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ON SUPER (a, d)-ANTI-EDGEMAGIC OF $C_m o\overline{k_{n,}}$ $C_n o K_1$, (n,2) – Kite AND $K_2 \cup C_n$

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Abstract

An edge-magic total labeling on a graph G with vertex set V(G) and edge set E(G) is one-to-one function λ from V (G) ∪ E(G) onto the set of integers 1, 2, 3, ..., |V (G)| ∪ |E(G)| with the property that for every edge (x, y), λ(x) + λ(y) + λ(xy) is called (a, d)-edge antimagic total labeling of G if the edge weights w(xy) = λ(x)+λ(y)+λ(xy) form an arithmetic progress with initial term a and common difference d.

An (a, d)-edge antimagic total labeling is called super (a, d)-edge-antimagic total if λ(V (G)) ={1, 2, 3, ...|V (G)|} .

In this paper, we find Super (a, d)-edge antimagic property of $C_m o\overline{k_{n,}}$ $C_n o K_1$, (n,2) – Kite and$K_2 \cup C_n$

if n is even.

Keywords: Vertex Labeling, Edge Labeling, Edge-Antimagic, Edge-Weight.

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1. INTRODUCTION

We consider finite undirected graphs without loops and multiple edges for other definitions see [7]. In [4]Kotzig and Rosa and Wallies et al [6] proved the edge-antimagicness of cycles C_n and some variants of C_n . Also Enomoto et al [2] established that a cycle C_n is super edge magic if and only if n is odd. Further M Baca et al [1] obtained that a cycle C_n is super (a, d)-edgeantimagic total labeling. An (n, 2)-Kite is acycle of length n = {v_1, v_2, ..., v_{n-1}x} with a 2-edge path x − y − z (the tail) attached to one vertex say x. In [6], Wallis et al proved that (n, 1)-Kite is edge magic if and only if n is even.

The edge-weight of an edge xy under a labeling is the sum of labels of edge and the vertices x, y incident withxy. By an (a,d)-edge-antimagic total labeling is defined as a one-to-one mapping from V (G) ∪ E(G) into the set {1, 2, ..., |V (G)| + |E(G)|}, so that the set of edge-weights of all edges in G is equal to {a, a + d, ..., a + (|E(G) − 1|)d}, where integers a > 0 and d ≥ 0. An (a, d)-edge-antimagic total labeling f is called super (a, d)-edge antimagic total if f(V (G)) = {1, 2, ...|V (G)|} . A graph G is called super (a, d)-edge antimagic total if there exists a super (a, d)-edge-antimagic total of G. Simanjuntak et al [8] introduced (a, d)-edge antimagic total labeling and super (a, d)-edge antimagic total labeling and give constructions of (a, d)-edge antimagic total labelings for cycles and paths.

In this paper we study super (a, d)-edge-antimagic properties of certain classes of graphsincludingcorona
$C_m o\overline{k_{n,}}$, $C_n o K_1$, (n,2) – Kite and $C_n \cup K_2$.

2. MAGIC CORONATION OF N-CROWN $C_m o\overline{k_{n,}}$

n-crown $C_m o\overline{k_{n,}} m \geq 3, n \geq 1$ a cycle C_m with n pendant edges attached at each vertex. It has vertex set. $V\left(C_m o\overline{k_{n,}}\right) = \{u_i / 1 \leq i \leq m\} \cup \{u_{ij} / 1 \leq i \leq m, 1 \leq j \leq n\}$ and edge set.$\mathrm{E(G)} = \{u_1 u_m\} \cup \{u_i u_{i+1} / 1 \leq i \leq m-1\} \cup \{u_1 u_{ij} / 1 \leq i \leq m, 1 \leq j \leq n\}$.

Thus $\left|V\left(C_m o\overline{k_{n,}}\right)\right| = m(n+1), \left|E\left(C_m o\overline{k_{n,}}\right)\right| = m(n+1)$

Theorem 1. If n-crown $G = C_m o\overline{k_{n,}}$ is a super (a, d)-edge-antimagic total labeling then common difference d ≤ 2.

Proof. Suppose G has a super (a, d)-edge antimagic total labeling

f: V (G) ∪ E(G) →{1,2,…., 2m (n+1)}

Thus

$\{w(uv): uv \in E(G)\} = \{a, a+d, \dots . a + (m(n+1)-1)d\}$is the set of edge weights. The sum of all theedge-weight is

(1) $\sum_{uv\in E(G)} w(uv) = \frac{m(n+1)}{2}(2a + (m(n+1)-1)d)$

In the edge weights of G, each edge label used once and the label of vertices m used n+2 times eachand label of vertices mn used once each. The sum of all vertex labels and edge labels is.

(2) $(n+2)\sum_{i=1}^{m} f(u_i) + \sum_{i=1}^{mn} f\,(u_i) + \sum_{uv\in E(G)} f(uv) = \frac{m(n+1)}{2}[(m+1) + 4m(n+1) + 2]$

Combining (1) and (2) we get

(3) 2a+[m(n+1) -1]d=4mn+5m+3

Since the minimum possible edge weight under the labeling `f` is atleast

1+2 +m(n+1) + 1= m(n+1) + 4,a≥ m(n+1)+ 4, then from (3),

(4) $\leq 2 + \frac{m-3}{mn+m-1}$

if n ≥ 1and m ≥ 3 then $\mathrm{mn} + \mathrm{m} - 1 = \mathrm{m}(\mathrm{n}+1) - 1 > 0$ *and* $\frac{\mathrm{m}-3}{\mathrm{m}(\mathrm{n}+1)-1} \leq 0$

Thus d ≤ 2,

If n=1 and m =3 the inequality (4) becomes d ≤ 2 +$\frac{m-3}{2m-1} = 2$

From (3) we get a = $\frac{(4mn+5m+3)-(mn+m-1)d}{2}$

If d = 0 then a = $\frac{4mn+5m+3}{2}$

If d = 1 then a = $\frac{3mn+4m+4}{2}$

If d = 2 then a = $\frac{2mn+3m+5}{2}$

In (3) R M Figueroa et al obtained super edge magic (ie super (a,0)-edge antimagic) of $C_m o\overline{k_{n,}}$,

In the following Theorems we proved Super (a,1) edge antimagicness of $C_m o\overline{k_{n,}}$ and Super,

(a,2) edge antimagic of $C_m o\overline{k_{n,}}$.

Theorem 2. Let m (even) ≥ 4 and n ≥ 1 be two integers. Then n-crown G ≈$C_m o\overline{k_{n,}}$ is super (a,1)- edge antimagic total with magic constant $\frac{[3mn+4(m+1)}{2}$**.**

Proof. Let G ≈ $C_m o\overline{k_{n,}}$be the n-crown with
$V(G) = \{u_i: 1 \leq i \leq m\} \cup \{v_{ij}: 1 \leq i \leq m; 1 \leq j \leq n\}$ *and*
$E(G) = \{u_i u_{i+1} : 1 \leq i \leq m-1\} \cup \{u_1 u_m\}\ U\ \{u_1 v_{ij} : 1 \leq i \leq m; 1 \leq j \leq n\}$

For even m ≥ 4 and n ≥ 1 integers define the vertex labeling and edge labeling

$f(u_i) = i \qquad 1 \leq i \leq m$

$\boldsymbol{f}(v_{ij}) = (m+1)j + (i-1) - (j-1), \qquad 1 \leq i \leq m; 1 \leq j \leq n$

$\boldsymbol{f}(u_i u_{i+1}) = \frac{3mn}{2} + 2m - i, \qquad 1 \leq i \leq m-1; 1 \leq j \leq n$

$\boldsymbol{f}(u_n u_1) = \frac{3mn}{2} + 2m$

$\boldsymbol{f}(u_i v_{ij}) = 2m(n+1) + 1 - i - \frac{m}{2}(j-1), \qquad 1 \leq i \leq \frac{m}{2}; 1 \leq j \leq n$

$f(u_i v_{ij}) = \frac{3m}{2}(n+1) + 1 - i - \frac{m}{2}(j-1), \qquad \frac{m}{2} + 1 \leq i \leq m; 1 \leq j \leq n$

we can see that the resulting labeling is total sper (a,1)-edge antimagic and the set of edge-weights consists of the consecutive integers {m (n+1) + 1, m (n+1) + 2,…., 2n (n+1) }.

Theorem 3. For every two integers m ≥ 3 and n ≥ 1 the n-crown G ≈ $C_m o\overline{k_{n,}}$ is super ($\frac{2mn+3m+5}{2}$, 2) edge – antimagic total labeling.

Proof. Let G ≈$C_m o\overline{k_{n,}}$ be the n-crown with
$V(G) = \{u_i: 1 \leq i \leq m\} \cup \{v_{ij}: 1 \leq i \leq m; 1 \leq j \leq n\}$ *and*

$E(G) = \{u_1 u_m \cup \{u_i u_{i+1} : 1 \leq i \leq m-1\} \cup \{u_i u_{ij}\}: 1 \leq i \leq m; 1 \leq j \leq n\}$

Since a $= \frac{3mn+3m+5}{2}$ is a rational number that is not a integer. Thus m must be an odd.

Define the vertex labeling f : V (G) → {1,2,…., m(n+1)} andedge labeling f : E(G) → {m (n+1) +1,…..,2m (n+1)}

Such that $f(u_i) = \frac{m+1}{2} + \frac{i-1}{2}$; $1 \leq i \leq m, i is odd$ $f(u_i) = \frac{i}{2}$; $1 \leq i \leq m, i is even$ $f(v_{ij}) = \frac{2m+n+1}{2} + (i-1) + m(j-1)$; $1 \leq i \leq m, 1 \leq j \leq n$ $f(u_i u_{i+1}) = m(n+1) + i$; $1 \leq i \leq m$ $f(u_i v_{ij}) = m(n+2) + \frac{i-1}{2} + m(j-1)$; $1 \leq i \leq m, 1 \leq j \leq n, i\ is\ odd$ $f(u_i v_{ij}) = m(n+2) + \frac{2n+1}{2} + m(j-1)$; $1 \leq i \leq m, 1 \leq j \leq n,\ i\ is\ even$

Hence f is a super (a,d)-edge antimagic total labeling with first term $\frac{2mn+3m+5}{2}$ and common difference 2.

Theorem 4. If $C_n^+, n \geq 3$ is a super (a,d)-edge-antimagic total then d < 3

Proof. Suppose that $C_n^+, n \geq 3$ has a super (a,d)-edge-antimagic total labeling f: $V(C_n^+) \cup E(C_n^+) \to \{1,2,\ldots,4n\}$. The minimum edge weight is more than (2n-1) +2n +4n.

Thus a + (q-1) d = a + (2n - 1) d ≤ 8n-1

On the other hand the minimum possible edge-weight is atleast 1+2 + (2n + 1)

Since the minimum possible weight is (2n + 4), it gives that a > 2n + 3 $a + (q-1)d \geq (2n+4) + (2n-1)d$

$$Thus\ \ d \leq \frac{6n-5}{2n-1} < 3\ foralln > 2.$$

Theorem 5. Let $C_n^+,\ n \geq 3\ besuper(a\ ,d) - edge - antimagic\ totallabeling.$

If n is even then d = 1 and a = $\frac{7n+4}{2}$

If n is odd then $d = 0\ anda = \frac{9n+3}{2} ord = 2\ anda = \frac{5n+5}{2}$

Proof. Assume that a one-to-one and onto mapping $f: V(C_n^+) \cup E(C_n^+) \to \{1,2,\dots,4n\}$ is super (a,d)-edge-antimagic total and

$$W = \{w(e): e \in E(C_n^+)\} = \{a, a+d, \dots, a+(q-1)d\}$$

is the set of edge-weights.

The sum of all vertex labels and edge labels used to calculate the edge-weights is equal to

$$3\sum_{v \in V(C_n^+)} f(v) + \sum_{v_i \in V(C_n^+)} f(v_i) + \sum_{e \in E(C_n^+)} f(e)$$

$$3\{1,2,\dots,+n\} + \{(n+1)+(n+2)+\dots+2n\} + \{(2n+1)+(2n+2)+.\dots.+4n\}$$

$=qn^2 + 3n$……………………………………………….(1)

The sum of edge-weights in the set W is

$\sum_{e \in E(C_n^+)} w(e) = 2na + n(2n-1)d$……………………(2)

Combining (1) and (2) we get

$a = \frac{1}{2}\{9n + 3 - (2n-1)d\}$……………………………(3)

By Theorem 1, d < 3 then from (3) it follows:

If d = 0 then $a = \frac{9n+3}{2}$.The parameter a is integer if and only if n is odd.

If d = 1 then $a = \frac{7n+4}{2} andniseven$

If d = 2 then $a = \frac{5n+5}{2} andnisodd.$

Theorem 6. For n odd, n ≥ 3, the cycle C_n^+ has super (a,0)-edge-antimagic total labeling andsuper (a,2)-edge-antimagic total labeling.

Proof. Let the cycleC_n^+ has $(C_n^+) = \{v_{1,}v_{2,\dots,}v_n;\ v_1^l, v_{2,}^l \dots, v_n^l\}\ and$

$E(C_n^+) \to \{v_i v_{i+1}: i = 1,2,\dots n-1\} \cup \{v_n v_1\} \cup \{v_i v_{i+1}^1: i = 1,2,\dots,n\}$

Also define the vertex labeling $f_{1,}: V(C_n^+) = \{1,2,\dots,n\} \cup \{n+1,\dots,2n\}$

And the edge labeling $f_2: E(C_n^+) = \{2n+1, 2n+2, \dots, 4n\} in\ the\ following\ way$:

(i) $f_1(v_i) = \frac{1+i}{2}$ $1 \leq \text{i} \leq \text{n}, \text{i is odd.}$

$f_1(v_i) = \frac{n+i}{2} + \frac{i}{2}$ $1 \leq i \leq n, iiseven$

$f1(x_i^l) = 2n + 1 - i,$ $1 \leq i \leq n$

$f_2(x_i x_{i+1}) = 4n - i, 1 \leq i \leq n - 1$

$f_2\ (x_n x_1) = 4n$

$f_2(x_i x_i^l) = \frac{5n-1}{2} + \frac{i+1}{2} 1 \leq i \leq n, iisodd.$

$f_2(x_i x_i^l) = 2n + \frac{i}{2} 1 \leq i \leq n, iiseven$

Combining the vertex labeling $f_1$and the edge labeling $f_2$gives total labeling. It is obvious that the labeling the edge-weights of all edges of C_n^+ constitute an AP-sequence with common difference d = 0

(ii) Now, construct the vertex labeling f_3 and the edge labeling f_4 as follows:

$f_3(v_i) = f_1(v_i)\ \ for$ $1 \leq i \leq n$

$f_3(v_i^l) = \frac{3n+1}{2} + \frac{1+i}{2}\ for$ $1 \leq i \leq n - 2, i\ is\ odd.$

$f_3(v_i^l) = n + 1 + \frac{i}{2} for$ $1 \leq i \leq n - 1, i\ is\ even$

$f_3(v_n^l) = n + 1$

$f_4(x_i x_{i+1}) = 2(n + 1) - 1 + i\ for$ $1 \leq i \leq n - 1$

$f_4(x_n x_1) = 2n + 1$

$f_4(x_i x_i^l) = \frac{7(n-1)}{2} + i for$ $1 \leq i \leq n - 1$

$f_4(x_n x_n^l)$=3n+1

Label the vertices and the edges $C_n^+$by $f_3$andf_4 we can see that the resulting labeling is total labeling and the set of edge-weights has of the consecutive integers $\{\frac{5(n+1)}{2}, \frac{5(n+1)}{2} + 2, \dots, \frac{13n+1}{2}\}$

Theorem 7. For n even, n ≥ 4, the cycle C_n^+ has super (a,1)-edge-antimagic total labeling.

Proof. Consider $V(C_n^+)$=$\{v_1 v_{2,\dots\dots,} v_n\}$ ∪ $\{v_1^l, v_2^l, \dots., v_n^l\ \}\ and$

$$E(C_n^+) = \{v_i v_{i+1} : i = 1,2, \dots, (n - 1)\} \cup \{v_n v_2\} \cup \{v_i v_i^l : i = 1,2, \dots, n\}$$

Define the vertex labeling (bisection)

$f_5: V(C_n^+) \cup E(C_n^+) = \{1,2, \dots 4n\}$

$f_5(v_i) = i \quad for \qquad 1 \le i \le n$

$f_5(v_i^l) = n + 1 \quad for \qquad 1 \le i \le n$

$f_6(x_i x_{i+1}) = 4n + 1 = i for \qquad 1 \le i \le n - 1$

$f_6(x_n x_1) = 3n + 1$

$f_6\left(x_i x_i^l\right) = \frac{5n}{2} + 1 - i for \qquad 1 \le i \le \frac{n}{2}$

$f_6\left(x_i x_i^l\right) = \frac{7n}{2} + 1 - i \ for \qquad \frac{n}{2} + 1 \le i \le n$

From previous theorems (6) and (7) we get the following Theorem

Theorem 8. The cycle C_n^+ has super (a,d)-edge-antimagic total labeling if and only if either.

(i) d ∊ {0,2} and n is odd, n ≥ 3 or

(ii) d = 1 and n is even, n ≥ 4

3. ANTIMAGIC OF DISJOINT GRAPH ($\boldsymbol{K_n} \cup \boldsymbol{K_2}$)

Theorem 1. If the graph G = $K_n \cup K_2$ is super (a,d)-edge-antimagic total then d < 3.

Proof. Assume that G = $K_n \cup K_2, n \ge 3$ is super (a,d)-edge-antimagic total labeling

$f: V(G) \cup E(G) \rightarrow \{1,2, \dots, n + 2\}$

The maximum edge-weight is not more than (n+1) + (n+2) + (2n+3)

Thus a + (E(G) – 1) d= a + (n+1) d ≤ (4n + 6)

On the other hand, the minimum possible weight is atleast 1 + 2 + (n + 2) +1 = n+6 it follows

that a > n + 5

$a + (n + 1)d \ge n + 6 + (n + 1)d$

Thus $n + 6 + (n - 10d \le 4n + 6$

$$d \le \frac{3n}{n+1} = 3 - \frac{3}{n+1} < 3$$

Theorem 2. Let $G = K_n \cup K_2$ be super (a,d)-edge-antimagic total. If d=1,

a = 2n + 6 and $d = 2, a = \frac{3(n+4)}{2}$

Proof. Suppose that a bijection f from V(G) ∪ E(G) onto the set {1,2,…,n+2} is suppose (a,d)-edge-antimagic total and

$W = \{w(e): e \in E(G)\} = \{a, a+d, \dots\dots, a+(n+1)d\}$is the set of edge-weights is

(1) $2\sum_{v \in V(K_n)} f(u) + [f(v) + f(w)] + \sum_{e \in E(G)} f(e) = \frac{5n^2+19n+18}{2} - [f(v) + f(w)]$

The sum of the edge-weights in the set W is

(2) $\sum_{e \in E(G)} w(e) = (n+1)a + \frac{n(n+1)d}{2}$

Equating (1) and (2) we get

$$a = \frac{5n^2 + 19n + 18}{2(n+1)} - \frac{[f(v) + f(w)]}{n+1} - \frac{n}{2}d$$

Since $f(v) \le n+2\ andf(w) \le n+1\ orf(v) \le n+1\ \ andf(w) \le n+2$

(3) If $d = 0, f(v) + f(w) = \frac{5n^2+19n+18}{2} - a(n+1) = \frac{5n^2+19n+18}{2} - \frac{(5n+6)(n+1)}{2} = n+3$

Therefore $f(v) + f(w) = n+3$

Since $f(u) \le n+2\ andf(w) \le n+1$ or viceversa, $f(u), +f(w) \le 2n+3$

It is easy to see that $1 + 2 \le f(u) + f(w) \le 2n+3$

Implies $f(u) + f(w) = n+3$

If $d = 1, a = \frac{5n^2+19+18}{2} - \frac{f(u)+f(w)}{n+1} - \frac{n}{2}(1) = 2n+6$

If $d = 2, a = \frac{5n^2+19+18}{2} - \frac{f(u)+f(w)}{n+1} - \frac{n}{2}(2) = \frac{3(n+4)}{2}$

JiYeon Park et al [8] proved that super magic (ie super (a,0) edge antimagic) total labeling of $C_n \cup K_2$ with magic constant $k = \frac{5n+6}{2}$ where n is even and n ≠ 10.

In the following Theorem we proved super (a,d) , d = 1,2 edge antimagic total labeling of $C_n \cup K_2$

Theorem 3. The graph G=$C_n \cup K_2$ has super (2n + 6,1)-edge-antimagic labeling if n is even.

Proof. Let $v_1, v_2, \ldots\ldots, v_n$ be the vertex sequence of $C_n$and let u and w be the vertices of $K_{2.}$

$V(G) = \{v_1, v_2, \ldots., m, u, w\}$ and $E(G) = \{v_i v_{i+1} /0 \leq i \leq n-1\} \cup \{v_n, v_1\} \cup \{uw\}$
We define a labeling $f: V \cup E \rightarrow \{1,2, \ldots, 2n+3)$

Where $f(u) = 1,\ f(w) = n+2$

$f(v_{2i-1}) = i+1, \qquad 1 \leq i \leq n$

$f(v_{2i}) = \frac{n}{2} + 1 + i, \qquad 1 \leq i \leq n$

$f(v_i v_{i+1}) = \frac{3n}{2} + 2 + i, \qquad 1 \leq i \leq \frac{n}{2} - 1$

$f(v_i v_{i+1}) = n - 3 + i, \qquad \frac{n}{2} \leq i \leq n-1$

$f(v_n v_1) = 2n + 2$

$f(uw) = 2n + 3$

Combining the vertex labeling and edge labeling gives total labeling. It is easy to verify that under the total labeling the edge weights of all edges of $C_n \cup K_2$ constitue an arithmetic sequence with common difference d=1

Theorem 4. The disjoint graph $C_n \cup K_2$ where n ≥ 4 even has super ($\frac{3(n+4)}{2}$, 2)-edge-antimagic total labeling.

Proof. Now consider n ≥ 4, n is even

n ≡ 0 (mod4)

$f(v_i) = i, \qquad 1 \leq i \leq 3$

$f(v_{2i}) = \frac{n}{2} + 1 + i, \qquad 2 \leq i \leq \frac{n}{2} + 1$

$f(v_{2i-1}) = 2 + i, \qquad 2 \leq i \leq \frac{n}{2}$

$f(v_{2i-1}v_{2i}) = n + 2 + 2i, \qquad 2 \leq i \leq \frac{n}{4} - 1$

$f(v_{n+1}v_2) = n + 4$

$f(v_2v_3) = \frac{3n}{2} + 1$

$f(v_{2i}v_{2i+1}) = n + 2i - 1, \qquad 2 \leq i \leq \frac{n}{4} - 1$

$f(v_iv_{i+1}) = n + 3 + i, \frac{n}{2} - 1 \leq i \leq n$

$\underline{n \equiv 2(mod4)}$

$f(v_i) = i, \qquad 1 \leq i \leq 3$

$f(v_{2i}) = \frac{n}{2} + 4 - i, \qquad 2 \leq i \leq \frac{n}{2}$

$f(v_{2i+3}) = n + 2 - i, \qquad 1 \leq i \leq \frac{n}{2} - 1$

$f(v_{n+2}) = n + 2$

$f(v_iv_{i+1}) = 2n + 7 - i, \qquad 4 \leq i \leq \frac{n}{2} + 5$

$f(v_{2i-1}v_{2i}) = 2n + 4 - 2i, \qquad \left\lfloor \frac{n-3}{2} \right\rfloor or \left\lceil \frac{n-3}{2} \right\rceil \leq i \leq \frac{n}{2}$

$f(v_{2i}v_{2i+1}) = \frac{5n}{2} - 4 - 2i, \qquad \left\lfloor \frac{n-3}{2} \right\rfloor or \left\lceil \frac{n-3}{2} \right\rceil \leq i \leq \frac{n}{2}$

$f(v_{2i-1}v_{2i}) = \frac{3n}{2} - 4 - 2i, \qquad \left\lfloor \frac{i}{2} \right\rfloor or \left\lceil \frac{i}{2} \right\rceil \leq i \leq 2$

$f(v_2v_3) = \frac{3n}{2} + 1$

It is easily see that f is a super (a,2) edge antimagic total labeling $K_2 \cup C_n$.

4. ANTIMAGIC OF (n, 2)-KITE

Theorem1. Let G = (n, 2)-Kite, n ≥ 3. If the graph G is super (a,d)-edge-antimagic total then d<3.

Proof. Assume that G has a super (a, d)-edge-antimagic total labeling

$f : V(G) \cup E(G) \to \{1,2,\dots,|V(G) + |E(G)|\}$.

The maximum edge-weight is no more than

$|V(G)| - 1 + |V(G) + (|V(G)| + |E(G)|) = 4n + 7$

$Thus\ a + (|E(G) - 1|)d = a + (n+1)d \leq 4n + 7$

On the other side, the minimum possible edge-weight is atleast 1+2+|V(G)+1| = n + 6.

The minimum possible edge-weight is n+6, it follows that a> n+5.

Hence

$a + (n-1)d \leq n + 6 + (n-1)d \leq 4n + 7$

$$d \leq \frac{3n+1}{n+1} = 3 - \frac{2}{n+1} < 3$$

<u>**Second Method**</u>: Let $W = \{w(e): e \in E(G)\} = \{a, a+d, \dots, a + (E(G) - 1)d\}$

The sum of all vertex labels and edge labels used to calculate the edge weights is equal to

$2\sum f(v_i) + 3f(y) + f(z) + \sum_{xy \in E(G)} f(\text{xy})$

$$= \frac{5n^2 + 23n + 26}{2} + f(y) - f(z) \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots. (1)$$

The sum of edge-weights in the set W is

$$\sum_{xy \in E(G)} w(xy) = (n+2)a + \frac{(n+1)(n+2)d}{2} \dots\dots\dots\dots\dots\dots\dots\dots (2)$$

From (1) and (2)

$$\frac{5n^2 + 23n + 26}{2} + f(y) - f(z) = (n+2)a + \frac{(n+1)(n+2)d}{2} \dots\dots\dots (3)$$

$$d = \frac{(5n^2 + 23n + 26 + 2\big(f(y) - f(z)\big) - 2((n+2)(n+6))}{(n+1)(n+2)}$$

The label of the vertex y ≤ n+2 and z ≤ n +1 or y ≤ n +1

and z ≤ n + 2 and f(y) – f(z) ≥ 1 or -1

If $f(y) - f(z) \geq 1\ then d \leq \frac{3n+4}{n+2} = 3 - \frac{2}{n+2} < 3$

If $f(y) - f(z) \geq -1\ then d \leq \frac{3n^2+7n}{(n+1)(n+2)} = 3 - \frac{2(n+3)}{(n+1)(n+2)} < 3.$

In [8] JiYeon Park show that (n,2)- Kite super edge magic (super (a,0) edge-antimagic total)total labeling with magic constant $d = \frac{5n+12}{2} iea = \frac{5n+12}{2}$, $d = 0$.

From (3) we get

$$f(z) - f(y) = \frac{(n+2)(5n+12)}{2} - \frac{5n^2+23n+26}{2}$$

$$f(z) - f(y) = \frac{n+2}{2}$$

$$put\ d = 1, f(z) - f(y) = \frac{n+2}{2} in(3) we\ have\ \ a = \frac{4n+13}{2}$$

$$and\ d = 1, f(y) - f(z) = -\frac{n+2}{2} in(3) we\ have\ a = \frac{4n+11}{2}$$

In both cases `a` is not an integer.

Thus (n, 2)-Kite is not super (a,1)-edge-antimagic total labeling.

Similarly from (3) we get with $a = n + 6, d = 2$

$$f(y) - f(z) = -\frac{-n^2 - n + 2}{2}, \qquad f(z) - f(y) = \frac{n^2 + n - 2}{2}$$

$put\ d = 2, f(z)$ –f (y) =$\frac{n^2+n-2}{2} in(3) we\ have\ a = 2n + 5$

Put $d = 2, f(y) - f(z) = \frac{-n^2-n+2}{2} in\ (3)\ we\ have\ a = n + 6$

Difference of vertex weights f(y) and f (z) not possible.

Then (n, 2) is not super (a,d) , d ≥ 1 edge-antimagic total labeling.

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