

## **MHD Radiative Boundary Layer Flow of Nanofluid Past a Vertical Plate with Effects of Binary Chemical Reaction and Activation Energy**

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### **Abstract**

A mathematical analysis has been carried out to investigate the effects of internal heat generation/absorption, viscous and ohmic dissipations, effects of binary chemical reaction and activation energy on steady two-dimensional radiative MHD boundary-layer flow of a viscous, incompressible, electrically conducting nanofluid over a vertical plate. The governing nonlinear system of PDEs are converted into a set of nonlinear ODEs by employing suitable similarity transformations and then solved numerically by using NDSolve in Mathematica. The effects of various controlling parameters on the dimensionless velocity, temperature and nanoparticle volume fraction profiles are discussed graphically. The heat generation and magnetic field effect in addition to viscous dissipation accelerates the temperature and also decelerates the nanoparticle volume fraction profile. Comparisons have been made with bench mark solutions for a special case and found a very good agreement.

**Keywords :** MHD flow, Heat generation/absorption, Nanofluid, Ohmic dissipation, Viscous dissipation, Vertical plate.

## **1. INTRODUCTION**

Emission and absorption of heat energy is inevitable in a wide range of industrial processes. Speaking about any industrial facility, heat must be added, removed, or moved from one process stream to another. A source for energy recovery and fluid heating/cooling was provided by these processes. However, the emergence of modern materials technology provided the opportunity to produce nanometer-sized particles. The fluid containing nanometer-sized particles, called nanoparticles is known as nanofluid. Nanofluids have specific properties that make them potentially useful in many applications in heat transfer, including microelectronics, fuel cells, pharmaceutical processes, and hybrid-powered engines, engine cooling/vehicle thermal management, domestic refrigerator, chiller, heat exchanger, in grinding, machining and in boiler flue gas temperature reduction. They exhibit enhanced thermal conductivity and the convective heat transfer coefficient compared to the base fluid. Knowledge of the rheological behaviour of nanofluids is found to be very critical in deciding their suitability for convective heat transfer applications.

The situation changed when Choi [1] and Eastman in Argonne National Laboratory revisited this field with their nanoscale metallic particle and carbon nanotube suspensions; Nanofluid is a new kind of heat transfer medium, containing nanoparticles (1–100 nm) which are uniformly and stably distributed in a base fluid. These distributed nanoparticles, generally a metal or metal oxide greatly enhance the thermal conductivity of the nanofluid which increases conduction and convection coefficients by allowing for more heat transfer. Many of the publications on nanofluids are for understanding their behaviour so that they can be utilized for straight heat transfer enhancement which is paramount in many industrial applications like nuclear reactors, transportation, electronics as well as biomedicine and food industry. This concept attracted various researchers towards nanofluids, and various theoretical and experimental studies have been done to find the thermal properties of nanofluids. Boungiorno et al. [2] studied the thermal conductivity of nanofluids through experimental results. He also proposed an analytical model for convective transport in nanofluids taking into the account of Brownian diffusion and thermophoresis[3]. In recent years, the natural convection flow of nanofluid has been studied through the publications [4–6]. Kuznetsov and Nield [7] analyzed the natural convective boundary-layer flow of a nanofluid past a vertical plate using Boungiorno model. Gorla and Chamkha [8] reported the natural convective boundary layer flow of nanofluid in a porous medium. Khan and Pop [9] investigated the boundary layer flow of a nanofluid past a stretching sheet by considering the Brownian diffusion and thermophoresis effects. Khan and Aziz [10] represented the boundary layer flow of a nanofluid past a vertical surface with a constant heat flux. Aziz and Khan [11] analyzed the natural convective flow of a nanofluid over a convectively heated

vertical plate. In recent times, Rashad et al. [12] analyzed the natural convection flow of nanofluid over a vertical plate with stream wise temperature variation. Magnetohydrodynamic boundary layer flow is of considerable interest due to its wide usage in industrial technology and geothermal application, high temperature plasmas applicable to nuclear fusion energy conversion, MHD power production systems and liquid metal fluids. Due to its wide range of applications, the following researchers have investigated the magnetic field effect on the fluid flow problems [13–24]. Very freshly, Abdul Hakeem et al. [25] studied the magnetic field effect on second order slip flow of single phase nanofluid over a stretching/shrinking sheet. The interaction of natural convection with thermal radiation has increased greatly during the last decade due to its importance in many practical involvements. When free convection flows occur at high temperature, radiation effects on the flow become significant. Radiation effects on the free convection flow are important in context of space technology, processes in engineering areas occur at high temperature. Based on these applications, Olanrewaju et al. [26] investigated the boundary layer flow of nanofluids in the presence of radiation past a moving semi-infinite flat plate in a uniform free stream. Poornima et al. [27] analyzed the simultaneous effects of thermal radiation and magnetic on heat and mass transfer flow of nanofluids over a non-linear stretching sheet. Recently, Turkyilmazoglu and Pop [28] reported the thermal radiation effects on the flow of single phase nanofluid over an infinite vertical plate. Viscous dissipation is often a quite negligible effect, but its contribution towards the fluid viscosity is very high which might become important. It changes the temperature distributions by playing a role like an energy source, which leads to affected heat transfer rates. Anjali Devi and Ganga [29] reported the effects of viscous and Joules dissipation on MHD flow past a stretching porous surface embedded in a porous medium for ordinary fluid. The effects of thermal radiation and viscous dissipation on boundary layer flow of nanofluids over a moving flat plate were overlooked by Motsumi and Makinde [30]. Incredibly Makinde and Mutuku [31] investigated the hydromagnetic thermal boundary layer of nanofluids over a convectively heated flat plate with viscous dissipation and Ohmic heating effects. In cooling processes the study of heat generation or absorption effects are very important. Although, exact modeling of internal heat generation or absorption is quite difficult, some simple mathematical models can express its standard behavior for most substantial situations. Ahmed et al. [32] examined the effects of heat source/sink on the boundary layer flow of single phase nanofluid over a stretching tube. Very recently, Akilu and Narahari [33] explored the effects of internal heat generation/ absorption over an inclined plate of a nanofluid for a natural convection flow

Ganga *et al.* [34] investigated the effects of steady two dimensional radiative MHD boundary layer flow of viscous incompressible flow with internal heat

generation/absorption over a vertical plate by using HAM method and compared the results by using fourth order RK-shooting method. Anuradha *et al.*[35] have extended Ganga *et al.* work for two dimensional MHD boundary layer flow on a vertical plate with effects of radiation, chemical reaction, viscous and ohmic dissipation effects and heat source. The results for the dimensionless velocity, temperature, and nanofluid solid volume profiles are discussed with the help of graphs.

The main aim of this paper is to analyze the effects of binary chemical reaction and activation energy on two-dimensional radiative MHD boundary-layer flow of nanofluid over a vertical plate numerically. The coupled partial differential equations are solved numerically by using NDSolve in Mathematica.

## 2. FORMULATION OF THE PROBLEM

A steady two-dimensional boundary layer flow of a nanofluid over vertical plate in the presence of magnetic field intensity, thermal radiation, viscous dissipation, Ohmic dissipation and volumetric rate of heat generation/absorption is considered. A coordinate frame is selected in such a way that the x-axis is aligned vertically upwards. Also we consider a vertical plate at  $y = 0$ . The temperature  $T$  and the nanoparticle volume fraction  $\phi$  assume constant values  $T_w$  and  $\phi_w$  respectively at this boundary and it take values  $T_\infty$  and  $\phi_\infty$  respectively as  $y \rightarrow \infty$ . We are considering a steady state flow. We also consider influence of a constant magnetic field strength  $B_0$  which is applied normally to the plate. It is further assumed that the induced magnetic field is negligible in comparison to the applied magnetic field. Under the above assumptions, the boundary layer equations governing the flow, thermal and concentration fields can be written in dimensional form as [7].

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$\begin{aligned} \frac{\partial p}{\partial x} = & \mu \frac{\partial^2 u}{\partial y^2} - \rho f \left( u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) - \sigma B_0^2 u \\ & + \left[ (1 - \phi_\infty) \rho_{f\infty} \beta g (T - T_\infty) - (\rho_p - \rho_{f\infty}) g (\phi - \phi_\infty) \right] - \frac{v}{K} u \end{aligned} \quad (2)$$

$$\begin{aligned} u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = & \alpha \nabla^2 T + \frac{\mu}{\rho c} \left( \frac{\partial u}{\partial y} \right)^2 + \left( \frac{\sigma B_0^2}{\rho c} \right) u^2 + \tau \left[ D_B \frac{\partial \phi}{\partial y} \frac{\partial T}{\partial y} + \left( \frac{D_T}{T_\infty} \right) \left( \frac{\partial T}{\partial y} \right)^2 \right] - \frac{1}{(\rho c)_f} \left( \frac{\partial q_r}{\partial y} \right) \\ & + \frac{Q}{(\rho c)_f} (T - T_\infty) \end{aligned} \quad (3)$$

$$u \frac{\partial \phi}{\partial x} + v \frac{\partial \phi}{\partial y} = D_B \frac{\partial^2 \phi}{\partial y^2} + \left( \frac{D_T}{T_\infty} \right) \left( \frac{\partial^2 T}{\partial y^2} \right) - K_r^2 (\phi - \phi_\infty) \left( \frac{T}{T_\infty} \right)^n e^{\left( \frac{E_a}{\kappa T} \right)} \tag{4}$$

Where  $u$  and  $v$  are the velocity components along the  $x$  and  $y$  directions respectively.  $P$  is the fluid pressure,  $\rho_f$  is the density of base fluid,  $\rho_p$  is the nanoparticle density,  $\rho$  is the density of nanofluid,  $c$  is specific heat of nanofluid at constant pressure,  $\mu$  is the absolute viscosity of the base fluid,  $\alpha = \frac{k}{(\rho c)_f}$  is the thermal diffusivity of the base

fluid,  $\tau = \frac{(\rho c)_p}{(\rho c)_f}$  is the ratio of nanoparticle heat capacity and the base fluid heat

capacity,  $\phi$  is the local solid volume fraction of the nanofluid,  $\beta$  is volumetric thermal expansion coefficient of the base fluid,  $D_B$  is the Brownian diffusion coefficient,  $D_T$  is the thermophoretic diffusion coefficient,  $T$  is the local temperature and  $g$  is the acceleration due to gravity,  $B_0$  is the constant magnetic field and  $Q$  is the heat generation/absorption coefficient. The term  $K_r^2 (\phi - \phi_\infty) \left( \frac{T}{T_\infty} \right)^n e^{\left( \frac{E_a}{\kappa T} \right)}$  in equation (4)

represents the modified Arrhenius equation in which  $K_r^2$  is the reaction rate,  $E_a$  the activation energy,  $\kappa = 8.61 \times 10^{-5}$  eV/K the Boltzmann constant and  $n$  fitted rate constant which generally lies in the range  $-1 < n < 1$ .

Here the boundary conditions are considered to be,

$$u = 0, \quad v = 0, \quad T = T_w, \quad \phi = \phi_w \quad \text{at} \quad y = 0 \tag{5}$$

$$u = v = 0, \quad T \rightarrow T_\infty, \quad \phi \rightarrow \phi_\infty \quad \text{as} \quad y \rightarrow \infty \tag{6}$$

The radiative heat flux  $q_r$  is described by Rosseland approximation [34,35] such that

$$q_r = - \frac{4\sigma^*}{3\delta} \frac{\partial T^4}{\partial y} \tag{7}$$

Where  $\sigma^*$  and  $\delta$  are the Stefan–Boltzmann constant and the mean absorption coefficient, respectively. It is assumed that the temperature differences within the flow are sufficiently small so that the  $T^4$  can be expressed as a linear function after using Taylor series to expand  $T^4$  about the free stream temperature  $T_\infty$  and neglecting higher-order terms. This result is the following approximation:

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4 \tag{8}$$

Using(7) and (8) in (3), we obtain

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \nabla^2 T + \frac{\mu}{\rho c} \left( \frac{\partial u}{\partial y} \right)^2 + \frac{\sigma B_0^2}{\rho c} u^2 + \tau \left[ D_B \frac{\partial \phi}{\partial y} \frac{\partial T}{\partial y} + \frac{D_T}{T_\infty} \left( \frac{\partial T}{\partial y} \right)^2 \right] - \frac{1}{(\rho c)_f} \left( -\frac{16\sigma^* T_\infty^3}{3\delta} \frac{\partial^2 T}{\partial y^2} \right) + \frac{Q}{(\rho c)_f} (T - T_\infty). \quad (9)$$

### 3. SIMILARITY TRANSFORMATIONS

The Eqs. (2), (4) and (9) are transformed into ordinary differential equations [7] by introducing the following quantities,

$$\eta = \frac{y}{x} Ra_x^{1/4}, \quad \psi = \alpha Ra_x^{1/4} s(\eta), \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad f(\eta) = \frac{\phi - \phi_\infty}{\phi_w - \phi_\infty} \quad (10)$$

and the local Rayleigh number which is defined as

$$Ra_x = \frac{(1 - \phi_\infty) g \beta (T_w - T_\infty) x^3}{\nu \alpha} \quad (11)$$

The stream function  $\psi(x, y)$  is defined in such a way that

$$u = \frac{\partial \psi}{\partial y}, \quad v = \frac{\partial \psi}{\partial x} \quad (12)$$

So, the continuity equation Eq. (1) is identically satisfied. The momentum, energy and the solid volume fraction equations are obtained by employing some algebraic manipulations as follows,

$$s''' + \frac{1}{4Pr} (3ss'' - 2s'^2 - 4M\sqrt{Pr}s' - Gs') + \theta - Nrf = 0 \quad (13)$$

$$\left( 1 + \frac{4N}{3} \right) \theta'' + \frac{3}{4} s \theta' + Nbf' \theta' + Nt \theta'^2 + Ecs''^2 + \frac{EcM}{\sqrt{Pr}} s'^2 + \lambda \sqrt{Pr} \theta = 0 \quad (14)$$

$$f'' + \frac{3}{4} Lesf' + \frac{Nt}{Nb} \theta'' - Le\sigma(1 + \delta\theta)^n f e^{\left( \frac{E}{1+\delta\theta} \right)} = 0 \quad (15)$$

where primes denote differentiation with respect to  $\eta$  and the non-dimensional parameters, Prandtl number (Pr), Buoyancy-ratio parameter (Nr), Brownian motion parameter (Nb), thermophoresis parameter (Nt), Lewis number (Le), radiation parameter (N), Eckert number (Ec), heat generation or absorption parameter ( $\lambda$ ) and

Magnetic parameter (M), are defined as follows,

$$Pr = \frac{\nu}{\alpha}, Nr = \frac{(\rho_p - \rho_{f\infty})(\phi_w - \phi_\infty)}{\rho_{f\infty}\beta(T_w - T_\infty)(1 - \phi_\infty)}, Nb = \frac{(\rho c)_p D_B(\phi_w - \phi_\infty)}{(\rho c)_f \alpha}, E = \frac{E_a}{\kappa T},$$

$$Nt = \frac{(\rho c)_p D_T(T_w - T_\infty)}{(\rho c)_f \alpha T_\infty}, Le = \frac{\alpha}{D_B}, M = \frac{\sigma B_0^2 x^{\frac{1}{2}}}{\rho_f \sqrt{(1 - \phi_\infty) g \beta (T_w - T_\infty)}}, \sigma = \frac{K_r^2}{c}$$

$$N = \frac{4\sigma^* T_\infty^3}{k\delta}, Ec = \frac{(1 - \phi_\infty) g \beta x}{c}, \lambda = \frac{Qx^{\frac{1}{2}}}{(\rho c)_f \sqrt{(1 - \phi_\infty) g \beta (T_w - T_\infty)}}, \delta = \frac{T_w - T_\infty}{T_\infty}$$

The corresponding boundary conditions are as follows,

$$s(\eta) = 0, s'(\eta) = 0, \theta(\eta) = 1, f(\eta) = 1 \text{ at } \eta = 0 \tag{16}$$

$$s'(\eta) = 0, \theta(\eta) = 0, f(\eta) = 0 \text{ as } \eta \rightarrow \infty \tag{17}$$

A quantities of practical interest are the Nusselt number Nu and Sherwood number Sh which are defined by

$$Nu = \frac{xq_w''}{k(T_w - T_\infty)} \tag{18}$$

$$Sh = \frac{xq_m''}{D_B(\phi_w - \phi_\infty)} \tag{19}$$

where  $q_w''$  and  $q_m''$  are the wall heat flux and mass flux.

#### 4. NUMERICAL SOLUTION OF THE PROBLEM

The nonlinear coupled differential equations (13)–(15) along with the boundary conditions (16) and (17) form a two point boundary value problem. The coupled partial differential equations are solved numerically by using NDSolve in Mathematica. The effects of binary chemical reaction and activation energy on two-

dimensional radiative MHD boundary-layer flow of nanofluid over a vertical plate analyzed numerically and graphically.

## 5. RESULTS AND DISCUSSION

In order to have a detail of the boundary layer flow, viscous Ohmic dissipation, binary chemical reaction and activation energy, it is interested to investigate the role of all the non-dimensional parameters such as Magnetic parameter ( $M$ ), Heat generation/absorption parameter ( $\lambda$ ), Eckert number ( $Ec$ ), Brownian motion parameter ( $Nb$ ), Thermophoresis parameter ( $Nt$ ), Buoyancy-ratio parameter ( $Nr$ ), Radiation parameter ( $N$ ), respectively. The influence of the pertinent physical parameters on the velocity, temperature and concentration profiles of the nanofluid can be observed from the graphical results.

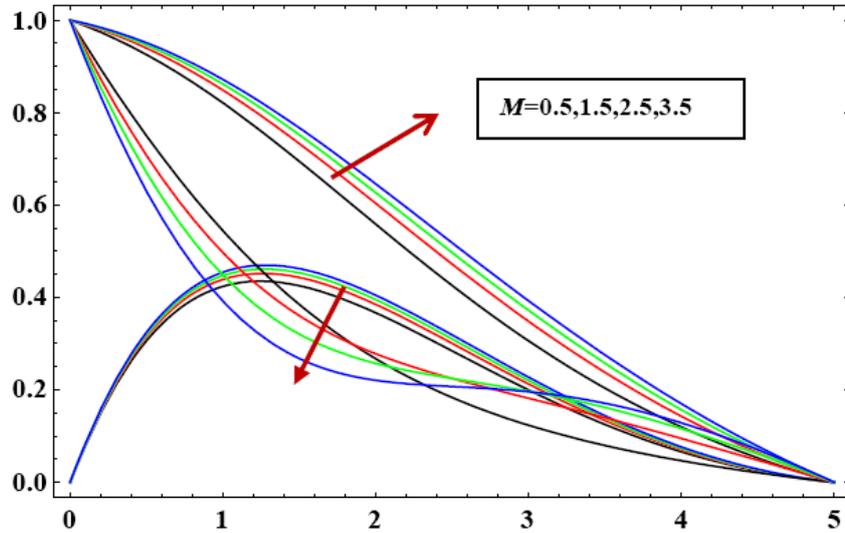
Fig.1 represents the effects of magnetic parameter ( $M$ ) on the dimensionless velocity, temperature and concentration profiles. It shows that the velocity and concentration field decreases with an increasing value of Magnetic parameter ( $M$ ) and the temperature field increases with an increasing value of Magnetic parameter ( $M$ ).

Fig.2 represents the effects of Heat generation/absorption parameter ( $\lambda$ ) on the dimensionless velocity, temperature and concentration profiles. A good variation has also given in these figures because of the heat generation ( $\lambda > 0$ )/absorption ( $\lambda < 0$ ) parameter. It shows that the velocity and temperature field increases in heat generation ( $\lambda > 0$ ) case, but decreases in heat absorption ( $\lambda < 0$ ) case. The concentration field decreases in heat generation ( $\lambda > 0$ ) case and enhances in heat absorption ( $\lambda < 0$ ) case.

Fig.3 represents the effects of values Brownian motion parameter ( $Nb$ ) and Thermophoresis parameter ( $Nt$ ) on the velocity and temperature profile enhances with an increasing values Brownian motion parameter ( $Nb$ ) and Thermophoresis parameter ( $Nt$ ) and concentration profile has an opposite effect in the values of Brownian motion parameter ( $Nb$ ) and Thermophoresis parameter ( $Nt$ ).

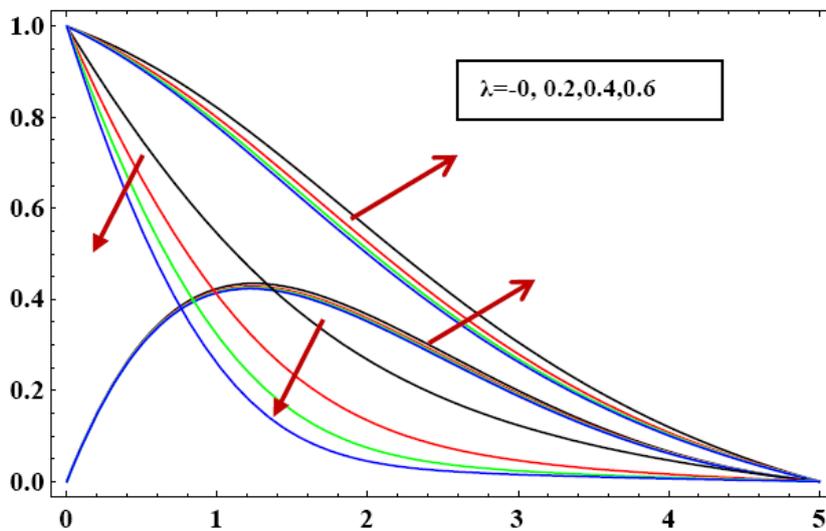
Fig.4 shows the case of Eckert number ( $Ec$ ), velocity and temperature profile enhances with an increasing values of Eckert number ( $Ec$ ) and concentration profile has an opposite effect. Fig.5 represents effects of values temperature difference parameter  $\delta$ , velocity and temperature profile increases with an increasing values of Eckert number ( $Ec$ ) and concentration profile decreases with the increasing value of  $Ec$ . Similarly in Fig.6 velocity and temperature profile increases with an increasing values of fitted rate constant  $n$  and concentration profile decreases with the increasing value of fitted rate constant  $n$

Fig.7 represents Effects of values reaction rate  $\sigma$  on the velocity, temperature and concentration profiles. It shows that the velocity and concentration field increases with an increasing values of reaction rate  $\sigma$  and the temperature field decreases with an increasing values of reaction rate  $\sigma$ .



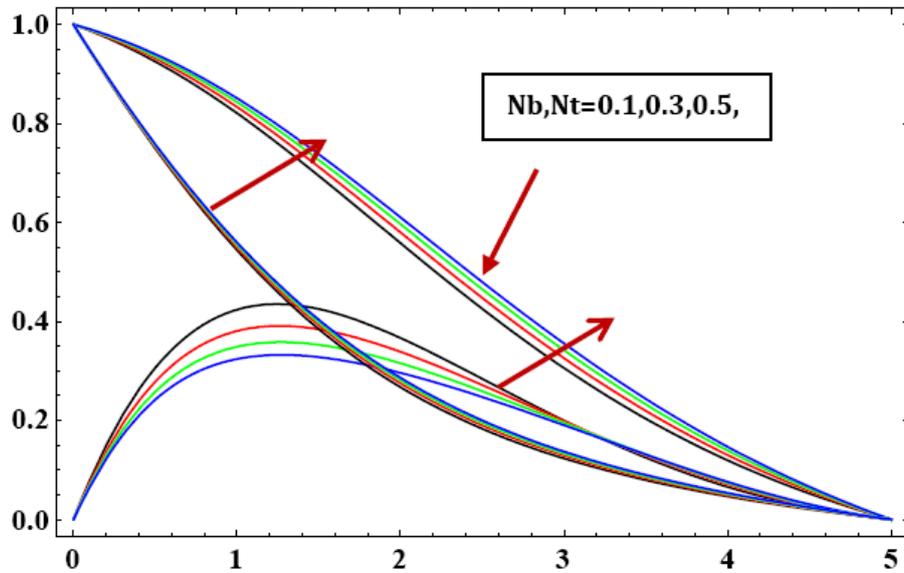
**Fig 1.** Effects of magnetic parameter(M) on the velocity, temperature and concentration profiles

$Pr=2, Ec=0.5, Nr=0.1, Nb=0.5, Le=1, M=0.5, \lambda=0.5, K=0.1, N=0.5,$

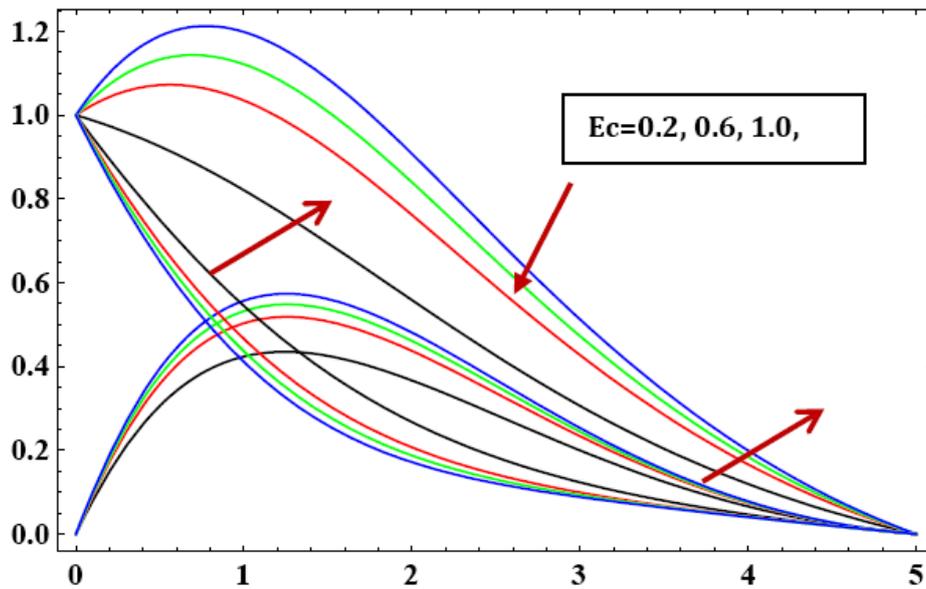


**Fig 2.** Effects of Heat generation/absorption parameter ( $\lambda$ ) on the velocity, temperature and concentration profiles

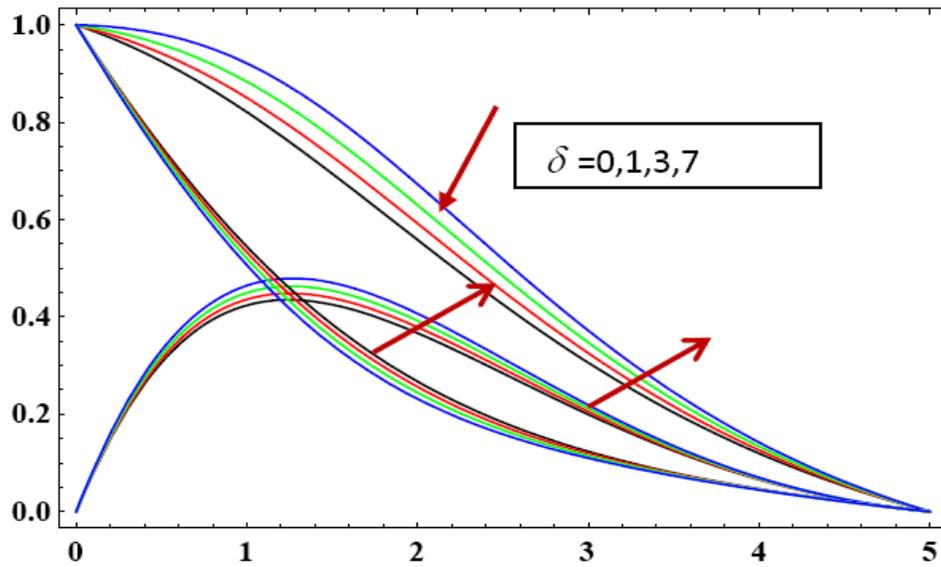
$Pr=2, Ec=0.5, Nr=0.1, Nb=0.5, Le=1, M=0.5, \lambda=0.5, K=0.1, N=0.5,$



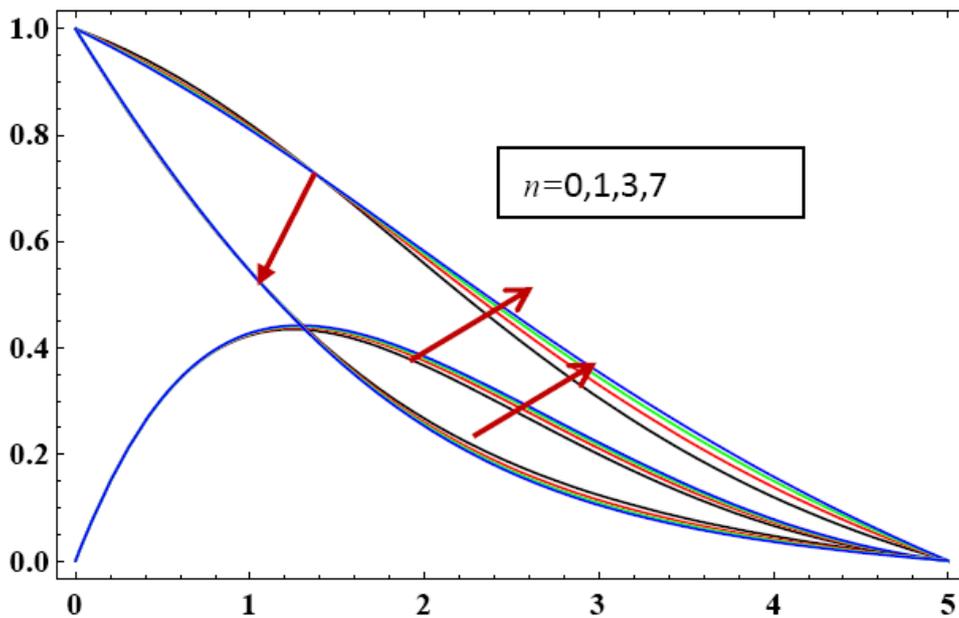
**Fig 3.** Effects of values Brownian motion parameter( $N_b$ ) and Thermophoresis parameter ( $N_t$ ) on the velocity, temperature and concentration profiles  $Pr=2, Ec=0.5, Nr=0.1, Nb=0.5, Le=1, M=0.5, \lambda=0.5, K=0.1, N=0.5,$



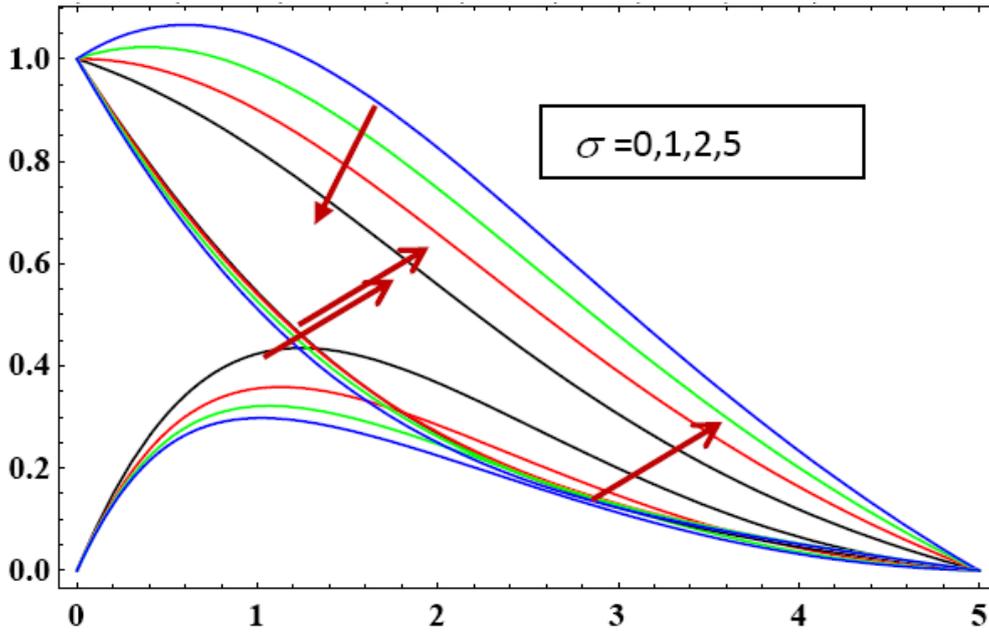
**Fig 4.** Effects of values Eckert number  $Ec$  on the velocity, temperature and concentration profiles  $Pr=2, Ec=0.5, Nr=0.1, Nb=0.5, Le=1, M=0.5, \lambda=0.5, K=0.1, N=0.5,$



**Fig 5.** Effects of values temperature difference parameter  $\delta$  on the velocity, temperature and concentration profiles  
 $Pr=2, Ec=0.5, Nr=0.1, Nb=0.5, Le=1, M=0.5, \lambda=0.5, K=0.1, N=0.5,$



**Fig 6.** Effects of values fitted rate constant  $n$  on the velocity, temperature and concentration profiles  
 $Pr=2, Ec=0.5, Nr=0.1, Nb=0.5, Le=1, M=0.5, \lambda=0.5, K=0.1, N=0.5,$



**Fig 7.** Effects of values reaction rate  $\sigma$  on the velocity, temperature and concentration profiles

$Pr=2, Ec=0.5, Nr=0.1, Nb=0.5, Le=1, M=0.5, \lambda=0.5, K=0.1, N=0.5,$

## 6. CONCLUSION

In this study, a mathematical model for a two dimensional MHD boundary layer flow on a vertical plate has been designed considering various physical parameters such as effects of radiation, chemical reaction, viscous and ohmic dissipation effects and binary chemical reaction and activation energy. The validity of the present computations has been confirmed via benchmarking based on several earlier studies

- The increase in magnetic parameter is to increase both velocity and temperature profiles and decrease in concentration profiles.
- The increasing values of Eckert number accelerate both the velocity and temperature of nanofluid.
- The increase in temperature difference parameter  $\delta$  is to increase in temperature and decrease in concentration profiles.
- The increase of fitted rate constant  $n$  is to increases in velocity and temperature profiles.
- The increasing values reaction rate  $\sigma$  increases velocity and temperature profiles.

- Brownian motion and thermophoresis parameters increase both the velocity and temperature profiles and decrease the concentration profile.

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