean a finite undirected graph without loops or multiple lines. For a graph $G = (V, E)$, let V and E respectively denote the vertex set and the edge set of graph G. For any vertex $v \in V$, the open neighborhood of v is the set $N(v) = \{u \in V : uv \in E\}$ and for a set $S \subseteq V$, $N(S) = \cup_{v \in S} N(v)$. The closed neighborhood of a vertex $v \in V$ is the set $N[v] = N(v) \cup \{v\}$ and for a set $S \subseteq V$, $N[S] = N(S) \cup \{S\}$.

A set $D \subseteq V$ is dominating set of G, if every vertex in $V - D$ is adjacent to some vertex in D. The domination number $\gamma(G)$ of G is the minimum cardinality of a dominating set.

The PBIBD with m-association scheme which are arising from dominating sets has been studied extensively by many for example see [1], [6], [7]. In this paper, We study the PBIBD and the association scheme which can be obtained from the minimum dominating sets in $H \cong G \circ K_m$ graph. Finally we generalize the results for the graph $H \cong G \circ K_m$.

2. PBIBD arising from minimum dominating sets of $H \cong G \circ K_m$

Definition 2.1. Given v objects a relation satisfying the following conditions is said to be an association scheme with m classes:

(i) any two objects are either first associates, or second associates,..., or mth associates, the relation of association being symmetric.

(ii) each object α has n_i ith associates, the number n_i being independent of α.

(iii) if two objects α and β are ith associates, then the number of objects which are jth associates of α and kth associates of β is p^i_{jk} and is independent of the pair of ith associates α and β. Also $p^i_{jk} = p^i_{kj}$.

If we have association scheme for the v objects we can define a PBIBD as the following definition.

Definition 2.2. The PBIBD design is arrangement of v objects into b sets (called blocks) of size k where $k < v$ such that

(i) every object is contained in exactly r blocks.

(ii) each block contains k distinct objects.

(iii) Any two objects which are ith associates occur together in exactly λ_i blocks.

Theorem 2.3. Let $H \cong G \circ K_m$ be a graph. Then the number of minimum dominating sets of the graph is $(m+1)^n$.

Proof. Let $H \cong G \circ K_m$. Then $\gamma(G) = n$,we need to find out all the sets of size n. For this we have three cases:

case 1. All the vertices of the minimum dominating set are selected from the vertices of G. Then there is only one minimum dominating set.

case 2. All the vertices of the minimum dominating set are not from the vertices of G. Then number of ways to select minimum dominating sets of size n m^n.

case 3. By selecting some vertices from G and other vertices from outside G. So the we select one vertex from G and $(n-1)$ vertices from outside G then there are $\binom{n}{1} m^{n-1}$ ways. Similarly to select two vertices from G and $(n-2)$ vertices from outside G then there are $\binom{n}{2} m^{n-2}$ ways and so on.

From Case 1, Case 2 and case 3 the total number of minimum dominating sets in H are

$$\begin{aligned}&(m)^n + \binom{n}{1}m^{n-1} + \binom{n}{2}m^{n-2} + \cdots + \binom{n}{n-1}m + 1\\ &= \sum_{i=0}^{n}\binom{n}{i}m^{n-i}\\ &= (m+1)^n.\end{aligned}$$

■

Theorem 2.4. Let $H \cong G \circ K_m$ be a graph. Any two vertices in G either belong to zero minimum dominating set or $(m+1)^{n-2}$ minimum dominating sets.

Proof. By labelling the vertices of the graph
$H \cong G \circ K_m = \{v_1, v_2, ..., v_n, v_1^{'}, v_1^{''}, v_1^{'''} ...v_1^m, v_2^{'}, v_2^{''}, v_2^{'''}, ...v_2^m ...v_n^{'}, v_n^{''}, v_n^{'''}, ...v_n^m\}$
where $\{v_1, v_2, ..., v_n\}$ are the vertices of G and
$\{v_1^{'}, v_1^{''}, v_1^{'''} ...v_1^m, v_2^{'}, v_2^{''}, v_2^{'''}, ...v_2^m ...v_n^{'}, v_n^{''}, v_n^{'''}, ...v_n^m\}$ are the vertices of k_m.

Suppose A = $\{v_1, v_2, ..., v_n\}$ and B = $\{v_1^{'}, v_1^{''}, v_1^{'''} ...v_1^m, v_2^{'}, v_2^{''}, v_2^{'''}, ...v_2^m ...v_n^{'}, v_n^{''}, v_n^{'''}, ...v_n^m\}$. Let u, v be any two vertices, we have the following cases:

Case 1. u and v belong to A then there are $(m+1)^{n-2}$ minimum dominating sets containing u and v.

Case 2. u and v belong to B then there are $(m+1)^{n-2}$ways to select minimum dominating sets containing u and v.

Case 3. Let $u \in A$ and $v \in B$ we have two subcases:

Case (i). Let u and v in the same triangle then there does not exists any minimum dominating sets containing u and v.

Case (ii). If u and v are from the different triangle then there are $(m+1)^{n-2}$ ways to select minimum dominating sets. ■

Theorem 2.5. Let $H \cong G \circ K_m$. Then every vertex $v \in V(G)$ contained in $(m+1)^{n-1}$ minimum dominating sets.

Proof. Let $H \cong G \circ K_m$. The vertices of H can be partitioned into n sets, each set containing m vertex as the triangles $\Delta_1, \Delta_2 .... \Delta_n$. Let $v \in V(G)$ be any vertex such that $v \in \Delta_i$ for some $1 \leq i \leq n$. Any minimum dominating set containing v will contain $(n-1)$ vertices from the other triangle Δ_j where $i \neq j$. But it is not allowed to take two vertex from the same triangle so we need to take one vertex from each triangle. ■

Hence the ways to select $n-1$ vertices from the Δ_j triangles $i \neq j$ is

$$\underbrace{\binom{m+1}{1}\binom{m+1}{1}\binom{m+1}{1}\dots\binom{m+1}{1}}_{n-1} = (m+1)^{n-1}.$$

Finally we generalize theorem as:

Theorem 2.6. For any graph $H \cong G \circ K_m$, we can get PBIBD with parameters, ($v = mn, k = m, r = (m+1)^{n-1}, b = (m+1)^n, \lambda_1 = 0, \lambda_2 = (m+1)^{n-2}$) and association scheme of 2-classes with:

$$P_1 = \begin{bmatrix} p^1_{11} & p^1_{12} \\ p^1_{21} & p^1_{22} \end{bmatrix} = \begin{bmatrix} (m-1) & 0 \\ 0 & (m+1)(n-1) \end{bmatrix}$$

and

$$P_2 = \begin{bmatrix} p^2_{11} & p^2_{12} \\ p^2_{21} & p^2_{22} \end{bmatrix} = \begin{bmatrix} 0 & m \\ m & (m+1)(n-2) \end{bmatrix}.$$

Proof. Above theorem follows from the previous theorems. ■

Acknowledgment

I thank the college and UGC for supporting this minor project MRP(S)-0154/12-13/ KAMY008/UGC-SWRO by UGC grants.

References

[1] Anwar Alwardi and N. D. Soner, Partial balanced incomplete block designs arising from some minimal dominating sets of SRNT graphs, International Journal of Mathematical Archive 2(2) (2011), 233–235.

[2] P. J . Cameron and J . H. Van Lint, Designs, graphs, Codes and their links, vol. 22 of London Mathematical Society Student Texts, Cambridge University Press, Cambridge, 1991.

[3] F. Harary, Graph theory, Addison-Wesley, Reading Mass (1969).

[4] V. R. Kulli and S. C. Sigarkanti, Further results on the neighborhood number of a graph. Indian J. Pure and Appl. Math., 23 (8) (1992) 575–577.

[5] Sharada. B, and Soner Nandappa. D, Partially balanced incomplete block designs arising from minimum efficient dominating sets of graph, Bull. Pure Appl. Math, Vol. 2, No. 1 (2008), 47–56.

[6] Sumathi. M.P. and N. D. Soner, Association scheme on some cycles related with minimum neighbourhood sets. My Science Vol V(1-2), Jan-Jul (2011), 23–27.

[7] H. B. Walikar, H. S. Ramane, B. D. Acharya, H. S. Shekhareppa and, S. Arumugum, Partially balanced incomplete block design arising from minimum dominating sets of paths and cycles. AKCE J. Graphs Combin., 4(2) (2007), 223–232.