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Permuting tri- (σ, τ)- Generalized Derivations in Prime Near-Rings

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Abstract

Let N be a 2,3- torsion free prime near ring, F a permuting tri-additive mapping of N and F be a permuting tri- (σ, τ) -generalized derivation associated with a nonzero permuting tri-(σ, τ)-derivation D of N. (i) If $F(N, N, N) \subseteq Z$, then N is a commutative ring.

(ii) If $f(x), f(x) + f(x) \in C(D(y, z, w))$ for all $x, y, z, w \in N$, then $(N, +)$ is an abelian and $f(x) \in Z$, for all $x \in N$.

Keywords: Prime ring, Derivation, Generalized derivation, (σ, τ)-derivation, Permuting tri-derivation, Permuting tri-(σ, τ)-derivation, Permuting tri-(σ, τ)-generalized derivation.

Introduction: The concept of a permuting tri-derivation has been introduced Ozturk in [4]. Some recent results on properties of prime rings, semiprime rings and near rings with derivations have been investigated in several ways [1-4, 7]. Kyoo-Hong Park et al. in [6] have introduced the concept of permuting tri-derivation of a near ring and investigated the conditions for a near ring to be commutative ring. Further Ozturk et al. in [5] introduce the concepts of permuting tri-(σ, τ)-derivation and permuting tri-generalized derivation of near ring and gave some properties. In this paper we proved some results on permuting tri-(σ, τ)-generalized derivations in prime near rings.

Preliminaries: Throughout this paper N will be a nonzero-symmetric left near ring with multiplicative center Z, σ and τ be a automorphisms of N. Recall that a ring N is prime if $xNy = \{0\}$ implies $x = 0$ or $y = 0$. A ring N is said to be characteristic not two if $2x = 0$ implies $x = 0$, for all $x \in N$. For any $x, y \in N$, the symbol $[x, y]$ stands for the commutator $xy - yx$ and the symbol (x, y) denotes the additive commutator $x + y - x - y$. A mapping $D(.,.,.): N \times N \times N \to N$ is called permuting tri-additive if $D(x, y, z) = D(y, x, z) = D(z, y, x) = D(x, z, y) = D(y, z, x) = D(z, x, y)$, for all $x, y, z \in N$. A mapping $d: N \to N$ is said to be trace of D if $d(x) = D(x, x, x)$, for all $x \in N$, where $D(.,.,.): N \times N \times N \to N$ is a permuting tri-additive mapping. The trace of D satisfies the relation $d(x + y) = d(x) + 3D(x, x, y) + 3D(x, y, y) + d(y)$, for all $x, y \in N$. An additive mapping $d: N \to N$ is called a derivation if $d(xy) = d(x)y + xd(y)$ holds for all $x, y \in N$. A permuting tri-additive mapping $D(.,.,.): N \times N \times N \to N$ is called a permuting tri-derivation if $D(xw, y, z) = D(x, y, z)w + xD(w, y, z)$, for all $x, y, z, w \in N$. An additive mapping $d: N \to N$ is called a (σ, τ)-derivation if $d(xy) = d(x)\sigma(y) + \tau(x)d(y)$ holds for all $x, y \in N$, where σ and τ to be a automorphisms of N. A permuting tri-additive mapping $D: N \times N \times N \to N$ is called a permuting tri-(σ, τ)-derivation if there exists functions $\sigma, \tau: N \to N$ such that $D(xw, y, z) = D(x, y, z)\sigma(w) + \tau(x)D(w, y, z)$, for all $x, y, z, w \in N$. An additive mapping $F: N \to N$ is called a generalized derivation, if there exists a derivation $d: N \to N$ such that $F(xy) = F(x)y + xd(y)$ holds for all $x, y \in N$.

An additive mapping $F: N \to N$ is said to be a (σ, τ)-generalized derivation of N, if there exists a (σ, τ)-derivation $d: N \to N$ such that $F(xy) = F(x)\sigma(y) + \tau(x)d(y)$ holds for all $x, y \in R$. Let N be a near ring and $D(.,.,.): N \times N \times N \to N$ a tri-(σ, τ)-derivation of N. A permuting tri-additive map $F(.,.,.): N \times N \times N \to N$ is said to be a permuting tri-right (resp. left) (σ, τ)-generalized derivation of N associated with D if $F(xw, y, z) = F(x, y, z)\sigma(w) + \tau(x)D(w, y, z)$ (resp. $F(xw, y, z) = D(x, y, z)\sigma(w) + \tau(x)F(w, y, z)$), for all $x, y, z, w \in N$. Also F is said to be a permuting tri-(σ, τ)-generalized derivation of N associated with D if it is both a permuting tri-right (σ, τ)-generalized derivation and permuting tri-left (σ, τ)-generalized derivation of N associated with D.

Lemma 1: [1, Lemma 3] Let N be a prime near ring. (i) If $z \in Z - \{0\}$, then z is not a zero devisor. (ii) If $Z - \{0\}$ contains an element z for which $z + z \in Z$, then $(N, +)$ is abelian.

Lemma 2: Let N be a 3 and 2-torsion free prime near ring, D a permuting tri-(σ, τ)-derivation of N, F be a permuting tri-additive mapping of N and f the trace of F.

Then (i) If F is a permuting tri-(σ,τ)-generalized derivation of N associated with D and $xf(y)=0$, for all $y\in N$, then $x=0$ or $D=0$. (ii) If F is a permuting tri-(σ,τ)-generalized derivation of N associated with D and $xf(y)=0$, for all $y\in N$, then $x=0$ or $F=0$.

Proof: (i) Given that $xf(y)=0$, for all $y\in N$. (1)

We replacing y by $y+z$ in equation (1), we get

$xf(y+z)=0$

$x\big(f(y)+3F(y,y,z)+3F(y,z,z)+f(z)\big)=0$

$xf(y)+3xF(y,y,z)+3xF(y,z,z)+xf(z)=0$, for all $y,z\in N$. (2)

Using equation (1) in the equation (2), we get

$3xF(y,y,z)+3xF(y,z,z)=0$, for all $y,z\in N$. (3)

We replacing y by $-y$ in equation (3), we get

$3xF(-y,-y,z)+3xF(-y,z,z)=0$

$3xF(y,y,z)-3xF(y,z,z)=0$, for all $y,z\in N$. (4)

We subtracting equation (4) form equation (3), we get

$6xF(y,z,z)=0$, for all $y,z\in N$.

Since N be a 3 and 2-torsion free ring, we get

$xF(y,z,z)=0$, for all $y,z\in N$. (5)

We replacing z by $z+w$ in equation (5), we get

$xF(y,z+w,z+w)=0$, for all $y,z,w\in N$.

$xF(y,z+w,z)+xF(y,z+w,w)=0$

$xF(y,z,z)+xF(y,w,z)+xF(y,z,w)+xF(y,w,w)=0$, for all $y,z,w\in N$.

Using equation (5) in the above equation, we get

$2xF(y,z,w)=0$, for all $y,z,w\in N$.

Since N be a 2-torsion free ring, we get

$xF(y,z,w)=0$, for all $y,z,w\in N$. (6)

We replacing y by yv in equation (6), we get

$xF(yv,z,w)=0$

$xF(y,z,w)\sigma(v) + x\tau(y)D(v,z,w) = 0$, for all $y,z,w,v \in N$. (7)

Using equation (6) in equation (7), we get

$x\tau(y)D(v,z,w) = 0$

Since τ is an automorphism, we have

$xND(v,z,w) = \{0\}$, for all $z,w,v \in N$.

Since N is prime near ring, we have $x = 0$ or $D = 0$.

(ii) Let x be a nonzero element of N. Thus from (i), we obtain $D = 0$.

We replacing y by yv, $v \in N$ in equation (6), we get

$xF(yv,z,w) = 0$

$xD(y,z,w)\sigma(v) + x\tau(y)F(v,z,w) = 0$, for all $y,z,w,v \in N$.

Using $D = 0$ in the above equation, we get

$x\tau(y)F(v,z,w) = 0$, for all $y,z,w,v \in N$.

Since N is prime and $x \neq 0$, we have that $F = 0$.

Lemma 3: Let N be a prime near ring, F a permuting tri-additive mapping of N and F be a permuting tri-(σ,τ)-generalized derivation associated with a nonzero permuting tri-(σ,τ)-derivation D of N. Then for all $y,z,x,w \in N$, the following are equivalent:

(i) $F(xw,y,z) = F(x,y,z)\sigma(w) + \tau(x)D(w,y,z)$

(ii) $F(xw,y,z) = \tau(x)D(w,y,z) + F(x,y,z)\sigma(w)$

Proof: (i) ⇒ (ii) :

We assumed that $F(xw,y,z) = F(x,y,z)\sigma(w) + \tau(x)D(w,y,z)$, for all $y,z,x,w \in N$. (8)

$F(x(w+w),y,z) = F(x,y,z)\sigma(w+w) + \tau(x)D(w+w,y,z)$

$= F(x,y,z)\sigma(w) + F(x,y,z)\sigma(w) + \tau(x)D(w,y,z) + \tau(x)D(w,y,z)$, for all $y,z,x,w \in N$. (9)

On the other hand

$F(x(w+w),y,z) = F(xw+xw,y,z)$

$= F(xw,y,z) + F(xw,y,z)$

$= F(x,y,z)\sigma(w) + \tau(x)D(w,y,z) + F(x,y,z)\sigma(w) + \tau(x)D(w,y,z)$, for all $y,z,x,w \in N$. (10)

From equation (9) and equation (10), we get

$F(x,y,z)\sigma(w) + \tau(x)D(w,y,z) = \tau(x)D(w,y,z) + F(x,y,z)\sigma(w)$

Hence we have $F(xw,y,z) = \tau(x)D(w,y,z) + F(x,y,z)\sigma(w)$, for all $y,z,x,w \in N$.

(ii) ⇒ (i) : This is proved in similar way.

Lemma 4: Let N be a prime near ring, F a permuting tri-additive mapping of N, f be the trace of F and F be a permuting tri-(σ,τ)-generalized derivation associated with a nonzero permuting tri-(σ,τ)-derivation D of N. Then, for all $x,y,z,w,v \in N$,

(i) $[F(x,y,z)\sigma(v) + \tau(x)D(v,y,z)]w = F(x,y,z)\sigma(v)w + \tau(x)D(v,y,z)w$,

(ii) $[f(x)\sigma(v) + \tau(x)D(x,x,v)]w = f(x)\sigma(v)w + \tau(x)D(x,x,v)w$,

(iii) $[\tau(x)D(v,y,z) + F(x,y,z)\sigma(v)]w = \tau(x)D(v,y,z)w + F(x,y,z)\sigma(v)w$,

(iv) $[\tau(x)D(x,x,v) + f(x)\sigma(v)]w = \tau(x)D(x,x,v)w + f(x)\sigma(v)w$.

Proof: (i) Suppose that $F(xv,y,z) = F(x,y,z)\sigma(v) + \tau(x)D(v,y,z)$, for all $x,y,z,v \in N$.

(11)

From the associative law

$F\big((xv)w,y,z\big) = F(xv,y,z)\sigma(w) + \tau(xv)D(w,y,z)$

$= [F(x,y,z)\sigma(v) + \tau(x)D(v,y,z)]\sigma(w) + \tau(x)\tau(v)D(w,y,z)$, for all $x,y,z,v \in N$.

(12)

On the other hand

$F(x(vw),y,z) = F(x,y,z)\sigma(vw) + \tau(x)D(vw,y,z)$

$= F(x,y,z)\sigma(vw) + \tau(x)D(v,y,z)\sigma(w) + \tau(x)\tau(v)D(w,y,z)$

$= F(x,y,z)\sigma(v)\sigma(w) + \tau(x)D(v,y,z)\sigma(w) + \tau(x)\tau(v)D(w,y,z)$, for all $x,y,z,v \in N$. (13)

From equation (12) and equation (13), we get

$[F(x,y,z)\sigma(v) + \tau(x)D(v,y,z)]\sigma(w) = F(x,y,z)\sigma(v)\sigma(w) + \tau(x)D(v,y,z)\sigma(w)$

We replacing $\sigma(w)$ by w in the above equation, we get

$[F(x,y,z)\sigma(v) + \tau(x)D(v,y,z)]w = F(x,y,z)\sigma(v)w + \tau(x)D(v,y,z)w$, for all

$x, y, z, v, w \in N$. (14)

(ii) We replacing y, z by x in the equation (14), we get

$[F(x,x,x)\sigma(v) + \tau(x)D(v,x,x)]w = F(x,x,x)\sigma(v)w + \tau(x)D(v,x,x)w$

$[f(x)\sigma(v) + \tau(x)D(x,x,v)]w = f(x)\sigma(v)w + \tau(x)D(x,x,v)w$, for all $x, v, w \in N$. (15)

(iii) Suppose that $F(xv,y,z) = \tau(x)D(v,y,z) + F(x,y,z)\sigma(v)$, for all $x, y, z, v \in N$. (16)

From the associative law

$F\big((xv)w,y,z\big) = \tau(xv)D(w,y,z) + F(xv,y,z)\sigma(w)$

$= \tau(x)\tau(v)D(w,y,z) + [\tau(x)D(v,y,z) + F(x,y,z)\sigma(v)]\sigma(w)$, for all $x, y, z, v \in N$. (17)

And other hand

$F(x(vw),y,z) = \tau(x)D(vw,y,z) + F(x,y,z)\sigma(vw)$

$= \tau(x)\tau(v)D(w,y,z) + \tau(x)D(v,y,z)\sigma(w) + F(x,y,z)\sigma(vw)$

$= \tau(x)\tau(v)D(w,y,z) + \tau(x)D(v,y,z)\sigma(w) + F(x,y,z)\sigma(v)\sigma(w)$, for all $x, y, z, v \in N$.

(18)

From equation (17) and equation (18), we get

$[\tau(x)D(v,y,z) + F(x,y,z)\sigma(v)]\sigma(w) = \tau(x)D(v,y,z)\sigma(w) + F(x,y,z)\sigma(v)\sigma(w)$

We replacing $\sigma(w)$ by w in the above equation, we get

$[\tau(x)D(v,y,z) + F(x,y,z)\sigma(v)]w = \tau(x)D(v,y,z)w + F(x,y,z)\sigma(v)w$, for all $x, y, z, v, w \in N$. (19)

(iv) We replacing y, z by x in the equation (19), we get

$[\tau(x)D(v,x,x) + F(x,x,x)\sigma(v)]w = \tau(x)D(v,x,x)w + F(x,x,x)\sigma(v)w$

$[\tau(x)D(x,x,v) + f(x)\sigma(v)]w = \tau(x)D(x,x,v)w + f(x)\sigma(v)w$, for all $x, v, w \in N$.

(20)

Lemma 5: Let N be a prime near ring, F a permuting tri-additive mapping of N and F be a permuting tri-(σ,τ)-generalized derivation associated with a nonzero permuting tri-(σ,τ)-derivation D of N and $x \in N$. If $xF(N,N,N) = \{0\}$, then $x = 0$.

Proof: By our hypothesis we have

$xF(y,z,w)=0$, for all $x,y,z,w \in N$. (21)

We replacing y by yv in equation (21), we get

$xF(yv,z,w)=0$

$xF(y,z,w)\sigma(v)+x\tau(y)D(v,z,w)=0$, for all $x,y,z,w \in N$. (22)

Using equation (21) in equation (22), we get

$x\tau(y)D(v,z,w)=0$, for all $x,y,z,w,v \in N$

Since τ is an automorphism, then we have

$xND(v,z,w)=0$, for all $x,z,w,v \in N$.

Since $D \neq 0$ and N is a prime ring implies that $x=0$.

Theorem 1: Let N be a 2,3- torsion free prime near ring, F a permuting tri-additive mapping of N and F be a permuting tri-(σ,τ)-generalized derivation associated with a nonzero permuting tri-(σ,τ)-derivation D of N and $x \in N$. If $F(N,N,N) \subseteq Z$, then N is a commutative ring.

Proof: Since $F(N,N,N) \subseteq Z$ and F is a nonzero, there exist nonzero elements $x,y,z \in N$ such that $F(x,y,z) \in Z-\{0\}$.

Then $F(x+x,y,z)=F(x,y,z)+F(x,y,z) \in Z$ and hence $(N,+)$ is abelian by Lemma 1.

$F(x,y,z) \in Z$, for all $x,y,z \in N$

Implies that $F(x,y,z)w=wF(x,y,z)$, for all $w,x,y,z \in N$. (23)

We replacing x by vx in the equation (23), we get

$F(vx,y,z)w=wF(vx,y,z)$

$[F(v,y,z)\sigma(x)+\tau(v)D(x,y,z)]w=w[F(v,y,z)\sigma(x)+\tau(v)D(x,y,z)]$

$F(v,y,z)\sigma(x)w+\tau(v)D(x,y,z)w=wF(v,y,z)\sigma(x)+w\tau(v)D(x,y,z)$, for all $w,x,y,z,v \in N$. (24)

We replacing w by $\sigma(x)$ in equation (24), we get

$F(v,y,z)\sigma(x)\sigma(x)+\tau(v)D(x,y,z)\sigma(x)=\sigma(x)F(v,y,z)\sigma(x)+\sigma(x)\tau(v)D(x,y,z)$, for all $w,x,y,z,v \in N$. (25)

Using equation (23) in equation (25), we get

$\sigma(x)F(v,y,z)\sigma(x) + \tau(v)D(x,y,z)\sigma(x) = \sigma(x)F(v,y,z)\sigma(x) + \sigma(x)\tau(v)D(x,y,z)$

$\tau(v)D(x,y,z)\sigma(x) = \sigma(x)\tau(v)D(x,y,z)$, for all $x,y,z,v \in N$. (26)

We replacing v by uv in equation (26), we get

$\tau(uv)D(x,y,z)\sigma(x) = \sigma(x)\tau(uv)D(x,y,z)$

$\tau(u)\tau(v)D(x,y,z)\sigma(x) = \sigma(x)\tau(u)\tau(v)D(x,y,z)$

Using equation (26) in the above equation, we get

$\tau(u)\sigma(x)\tau(v)D(x,y,z) = \sigma(x)\tau(u)\tau(v)D(x,y,z)$

$\sigma(x)\tau(u)\tau(v)D(x,y,z) - \tau(u)\sigma(x)\tau(v)D(x,y,z) = 0$

$[\sigma(x),\tau(u)]\tau(v)D(x,y,z) = 0$

Since τ is an automorphism, then we have

$[\sigma(x),\tau(u)]ND(x,y,z) = \{0\}$, for all $x,y,z,u \in N$. (27)

Since N is a prime ring implies that for each $x \in N$ either $[\sigma(x),\tau(u)] = 0$, for all $x \in N$ or $D(x,y,z) = 0$, for all $x,y,z \in N$.

If $D(x,y,z) = 0$, for all $x,y,z \in N$. (28)

We substituting equation (28) in equation (24), we get

$F(v,y,z)\sigma(x)w = wF(v,y,z)\sigma(x)$, for all $w,x,y,z,v \in N$. (29)

We replacing x by ux in equation (29), we get

$F(v,y,z)\sigma(ux)w = wF(v,y,z)\sigma(ux)$

$F(v,y,z)\sigma(u)\sigma(x)w = wF(v,y,z)\sigma(u)\sigma(x)$

Using equation (29) in the above equation, we get

$F(v,y,z)\sigma(u)\sigma(x)w = F(v,y,z)\sigma(u)w\sigma(x)$

$F(v,y,z)\sigma(u)w\sigma(x) - F(v,y,z)\sigma(u)\sigma(x)w = 0$

$F(v,y,z)\sigma(u)[w,\sigma(x)] = 0$

Since σ is an automorphism, then we have

$F(v,y,z)N[w,\sigma(x)] = \{0\}$, for all $w,x,y,z,v \in N$. (30)

Since $F \neq 0$ and N is a prime ring implies that $[w,\sigma(x)] = 0$, for all $w \in N$

But σ is an automorphism, then we conclude that $x \in Z$.

On the other hand, if $[\sigma(x), \tau(u)] = 0$, for all $x \in N$, then again $x \in Z$

Hence we find that $N = Z$, and N is a commutative ring.

Theorem 2: Let N be a 2,3- torsion free prime near ring, F a permuting tri-additive mapping of N and F be a permuting tri-(σ,τ)-generalized derivation associated with a nonzero permuting tri-(σ,τ)-derivation D of N. If $f(x)$, $f(x) + f(x) \in C(D(y,z,w))$, for all $x, y, z, w \in N$, then $(N, +)$ is an abelian and $f(x) \in Z$, for all $x \in N$.

Proof: Since $f(x)$, $f(x) + f(x) \in C(D(y,z,w))$, for all $x, y, z, w \in N$, if both u and $u + u$ commute element wise with $D(y,z,w)$, for all $y, z, w \in N$, then

$$[D(y,z,w) + D(v,z,w)](u + u) = D(y,z,w)(u + u) + D(v,z,w)(u + u)$$

$$[D(y,z,w) + D(v,z,w)]u + [D(y,z,w) + D(v,z,w)]u = D(y,z,w)u + D(y,z,w)u + D(v,z,w)u + D(v,z,w)u$$

$$D(y,z,w)u + D(v,z,w)u + D(y,z,w)u + D(v,z,w)u = D(y,z,w)u + D(y,z,w)u + D(v,z,w)u + D(v,z,w)u$$

$$D(v,z,w)u + D(y,z,w)u = D(y,z,w)u + D(v,z,w)u$$

$$D(y + v - y - v, z, w)u = 0$$

$$D\big((y,v), z, w\big)u = 0$$

We u by $f(x)$ in the above equation and using lemma 5, we get

$$D\big((y,v), z, w\big) = 0, \text{ for all } v, y, z, w \in N. \tag{31}$$

We replacing (y,v) by $s(y,v)$ in the equation (31), we get

$D(s(y,v), z, w) = 0$, for all $s, v, y, z, w \in N$.

$$D(s,z,w)\sigma\big((y,v)\big) + \tau(s)D\big((y,v), z, w\big) = 0$$

Using equation (31) in the above equation, we get

$D(s,z,w)\sigma\big((y,v)\big) = 0$, for all $s, v, y, z, w \in N$.

Since D is a nonzero permuting tri-(σ,τ)-derivation and N is prime ring, we have $\sigma\big((y,v)\big) = 0$. Since σ is an automorphism, then we have $(y,v) = 0$, for all $y, v \in N$.

Thus $(N, +)$ is abelian.

Since $f(x) \in C(D(y,z,w))$, for all $x, y, z, w \in N$ implies that

$f(x)D(y,z,w) = D(y,z,w)f(x)$, for all $x,y,z,w \in N$. (32)

We replacing y by yv in the equation (32), we get

$f(x)D(yv,z,w) = D(yv,z,w)f(x)$

$f(x)D(y,z,w)\sigma(v) + f(x)\tau(y)D(v,z,w) = D(y,z,w)\sigma(v)f(x) + \tau(y)D(v,z,w)f(x)$

We substitute equation (32) in the above equation, we get

$f(x)D(y,z,w)\sigma(v) + \tau(y)D(v,z,w)f(x) = D(y,z,w)\sigma(v)f(x) + \tau(y)D(v,z,w)f(x)$

$f(x)D(y,z,w)\sigma(v) = D(y,z,w)\sigma(v)f(x)$, for all $x,y,z,w,v \in N$. (33)

We replacing y by $d(y)$ in the equation (33), we get

$f(x)D(d(y),z,w)\sigma(v) = D(d(y),z,w)\sigma(v)f(x)$

We substitute equation (33) in the above equation, we get

$D(d(y),z,w)f(x)\sigma(v) = D(d(y),z,w)\sigma(v)f(x)$

$D(d(y),z,w)\sigma(v)f(x) - D(d(y),z,w)f(x)\sigma(v) = 0$

$D(d(y),z,w)[\sigma(v),f(x)] = 0$, for all $x,y,z,w,v \in N$. (34)

We replacing z by zt in the equation (34), we get

$D(d(y),zt,w)[\sigma(v),f(x)] = 0$

$D(d(y),z,w)\sigma(t)[\sigma(v),f(x)] + \tau(z)D(d(y),t,w)[\sigma(v),f(x)] = 0 = 0$

We substitute equation (34) in the above equation, we get

$D(d(y),z,w)\sigma(t)[\sigma(v),f(x)] = 0$

Since τ is an automorphism, then we have

$D(d(y),z,w)N[\sigma(v),f(x)] = 0$, for all

Since N is a prime ring implies that for each $x \in N$ either $D(d(y),z,w) = 0$, for all $y,z,w \in N$ or $[\sigma(v),f(x)] = 0$, for all $x,v \in N$.

Suppose that $D(d(y),z,w) = 0$, for all $y,z,w \in N$. (35)

We replacing y by $y + v$ in the equation (35), we get

$D(d(y+v),z,w) = 0$

$D(d(y) + d(v) + 3D(y,y,v) + 3D(y,v,v),z,w) = 0$

$D(d(y),z,w)+D(d(v),z,w)+3D(D(y,y,v),z,w)+3D(D(y,v,v),z,w)=0$

Using equation (35) in the above equation, we get

$3(D(D(y,y,v),z,w)+D(D(y,v,v),z,w))=0$

Since N is a 3- torsion free, then we have

$D(D(y,y,v),z,w)+D(D(y,v,v),z,w)=0$, for all $v,y,z,w\in N$. (36)

We replacing y by $-y$ in the equation (36), we get

$D(D(y,y,v),z,w)-D(D(y,v,v),z,w)=0$, for all $v,y,z,w\in N$. (37)

We subtracting equation (37) from equation (36), we get

$2D(D(y,v,v),z,w)=0$

Since N is a 2- torsion free, then we have

$D(D(y,v,v),z,w)=0$, for all $v,y,z,w\in N$. (38)

We replacing y by yx in the equation (38), we get

$D(D(yx,v,v),z,w)=0$

$D(D(y,v,v)\sigma(x)+\tau(y)D(x,v,v),z,w)=0$

$D(D(y,v,v)\sigma(x),z,w))+D(\tau(y)D(x,v,v),z,w))=0$

$D(D(y,v,v),z,w)\sigma(\sigma(x))+\tau(D(y,v,v))D(\sigma(x),z,w)+$
$D(\tau(y),z,w)\sigma(D(x,v,v))+\tau(\tau(y))D(D(x,v,v),z,w)=0$

Using equation (38) in the above equation, we get

$\tau(D(y,v,v))D(\sigma(x),z,w)+D(\tau(y),z,w)\sigma(D(x,v,v))=0$, for all $x,y,z,w\in N$. (39)

We replacing x by xt in the equation (39), we get

$\tau(D(y,v,v))D(\sigma(xt),z,w)+D(\tau(y),z,w)\sigma(D(xt,v,v))=0$

$\tau(D(y,v,v))D(\sigma(x),z,w)\sigma(\sigma(t))+\tau(D(y,v,v))\tau(\sigma(x))D(\sigma(t),z,w)+$
$D(\tau(y),z,w)\sigma(D(x,v,v))\sigma(t)+D(\tau(y),z,w)\sigma(\tau(x))\sigma(D(t,v,v))=0$

We substitute equation (39) in the above equation, we get

$\tau(D(y,v,v))\tau(\sigma(x))D(\sigma(t),z,w)+D(\tau(y),z,w)\sigma(\tau(x))\sigma(D(t,v,v))=0$

We replacing $\sigma(x)$ by x, $\sigma(t)$ by t, $\tau(y)$ by y and $\tau(x)$ by x in the above equation, we get

$\tau(D(y,v,v))\tau(x)D(t,z,w)+D(y,z,w)\sigma(x)\sigma(D(t,v,v))=0$

We replacing y, x, z, w, t by v in the above equation, we get

$\tau\big(D(v,v,v)\big)\tau(v)D(v,v,v) + D(v,v,v)\sigma(v)\sigma\big(D(v,v,v)\big) = 0$

$\tau\big(d(v)\big)\tau(v)d(v) + d(v)\sigma(v)\sigma\big(d(v)\big) = 0$

We replacing $\tau(d(v))$ by $d(v)$, $\sigma(d(v))$ by $d(v)$ and $\tau(v)$ by $\sigma(v)$ in the above equation , we get

$2d(v)\sigma(v)d(v) = 0$

Since N is a 2- torsion free, then we have

$d(v)\sigma(v)d(v) = 0$

Since N is prime near ring $d(v) = 0$, and so $D = 0$.

It is contradiction to $D \neq 0$.

Therefore $[\sigma(v), f(x)] = 0$, for all $x, v \in N$, then $f(x) \in Z$.

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