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Ranking Method of Intuitionistic Fuzzy Numbers

S. K. Bharati

Department of Mathematics, Kamala Nehru College
University of Delhi, New Delhi-110049, India.

Abstract

Ranking of intuitionistic fuzzy numbers plays an important role in comparison of intuitionistic fuzzy numbers decision making problems. Though several methods for ranking of intuitionistic fuzzy numbers are available in literature but each of them has some limitations. The paper considers a fuzzy origin and measures distance of each intuitionistic fuzzy number from fuzzy origin and then compare distance between them. Thus we have defined new ranking function particularly for triangular intuitionistic fuzzy numbers (TFN) and have verified its axioms.

Keywords: Intuitionistic fuzzy sets, fuzzy numbers, Sign distance, triangular intuitionistic fuzzy numbers, ranking function.

INTRODUCTION

Decision making problems of financial management in general utilizes information from the knowledge base of experts in view of their perception about that system. In such problems often the availability of resources and financial requirements are also flexible. Thus modelling of such real life decision making problems are extensively utilizing fuzzy set given by Zadeh (1965). The reason of its popularity is quite obvious as most of financial decision making problems involve some kind of imprecision in parameters affecting the process. These imprecise parameters are often represented by fuzzy numbers and consequently the alternatives available with decision makers also become imprecise. In such situations the comparison of various alternatives are not straight forward. Thus ranking of fuzzy numbers is needed for data analysis and decision making problems of financial, industrial and socio economical problems having parameters as fuzzy numbers. Here unlike to crisp numbers, the comparison of two fuzzy numbers do not follow strict equality or strict inequality. Since they obey the fuzzy inequality, some alternative method is needed for their quantitative comparison.

In such situations fuzzy ranking is used to deal with ordering of fuzzy numbers. Thus fuzzy ranking become one of the critical tool being used in decision making process of solving a fuzzy linear programming problem or a multi objective fuzzy linear programming problem. Out of many ranking methods available in literature some of the extensively used methods are given by Dubois and Prade (1983), Bortlan and Degani (1985), Raj and Kumar (1999), Lion and Wang (1992), Tran and Duckstein(2002), Mitchell(2004), Abbasbandy and Asadi(2006), Tsai(2012). Asadi and Zendehnam (2007) proposed method to find the nearest point to a fuzzy number and hence used it to develop a ranking method for fuzzy numbers. Kumar et al (2010) proposed a ranking method for generalized trapezoidal fuzzy numbers and claimed that his method provide a correct ordering of generalized and normal fuzzy number. However, Bakar et al (2012) proposed a new method of ranking fuzzy numbers using distance based spread of fuzzy number approach and its height. The difference in basic philosophy of various developed ranking methods may be reviewed in the work of Brunelli and Mezei (2013). A nice application of ranking of fuzzy number in determining critical path method can be found in the study of Yao and Lin (2000). Thus these fuzzy ranking methods have been used by various authors to solve the fuzzy linear programming problems. But in many case it has been observed that it is not possible to denote all such imprecise numbers by a fuzzy number. The difficulty in such problems is to find an exact membership function for belonging as it also involves some kind of hesitation factor.

In order to overcome this difficulty of hesitation factor, Atanassov (1986) generalized the fuzzy set to intuitionistic fuzzy set to accommodate hesitation factor along with grade of belonging and non belonging. Later, Atanassov (1994) defined various operators on intuitionistic fuzzy set which further enriched the theory for its applications to various area of decision sciences. This generalization of fuzzy set to intuitionistic fuzzy set gave new dimension to optimization under uncertainty and envisaged a new area of optimization under intuitionistic fuzzy environment. Angelov (1997) extended the Bellman and Zadeh (1970) approach of fuzzy optimization to intuitionistic fuzzy optimization. Here the degree of acceptance and degree of rejection is optimized in such a way that the degree of acceptance is maximized and the degree of rejection is minimized. Several authors have studied the problem of optimization in intuitionistic fuzzy environment such as Jana and Roy (2007), Mahapatra et al (2010), Dubey et al (2012), Basu and Mukherjee (2012). In modelling the decision making problem with imprecise quantity, one approach is to deal such imprecise quantity as intuitionistic fuzzy number. Thus intuitionistic fuzzy optimization problem involves the comparison of fuzzy numbers. The comparison of these imprecise numbers needs to develop the ranking methods for intuitionistic fuzzy numbers. Atanassov defined four basic distances between intuitionistic fuzzy sets: Hamming distance, normalized Hamming distance, Euclidean distance and normalized Euclidean distance. Szmidt and Kacprzyk (2000) enriched the theory and proposed a new definition of distance between intuitionistic fuzzy sets. Wang and Xin (2005) also analysed the similarity measure and distance between intuitionistic fuzzy set by proposing some new axioms. Further, Nayagam et al (2008) and Nehi (2010) have also studied ranking of intuitionistic fuzzy numbers. Li (2010) defined a ratio ranking method for triangular intuitionistic fuzzy numbers and applied it to MADM . More ranking methods can be found in literature as developed by Kumar and Kaur (2013), Zhang and Yu (2013), Esmailzadeh and Esmailzadeh (2013) and Papakostas et al (2013). Recently De and Das (2014) defined a value and ambiguity index of trapezoidal intuitionistic fuzzy number and used it to describe a ranking function by taking sum of the value and ambiguity index. Nishad et al (2014) have presented centroid method of ranking of intuitionistic fuzzy numbers. Recently, Bharati and Singh (2014, 2015) have studied the intuitionistic fuzzy multiobjective programming and applied it in agricultural production planning. Bharati and Malhotra (2016, 2017) have used intuitionistic fuzzy sets in two stage time minimizing transportation problem.

1. PRELIMINARIES

1.1. Intuitionistic Fuzzy Set

An intuitionistic fuzzy set (IFS) A in X is defined as object of the following form $\tilde{A}^I = \{(x, \mu_{\tilde{A}^I}(x), \nu_{\tilde{A}^I}(x)) : x \in X\}$, where $\mu_{\tilde{A}^I}: X \longrightarrow [0,1]$ and $\nu_{\tilde{A}^I}: X \longrightarrow [0,1]$ define the degree of membership and the degree of non membership of the element $x \in X \;\; in\; \tilde{A}^I,$ respectively and for every $x \in X,\; 0 \le \mu_{\tilde{A}^I}(x) + \nu_{\tilde{A}^I}(x) \le 1.$

The value of $\pi_{\tilde{A}^I}(x) = 1 - \mu_{\tilde{A}^I}(x) - \nu_{\tilde{A}^I}(x)$, is called the degree of non-determinacy (or uncertainty or hesitation factor) of the element x ∈ X to the intuitionistic fuzzy set A.

When $\pi_{\tilde{A}^I}(x) = 0$, then an intuitionistic fuzzy set becomes fuzzy set.

$$\left\{\left(x, \mu_{\tilde{A}^I}(x), 1 - \mu_{\tilde{A}^I}(x)\right) : x \in X\right\}.$$

1.2.Some operations and relations on Intuitionistic Fuzzy Sets

(i) $\tilde{A}^I \cap \tilde{B}^I = \left\{\langle x, Min\left(\mu_{\tilde{A}^I}(x), \mu_{\tilde{B}^I}(x)\right), Max\left(\nu_{\tilde{A}^I}(x), \nu_{\tilde{B}^I}(x)\right)\rangle : x \in X\right\}$.

(ii) $\tilde{A}^I \cup \tilde{B}^I = \left\{\langle x, Max\left(\mu_{\tilde{A}^I}(x), \mu_{\tilde{B}^I}(x)\right), Min\left(\nu_{\tilde{A}^I}(x), \nu_{\tilde{B}^I}(x)\right)\rangle : x \in X\right\}$.

(iii) $\tilde{A}^I \subseteq \tilde{B}^I$ iff $\mu_{\tilde{A}^I}(x) \leq \mu_{\tilde{B}^I}(x)$ and $\nu_{\tilde{A}^I}(x) \geq \nu_{\tilde{B}^I}(x)\ \forall x \in X$.

(iv) $\tilde{A}^I = \tilde{B}^I$ iff $\mu_{\tilde{A}^I}(x) = \mu_{\tilde{B}^I}(x)$ and $\nu_{\tilde{A}^I}(x) = \nu_{\tilde{B}^I}(x)\ \forall x \in X$.

1.3. Intuitionistic fuzzy number

An intuitionistic fuzzy set $\tilde{A}^I = \{(x, \mu_{\tilde{A}^I}(x), \nu_{\tilde{A}^I}(x)) : x \in \mathbb{R}\}$ of the real number is called intuitionistic fuzzy number if

(i) There exit a real numbers $x_0 \in \mathbb{R}$ such that $\mu_{\tilde{A}^I}(x_0) = 1$, and $\nu_{\tilde{A}^I}(x_0) = 0$,

(ii) Membership $\mu_{\tilde{A}^I}$ of $\tilde{A}^I$ is fuzzy convex and non-membership $\nu_{\tilde{A}^I}$ of A is fuzzy concave.

(iii) $\mu_{\tilde{A}^I}$ is upper semi-continuous and $\nu_{\tilde{A}^I}$ is lower semi-continuous.

(iv) Support $\left(\tilde{A}^I\right) = \overline{(\{x \in \mathbb{R} : \nu_{\tilde{A}^I}(x) < 1\})}$ is bounded.

In other words, Triangular intuitionistic fuzzy number is denoted by $\tilde{A}^I = \{(a_1, a_2, a_3), (b_1, a_2, b_3)\}$, where $a_1,\ a_2, a_3, b_1, , b_3 \in \mathbb{R}$ suchthat $b_1 \leq a_1 \leq\ a_2 \leq\ a_3 \leq\ b_3$ is an intuitionistic fuzzy numbers having membership and non-membership functions is of form

$$\mu_{\tilde{A}^I}(x) = \begin{cases} 1, & x = a_2 \\ 0, & x \leq a_1, x \geq a_3 \\ \varphi_{\tilde{A}^I}(x), & a_1 < x < a_2 \\ \chi_{\tilde{A}^I}(x), & a_2 < x < a_3 \end{cases} \quad \text{and} \quad \nu_{\tilde{A}^I}(x) = \begin{cases} 1, & x = a_2 \\ 0, & x \leq a_1, x \geq a_3 \\ \eta_{\tilde{A}^I}(x), & b_1 < x < a_2 \\ \xi_{\tilde{A}^I}(x), & b_2 < x < b_3 \end{cases}$$

Where $\varphi_{\tilde{A}^I} : (a_1, a_2) \rightarrow [0, 1]$, $\chi_{\tilde{A}^I} : (a_2, a_3) \rightarrow [0, 1]$, $\eta_{\tilde{A}^I} : (b_1, a_2) \rightarrow [0, 1]$, $\xi_{\tilde{A}^I} : (a_2, b_3) \rightarrow [0, 1]$.

1.4.Symmetric triangular intuitionistic fuzzy number

Symmetrical intuitionistic fuzzy number is a intuitionistic fuzzy number whose membership function ($\mu_{\tilde{A}^I}$) and non-membership function ($\nu_{\tilde{A}^I}$) are given by

$$\mu_{\tilde{A}^I}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1}, a_1 < x < a_2 \\ 1, \qquad\qquad x = a_2 \\ 0, \qquad x > a_3, x < a_1 \\ \dfrac{a_3 - x}{a_3 - a_1}, a_2 < x < a_3 \end{cases},$$

and

$$\nu_{\tilde{A}^I}(x) = \begin{cases} \dfrac{x - a_2}{b_3 - a_2}, b_2 < x < b_3 \\ 0, \qquad\qquad x = a_2 \\ 1, \qquad x > b_3, x < b_1 \\ \dfrac{a_2 - x}{a_2 - b_1}, b_1 < x < a_2 \end{cases}$$

Where $b_1 \leq a_1 \leq a_2 \leq a_3 \leq b_3$ and $a_2 - a_1 = a_3 - a_2, a_2 - b_1 = b_3 - a_2$, $a_1, a_2, a_3, b_1, b_2, b_3 \in \mathbb{R}$.

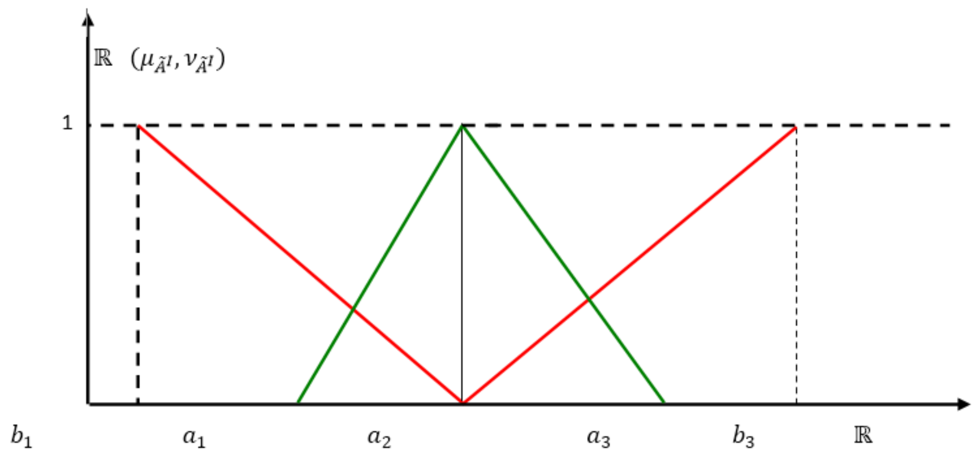

Fig. 1. Symmetric triangular intituitonistic fuzzy numbers: $\tilde{A}^I = \{(a_1, a_2, a_3), (b_1, a_2, b_3)\}$.

2. RANKING OF INTUITIONISTIC FUZZY NUMBER

2.1 Intuitionistic fuzzy origin

When in an intuitionistic fuzzy number $a_1 = a_2 = a_3 = b_1 = a_2 = b_3 \in \mathbb{R}$, then $\tilde{A}^I = \{(a_1, a_2, a_3), (b_1, a_2, b_3)\}$ is called intuitionistic fuzzy origin. Hence forth through out the study, we have denoted intuitonistic fuzzy origin by $\tilde{0}^I$.

2.2 Positive triangular intituitonistic fuzzy numbers

Triangular intituitonistic fuzzy numbers: $\tilde{A}^I = \{(a_1, a_2, a_3),\ (b_1,\ a_2, b_3)\}$ is said to be positive triangular intuitionitsic fuzzy number if $b_1 > 0$.

2.3 Properties of triangular intuitionistic fuzzy numbers

Property (P1)

Let $\tilde{A}^I = \{(a_1, a_2, a_3), (b_1, a_2, b_3)\}$ and $\tilde{C}^I = \{(c_1, c_2, c_3), (d_1, c_2, d_3)\}$ be two triangular intuitionistic fuzzy numbers. Then addition of $\tilde{A}^I$ and $\tilde{B}^I$ is also a triangular intuitionistic fuzzy number and is defined as

$$\tilde{A}^I \oplus \tilde{C}^I = \{(a_1 + c_1, a_2 + c_2, a_3 + c_3), (b_1 + d, a_2 + c_2, b_3 + d_3)\}.$$

Property (P2)

Let $\tilde{A}^I = \{(a_1, a_2, a_3), (b_1, a_2, b_3)\}$ be an intuitionistic fuzzy number and $k \in \mathbb{R}$, then scalar multiplication is also an intuitionistic fuzzy number and is given by

$$k\tilde{A}^I = \begin{cases} \{(ka_1, ka_2, ka_3), (kb_1, ka_2, kb_3)\}, k > 0 \\ \{(ka_3, ka_2, ka_1), (kb_3, ka_2, kb_1)\}\ k < 0 \end{cases}.$$

Property (P3)

Two intuitionistic fuzzy numbers $\tilde{A}^I = \{(a_1, a_2, a_3), (b_1, a_2, b_3)\}$ and $\tilde{C}^I = \{(c_1, c_2, c_3), (d_1, c_2, d_3)\}$ are said to be equal iff $a_1 = c_1, a_2 = c_2, a_3 = c_3, b_1 = d_1, a_2 = c_2, b_3 = d_3$

2.4 Sign Distance function

Let $a, b \in \mathbb{R}$, be two real numbers, then sign distance between a, b is defined as

$D(a, b) = a - b$.

Since $D(a, 0) = a$ and $D(b, 0) = b$,

Therefore, $D(a, b) = D(a, 0) + D(b, 0)$.

If $a > 0, D(a, 0) = a \Rightarrow a$ is the right- hand side of 0 with sign distance a.

If $a < 0, D(a, 0) = a \Rightarrow a$ is the left- hand side of 0 with sign distance $-a$.

2.5 Ranking of real numbers using sign distance

Let $a, b \in \mathbb{R}$, then ranking function for real numbers a, b is defined by saying

$(\boldsymbol{CR_1})$. $D(a, b) > 0 \iff D(a, 0) > D(b, 0) \iff a > b$,

$(\boldsymbol{CR_2})$. $D(a, b) < 0 \iff D(a, 0) < D(b, 0) \iff b < a$,

$(\boldsymbol{CR_3})$. $D(a, b) = 0 \iff D(a, 0) = D(b, 0) \iff b = a$,

2.6 α −cut of intuitionistic fuzzy number

Let $\tilde{A}^I = \{(x, \mu_A(x), \nu_A(x)) : x \in \mathbb{R}\}$ be an intuitionistic fuzzy number, then α −cut are given by

$$({\tilde{A}^{I}}^{+})_\alpha = \{x \in X :\ \mu_{\tilde{A}^I}(x) \geq \alpha\} = {\tilde{A}^{I}}^{\alpha}$$

$(\tilde{A}^{I^{-}})_{\alpha} = \{x \in X :\ 1 - \nu_A(x) \geq \alpha\} = (\tilde{A}^{I^{-}})_{\alpha}$
$= \{x \in X :\ \nu_{\tilde{A}^I}(x) \leq 1 - \alpha\} = \tilde{A}^{I^{1-\alpha}}$
and every α −cut of intuitionistic fuzzy number $(A^{+})_{\alpha}$or $(A^{-})_{\alpha}$ are closed intervals. Hence we get

$$(\tilde{A}^{I^{+}})_{\alpha} = [\tilde{A}^{I}{}_{L}^{+}(\alpha), \tilde{A}^{I}{}_{U}^{+}(\alpha)]$$
$$(\tilde{A}^{I^{-}})_{\alpha} = [\tilde{A}^{I}{}_{L}^{-}(\alpha), \tilde{A}^{I}{}_{U}^{-}(\alpha)]$$

Where $\tilde{A}^{I}{}_{L}^{+}(\alpha) = \inf\{x \in \mathbb{R} : \mu_{\tilde{A}^I}(x) \geq \alpha\}$,
$(\alpha) = \sup\{x \in \mathbb{R} : \mu_{\tilde{A}^I}(x) \geq \alpha\}$,
${}^{I}{}_{L}^{-}(\alpha) = \inf\{x \in \mathbb{R} : \nu_{\tilde{A}^I}(x) \leq 1 - \alpha\}$,
$\tilde{A}^{I}{}_{U}^{-}(\alpha) = \sup\{x \in \mathbb{R} : \nu_{\tilde{A}^I}(x) \leq 1 - \alpha\}$,

If the sides of intuitionistic fuzzy numbers are strictly monotone then $\tilde{A}^{I}{}_{L}^{+}(\alpha) = \varphi_{A}^{-1}(\alpha)$, $\tilde{A}^{I}{}_{U}^{+}(\alpha) = \chi_{\tilde{A}^I}^{-1}(\alpha)$, $\tilde{A}^{I}{}_{U}^{-}(\alpha) = \eta_{\tilde{A}^I}{}^{-1}(\alpha)$, $\tilde{A}^{I}{}_{U}^{-}(\alpha) = \xi_{\tilde{A}^I}{}^{-1}(\alpha)$.

2.7 Sign distance between intuitionistic fuzzy numbers

Let $\tilde{A}^{I} = \{(x, \mu_A(x), \nu_A(x)) : x \in \mathbb{R}\}$ and $\tilde{B}^{I} = \{(x, \mu_B(x), \nu_B(x)) : x \in \mathbb{R}\}$ be two intuitionistic fuzzy numbers, we distance between two intuitionistic fuzzy is given by

$$D^{s}(\tilde{A}^{I}, \tilde{B}^{I}) = \frac{1}{4}\Bigg[\int_{0}^{1}\left(\tilde{A}^{I}{}_{L}^{+}(\alpha) - \tilde{B}^{I}{}_{L}^{+}(\alpha)\right)d\alpha + \int_{0}^{1}\left(\tilde{A}^{I}{}_{U}^{+}(\alpha) - \tilde{B}^{I}{}_{U}^{+}(\alpha)\right)d\alpha + \int_{0}^{1}\left(\tilde{A}^{I}{}_{L}^{-}(\alpha) - \tilde{B}^{I}{}_{L}^{-}(\alpha)\right)d\alpha + \int_{0}^{1}\left(\tilde{A}^{I}{}_{U}^{-}(\alpha) - \tilde{B}^{I}{}_{U}^{-}(\alpha)\right)d\alpha\Bigg] \quad (1)$$

2.8 Sign distance of triangular intuitionistic fuzzy number

Let $\tilde{A}^{I} = \{(a_1, a_2, a_3),\ (b_1,\ a_2, b_3)\}$ be triangular intuitionistic fuzzy number. Then sign distance of A can be calculated as

$$D^{s}(\tilde{A}^{I}, \tilde{0}^{I}) = \frac{1}{4}\Bigg[\int_{0}^{1}\left(\tilde{A}^{I}{}_{L}^{+}(\alpha)\right)d\alpha + \int_{0}^{1}\left(\tilde{A}^{I}{}_{U}^{+}(\alpha)\right)d\alpha + \int_{0}^{1}\left(\tilde{A}^{I}{}_{L}^{-}(\alpha)\right)d\alpha + \int_{0}^{1}\left(\tilde{A}^{I}{}_{U}^{-}(\alpha)\right)d\alpha\Bigg]$$

$$= \frac{1}{4}\left[\int_0^1 \{a_1 + \alpha(a_2 - a_1)\}d\alpha + \int_0^1 \{a_3 - \alpha(a_3 - a_2)\}d\alpha + \int_0^1 \{a_2 - (1-\alpha)(a_2 - b_1)\}d\alpha + \int_0^1 \{a_2 + (1-\alpha)(b_3 - a_2)\}d\alpha\right]$$

$$= \frac{a_1 + 2a_2 + a_3 + b_1 + 2a_2 + b_3}{8} = \frac{a_1 + 4a_2 + a_3 + b_1 + b_3}{8}$$

Therefore

$$D^s(\tilde{A}^I, \tilde{0}^I) = \frac{a_1 + a_3 + 4a_2 + b_1 + b_3}{8} \qquad (3)$$

3. SOME THEOREMS RELATED TO $D^s(\tilde{A}^I, \tilde{0}^I)$

Theorem 1

$D^s(k\tilde{A}^I, \tilde{0}^I) = kD^s(\tilde{A}^I, \tilde{0}^I)$, $k \in \mathbb{R}$, where D^s is the sign distance function of TIF number.

Proof:

Case (i) when $k = 0$. nothing to prove.

Case (ii) when $k > 0$.

Let $\tilde{A}^I = \{(a_1, a_2, a_3), (b_1, a_2, b_3)\}$ be a triangular intuitionistic fuzzy number and k be a real number.

$\tilde{A}^I = k\{(a_1, a_2, a_3), (b_1, a_2, b_3)\} = \{(ka_1, ka_2, k\, a_3), (kb_1, ka_2, kb_3)\}$, using property (P2)

$\therefore\ k\tilde{A}^I = \{(ka_1, ka_2, ka_3), (kb_1, kb_2, kb_3)\}$

Applying D^s function, we get

$$D^s(k\tilde{A}^I, \tilde{0}^I) = D^s(\{(ka_1, ka_2, ka_3), (kb_1, ka_2, kb_3)\}, \tilde{0}^I)$$

$$= \frac{ka_1 + 4ka_2 + ka_3 + kb_1 + kb_3}{8} = k.\frac{a_1 + a_3 + 4a_2 + b_1 + b_3}{8} = kD^s(\tilde{A}^I, \tilde{0}^I) \qquad \square$$

Case (iii) when $k < 0$.

Let $\tilde{A}^I = \{(a_1, a_2, a_3), (b_1, a_2, b_3)\}$ be a triangular intuitionistic fuzzy number and k be a real number.

$k\tilde{A}^I = k\{(a_1, a_2, a_3), (b_1, a_2, b_3)\} = \{(ka_3, ka_2, ka_1), (kb_3, ka_2, kb_1)\}$, using property (P2)

$\therefore\ k\tilde{A}^I = \{(ka_1, ka_2, ka_3), (kb_3, ka_2, kb_1)\}$

Applying D^s function, we get

$$D^s\left(k\tilde{A}^I, \tilde{0}^I\right) = D^s\left(\{(ka_3, ka_2, ka_1), (kb_3, ka_2, kb_1)\}, \tilde{0}^I\right),$$

$$= \frac{ka_3 + ka_1 + 4ka_2 + kb_3 + kb_1}{8} = k.\frac{a_1 + a_3 + 4a_2 + b_1 + b_3}{8} = kD^s\left(\tilde{A}^I, \tilde{0}^I\right)$$

□

Theorem 2

Let $\tilde{A}^I = \{(a_1, a_2, a_3), (b_1, a_2, b_3)\}$ and $\tilde{C}^I = \{(c_1, c_2, c_3), (d_1, c_2, d_3)\}$ be two TIF numbers. Then $D^s\left(\tilde{A}^I \oplus \tilde{C}^I, \tilde{0}^I\right) = D^s\left(\tilde{A}^I, \tilde{0}^I\right) + D^s\left(\tilde{C}^I, \tilde{0}^I\right)$.

Proof:

$\because\ \tilde{A}^I \oplus \tilde{C}^I = \{(a_1 + c_1, a_2 + c_2, a_3 + c_3), (b_1 + d_1, a_2 + c_2, b_3 + d_3)\}$, using Property (P1)

Now applying D^s function, we get

$$D^s\left(\tilde{A}^I \oplus \tilde{C}^I, \tilde{0}^I\right) = D^s\left(\{(a_1 + c_1, a_2 + c_2, a_3 + c_3), (b_1 + d_1, , a_2 + c_2, b_3 + d_3)\}, \tilde{0}^I\right)$$

$$= \frac{a_1 + c_1 + a_3 + c_3 + 4(a_2 + c_2) + b_1 + d_1 + b_3 + d_3}{8}$$

$$= \frac{a_1 + c_1 + a_3 + c_3 + 4a_2 + 4c_2 + b_1 + d_1 + b_3 + d_3}{8}$$

$$= \frac{a_1 + a_3 + 4a_2 + b_1 + b_3}{8} + \frac{c_1 + c_3 + 4c_2 + d_1 + d_3}{8}$$

$= D^s\left(\tilde{A}^I, \tilde{0}^I\right) + D^s\left(\tilde{C}^I, \tilde{0}^I\right)$. □

Theorem 3

$D^s\left(\tilde{A}^I, \tilde{0}^I\right) = D^s\left(\{(a_1, a_2, a_3), (b_1, a_2, b_3)\}, \tilde{0}^I\right)$ is a_2 if $a_2 - a_1 = a_3 - a_2$ and $a_2 - b_1 = b_3 - a_2$.

Proof

$D^s\left(\tilde{A}^I, \tilde{0}^I\right)$ of given TIF numbers is

$$D^s\left(\tilde{A}^I, \tilde{0}^I\right) = \frac{a_1 + a_3 + 4a_2 + b_1 + b_3}{8}, \dots, (i)$$

$$a_2 - a_1 = a_3 - a_2 \Rightarrow a_2 = \frac{a_1 + a_3}{2},$$

Similarly

$$a_2 - b_1 = b_3 - a_2 \Longrightarrow a_2 = \frac{b_1 + b_3}{2},$$

(i) can be rewritten as

$$D^s\left(\tilde{A}^I, \tilde{0}^I\right) = \frac{\frac{a_1 + a_3}{2} + 2a_2 + \frac{b_1 + b_3}{2}}{4}, \dots, (ii)$$

Putting the values of a_2 in (ii), we get

$$D^s(\tilde{A}^I, \tilde{0}^I) = \frac{a_2 + 2a_2 + a_2}{4} = a_2$$

□

Theorem 4

Let $\tilde{A}^I = \{(a_1, a_2, a_3), (b_1, a_2, b_3)\}$ be triangular intuitionistic fuzzy number. If $a_1 = b_1, b_3 = a_3.$ then

$$D^s(\tilde{A}^I, \tilde{0}^I) = \frac{a_1 + 2a_2 + a_3}{4}$$

which is ranking of triangular fuzzy numbers(a_1, a_2, a_3).

Proof

Let $\tilde{A}^I = \{(a_1, a_2, a_3), (b_1, a_2, b_3)\}$ be triangular intuitionistic fuzzy number

Given $a_1 = b_1, b_3 = a_3, \dots, (i)$

We know that $D^s(\tilde{A}^I, \tilde{0}^I)$ of triangular intuitionistic fuzzy numbers are

$$D^s(\tilde{A}^I, \tilde{0}^I) = \frac{a_1 + a_3 + 4a_2 + b_1 + b_3}{8}, \dots, (ii)$$

From (i) and (ii),we get the following result

$$D^s(\tilde{A}^I, \tilde{0}^I) = \frac{2a_1 + 4a_2 + 2a_3}{8} = \frac{a_1 + 2a_2 + a_3}{4}$$

□

4. PROPOSED METHOD FOR RANKING OF TRIANGULAR INTUITIONISTIC FUZZY NUMBERS

Let $\tilde{A}^I = \{(a_1, a_2, a_3), (b_1, a_2, b_3)\}$ be triangular intuitionistic fuzzy number. The ranking of a TIF numbers is defined by

$$\Re(\tilde{A}^I) = \left(\frac{a_3 - a_1}{b_3 - b_1}\right)\left(D^s(\tilde{A}^I, \tilde{0}^I)\right)$$

Theorem 5

If $\tilde{A}^I$ is a TIF number. Then $\Re(\tilde{A}^I) = k\left(\frac{a_3 - a_1}{b_3 - b_1}\right)D^s(\tilde{A}^I)$, $k \in \mathbb{R}$, where D^s is the sign distance function of TIF number.

*Proof***:**

Let $\tilde{A}^I = \{(a_1, a_2, a_3), (b_1, a_2, b_3)\}$ be a triangular intuitionistic fuzzy number and k be a real number.

$k\tilde{A}^I = k\{(a_1, a_2, a_3), (b_1, a_2, b_3)\} = \{(ka_1, ka_2, k\,a_3), (kb_1, ka_2, kb_3)\}$, using property (P2)

$\therefore\ k\tilde{A}^I = \{(ka_1, ka_2, ka_3), (kb_1, kb_2, kb_3)\}$

Applying D^s function, we get

$$\Re\left(k\tilde{A}^I\right) = \Re(\{(ka_1, ka_2, ka_3), (kb_1, kb_2, kb_3)\}) = \left(\frac{ka_3 - ka_1}{kb_3 - kb_1}\right) D^s\left(k\tilde{A}^I, \tilde{0}^I\right)$$

Using theorem 1, we get the following result

$D^s\left(k\tilde{A}^I, \tilde{0}^I\right) = kD^s\left(\tilde{A}^I, \tilde{0}^I\right)$, where $k \in \mathbb{R}$ is any real numbers, we get

$$\Re\left(k\tilde{A}^I\right) = \left(\frac{ka_3 - ka_1}{kb_3 - kb_1}\right) kD^s\left(\tilde{A}^I, \tilde{0}^I\right)$$

$$\Re\left(k\tilde{A}^I\right) = \frac{k^2}{k}\left(\frac{a_3 - a_1}{b_3 - b_1}\right) D^s\left(\tilde{A}^I, \tilde{0}^I\right)$$

$\Re\left(k\tilde{A}^I\right) = k\left(\frac{a_3 - a_1}{b_3 - b_1}\right) D^s\left(\tilde{A}^I, \tilde{0}^I\right)$ □

5. ILLUSTRATION OF PROPOSED RANKING FUNCTION

Here we apply the developed method for ranking of some of the intuitionistic fuzzy number for illustration of its suitability.

(i). Let $\tilde{A}^I = \{(2, 3, 4), (1, 3, 5)\}$, $\tilde{B}^I = \{(1, 2, 3), (0, 2, 6)\}$ be two TIF numbers. Then

$$\Re\left(\tilde{A}^I\right) = \Re(\{(2, 3, 4), (1, 3, 5)\}) = \left(\frac{4 - 2}{5 - 1}\right)\left(\frac{2 + 12 + 4 + 1 + 5}{8}\right) = 1.5$$

$$\Re\left(\tilde{B}^I\right) = \Re(\{(1, 2, 3), (0, 2, 6)\}) = \left(\frac{3 - 1}{6 - 0}\right)\left(\frac{1 + 8 + 3 + 0 + 6}{8}\right) = 0.75$$

$$\Re\left(\tilde{A}^I\right) > R\left(\tilde{B}^I\right) \Rightarrow \tilde{A}^I > \tilde{B}^I.$$

(ii). Let $\tilde{C}^I = \{(-4, -3, -2), (-6, -3, -1)\}$, $\tilde{D}^I = \{(-4, -3, -2), (-8, -3, \ 0)\}$ be two TIF numbers. Then

$$\Re\left(\tilde{C}^I\right) = \left(\frac{-2 + 4}{-6 + 1}\right)\left(\frac{-4 - 12 - 2 - 6 - 1}{8}\right) = 1.25$$

$$\Re\left(\tilde{D}^I\right) = \left(\frac{-2 + 4}{0 + 8}\right)\left(\frac{-4 - 12 - 2 - 8 + 0}{8}\right) = -.8125$$

$$\Re\left(\tilde{D}^I\right) < R\left(\tilde{C}^I\right) \Rightarrow \tilde{D}^I < \tilde{C}^I.$$

(iii). Let $\tilde{E}^I = \{(1, 2, 3), (0, 2, 4)\}$, $\tilde{F}^I = \{(-3, -2, -1), (-4, -2, \ 0)\}$ be two TIF numbers. Then

$$\Re\left(\tilde{E}^I\right) = \left(\frac{3 - 1}{4 - 0}\right)\left(\frac{1 + 8 + 3 + 0 + 4}{8}\right) = 1.00$$

$$\Re\left(\tilde{F}^I\right) = \left(\frac{-1 + 3}{0 + 4}\right)\left(\frac{-3 - 8 - 1 - 4 + 0}{8}\right) = -1.00$$

$$\Re\left(\tilde{E}^I\right) > R\left(\tilde{F}^I\right) \Rightarrow \tilde{E}^I > \tilde{F}^I.$$

6. CONCLUSION

Thus we have defined a new ranking function to rank the triangular intuitionistic fuzzy numbers. The method is based on defining a fuzzy origin and the measuring the distances of fuzzy numbers from the origin. Thus using these distances of two under taken intuitionistic fuzzy numbers from origin is used for their ranking. Further, we have illustrated the proposed ranking method on different types of TIF numbers. The developed method may be a suitable ranking method for intuitionistic fuzzy numbers and may be applied to decision making problems in intuitionistic fuzzy environment.

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