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On Controlling the Nonlinear Response of Vibrational Vertical Conveyor under Mixed Excitation

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Abstract

In this paper, we introduced a mathematical and numerical study on controlling the nonlinear response of vibrational vertical conveyor under mixed excitation. By studying the vibrating motion of vertical vibration conveyor, we wrote the equations of motion as a coupling of non-linear differential equations. Multiple scale perturbation method is applied to study the approximate mathematical solutions up to the second order approximation and we studied the stability of the steady state solution mathematically at the worst different resonance cases using frequency response equations. The resulting different resonance cases are reported and studied numerically. Also, we obtained the numerical solutions of vertical shaking conveyor applying Runge-Kutta of fourth order. The stability of the steady state solution near the selected resonance cases are investigated numerically using phase-plane technique. The effect of the different parameters of the vertical shaking conveyor is studied numerically. Comparison with the available published work is reported. In the future work, the system can be studied at another worst different resonance cases or we can use another controller.

Keywords: Vertical shaking conveyor, Vibrations, harmonic and parametric excitation.

1. INTRODUCTION

In the area of mechanics and electronics, the behaviors of mechanical systems under periodic loadings have been examined by many researchers. Vertical conveyors are effective examples observing various kinds of parameters of this problem. Vertical Vibratory conveyor is developed recently to convey the bulk materials. It has some advantages like simple structure, occupied less, long conveying road, low maintenances cost and energy consumed less. It uses to convey high temperature, wearing, poisoned and volatile materials if it is sealed. The screening, dryness, and cooling processes can be fulfilled at the conveying process. So it is used broadly in iron and steel industry, metallurgical industry, chemical plant. In vertical shaking conveyers, the load-carrying element performs double harmonic oscillations: linear along the vertical axis and rotational around that axis (i.e. longitudinal and torsional oscillations). Conveyer drives with centrifugal vibration exciters may have (1) a single unbalanced mass, (2) two equal unbalancing masses, (3) a pendulum-type unbalanced mass, (4) four unbalanced masses in two shafts, (5) four rotating unbalanced masses for three principal modes of oscillation, i.e. linear, elliptical, and circular.

Alısverisci [1] study the transitional behavior across resonance, during the starting of a single degree of freedom vibratory system excited by crank and-rod. A loaded vibratory conveyor is safer to start than an empty one. Shaking conveyers with cubic nonlinear spring and ideal vibration exciter have been analyzed analytically for primary resonance by the Method of Multiple Scales, and numerically. The approximate analytical results obtained in this study have been compared with the numerical results, and have been found to be well matched. Comparing the results obtained by applying the approximate analytic method with those obtained numerically it is concluded that the difference is negligible, proving the correctness of the analytic procedure used. The nonlinear analysis for the change of the parameters of the motion, stability condition, and the jump phenomena has been shown graphically the transition over resonance of a nonlinear vibratory system, excited by unbalanced mass [2], is important in terms of the maximum vibrational amplitude produced on the drive for the cross-over. Alısverisci et al. [3] analyzed analytically the working ranges of oscillating shaking conveyers with a non-ideal vibration exciter for primary resonance by the method of multiple scales with reconstitution, and numerically. The maximum amplitude of vibration is important in determining the structural safety of the vibrating members. The results of the numerical simulations, obtained from the analytical equations, showed that the important dynamic characteristics of the system such as damping, non-linearity and the amplitude excitations effects, and presented a periodic behavior for these situations. The jump phenomenon occurs in the motion of the system near resonance. The analytical results obtained in this study have been compared with the numerical results, and have been found to be well matched. Alısverisci et al. [4] analyzed the vibrating system analytically and numerically for superharmonic and subharmonic resonance by the method of multiple scales. Very often in the motion of the system near resonance the jump phenomenon occurs. The stable motions of the oscillator are shown with one peak in the power spectrum for superharmonic resonance and with two peaks in the power spectrum for subharmonic resonance. Both analytical and numerical results that we have obtained are in good agreement. The system studied here exhibits chaotic behavior in case of strong nonlinearity. Yuejing Zhao et al. [5] conducted the configuration and force analysis of vertical vibratory conveyor. The model of system with considering the friction between the materials and the spiral conveying trough is developed. The numerical simulations are done and the dynamical responses curves are given. Suitable configuration parameters of vertical vibratory conveyor and parameters of materials can make it work normally. Bayıroglu [6] Primary, subharmonic, and superharmonic responses have been investigated with multiple scales along with numerical methods for vertical conveyors. The change in the parameters of motion, stability condition, and jump phenomena has been shown graphically by Mathematica software for comparing the results. Both analytical and numerical results obtained had good agreement. Systems, excited by unbalanced mass, are important in terms of the maximum vibration amplitude produced on the drive for the cross-over. The maximum amplitude of vibration is then of interest in determining the structural safety of the vibrating members. Ektiar et al. [7] introduced a nonlinear dynamic analysis and mathematical study of the vibration behavior in vertical shaking conveyor under harmonic and parametric excitations. Multiple scale perturbation method applied to study the approximate mathematical solutions up to the second order approximation. They studied the stability of the steady state solution mathematically at the worst different resonance cases using frequency response equations. Hüseyin Bayıroğlu [8] analyzed the nonlinear analysis of unbalanced mass of vertical conveyor with non-ideal DC motor. The results of numerical simulation are plotted and Lyapunov exponents are calculated. Sayed and Bauomy [9] used the two positive position feedback controllers (PPF) are used to reduce the vertical vibration in the vertical conveyors. An investigation is presented of the response of a four-degree-of-freedom system (4-DOF) with cubic nonlinearities and external excitations at primary resonance.

Kamel and Hamed [10] studied the nonlinear behavior of an inclined cable subjected to harmonic excitation near the simultaneous primary and 1:1 internal resonance using multiple scale method. Hamed et al. [11-13] showed how effective is the passive vibration control reduction at resonance under multi-external or both multi-external and multi-parametric and both multi-external and tuned excitation forces. They reported that the advantages of using multi-tools are to machine different materials and different shapes at the same time. This leads to saving the time and higher machining efficiency. Hamed et al. [14] presented the behavior of the nonlinear string beam coupled system subjected to external, parametric and tuned excitations for case 1:1 internal resonance. The stability of the system studied using frequency response equations and phase-plane method. It is found from numerical simulations that there are obvious jumping phenomena in the frequency response curves. Sayed and Hamed [15] studied the numerical response and stability analyses of a two-degree-of-freedom system under harmonic and parametric excitation forces. They obtained the approximate solutions up to and including the second-order approximations using the method of multiple scale perturbation technique. Sayed and Kamel [16, 17] investigated the influence of different controllers on the vibration control system. They reported that, the saturation of non-linear vibration absorbers is used to reduce and control the movement due to rotor blade flapping. Amer and El-Sayed [18] investigated the nonlinear dynamics of a two-degree-of freedom vibration system with absorber when subjected to multi external forces at primary and internal resonance with ratio 1:3. They reported that the steady-state amplitude of the main system is reduced to 2.5% of its maximum value. Sayed and Mousa [19] investigated the influence of the quadratic and cubic terms on non-linear dynamic characteristics of the angle-ply composite laminated rectangular plate with parametric and external excitations. The method of multiple time scale perturbation is applied to solve the non-linear differential equations describing the system up to and including the second-order approximation. Two cases of the sub-harmonic resonances cases ($\Omega_2 \cong 2\omega_1$ and $\Omega_2 \cong 2\omega_2$) in the presence of 1:2 internal resonance $\omega_2 \cong 2\omega_1$ are considered. The stability of the system is investigated using both frequency response equations and phase-plane method. It is quite clear that some of the simultaneous resonance cases are undesirable in the design of such system as they represent some of the worst behavior of the system. Such cases should be avoided as working conditions for the system. Kamel et al. [20] studied a model subject to multi-external excitation forces. The model is represented by two-degree-of-freedom system consisting of the main system and absorber simulating ultrasonic machining. They used the passive vibration controller to suppress the vibration behavior of the system. Hamed et al. [21] investigated the nonlinear vibrations and stability of the MEMS gyroscope subjected to different types of parametric excitations. The averaging method is applied to obtain the frequency response equations for the case of sub-harmonic resonance in the presence of 1:1 internal resonances. The stability of the system is investigated with frequency response curves and phase-plane method. Some recommendations regarding the different parameters of the system are reported.

2. MATHEMATICAL ANALYSIS

The system is excited by linear and nonlinear external and parametric excitation forces. Proceeding as in Ref. [6], we can obtain the following nonlinear ordinary differential governing equation of motion for the Vertical shaking conveyor

$$\ddot{z}+2\varepsilon\mu_1\dot{z}_1+\omega_1^2 z_1+\varepsilon\alpha_1 z_1^3=\varepsilon f_1\left(\cos\Omega t+\sin\Omega t\right)+\varepsilon z_1 f_2\left(\cos\Omega_1 t+\sin\Omega_1 t\right)+R_1 \quad (1)$$

$$\ddot{z}_2+2\varepsilon\mu_2\dot{z}_2+\omega_2^2 z_2+\varepsilon\alpha_2 z_2^3=\varepsilon f_1\left(\cos\Omega t+\sin\Omega t\right)+\varepsilon z_2 f_2\left(\cos\Omega_1 t+\sin\Omega_1 t\right)+R_2 \quad (2)$$

where $R_1=-\varepsilon G_1\dot{z}_1$, $R_2=-\varepsilon G_2\dot{z}_2$ are negative velocity feedback controls, μ_1, μ_2 are the damping coefficients of vertical shaking conveyor system and controller, α_1 and α_2 are nonlinear coefficients of the vertical shaking conveyor system, f_1, f_2 are the excitation forcing amplitudes and Ω, Ω_1 are external and parametric excitation frequencies. The vertical shaking conveyor system natural frequencies are ω_1 and ω_2, ε is a small perturbation parameter and $0<\varepsilon<<1$.

3. PERTURBATION ANALYSIS

The MSPT method is used to obtain a uniformly valid, asymptotic expansion of the solutions for (1)–(2) is in the form:

$$z_1\left(t;\varepsilon\right)=z_{10}\left(T_0,T_1\right)+\varepsilon z_{11}\left(T_0,T_1\right)+O(\varepsilon^2) \quad (3)$$

$$z_2\left(t;\varepsilon\right)=z_{20}\left(T_0,T_1\right)+\varepsilon z_{21}\left(T_0,T_1\right)+O(\varepsilon^2) \quad (4)$$

The derivatives will be in the form

$$\left.\begin{aligned}\frac{d}{dt}&=D_0+\varepsilon D_1+...\\ \frac{d^2}{dt^2}&=D_0^2+2\varepsilon D_0 D_1+...\end{aligned}\right\} \quad (5)$$

For the first-order approximation, we introduce two time scales, where $T_n=\varepsilon^n t$ and the derivatives $D_n=\partial/\partial T_n$, (n= 0, 1). Substituting (3)–(5) into (1)–(2) and equating the coefficients of equal powers of (ε) leads to

$O(\varepsilon^0)$

$$(D_0^2+\omega_1^2)z_{10}=0 \quad (6a)$$

$$(D_0^2+\omega_2^2)z_{20}=0 \quad (6b)$$

$O(\varepsilon^1)$

$$\begin{aligned}(D_0^2+\omega_1^2)z_{11}&=-2D_0D_1z_{10}-2\mu_1D_0z_{10}-\alpha_1z_{10}^3+f_1\left(\cos\Omega t+\sin\Omega t\right)\\&+z_{10}\ f_2\left(\cos\Omega_1 t+\sin\Omega_1 t\right)-G_1D_0z_{10}\end{aligned} \quad (7a)$$

$$(D_0^2+\omega_2^2)z_{21}=-2D_0D_1z_{20}-2\mu_2D_0z_{20}-\alpha_2z_{20}^3+f_1\left(\cos\Omega t+\sin\Omega t\right)$$
$$+z_{20}\,f_2\left(\cos\Omega_1 t+\sin\Omega_1 t\right)-G_2D_0z_{20} \tag{7b}$$

The general solutions of (6) can be written in the form

$$z_{10}=A_1(T_1)\exp\left(i\omega_1T_0\right)+cc. \tag{8a}$$

$$z_{20}=A_2(T_1)\exp\left(i\omega_2T_0\right)+cc. \tag{8b}$$

Where A_m (m=1, 2) are complex function in T_1, cc represents the complex conjugate of the previous terms. Substituting (8) into (7) and eliminating the secular terms, the particular solutions of (7) will be in the form:

$$z_{11}=\left[\frac{\alpha_1A_1^3}{8\omega_1^2}\right]\exp\left(3i\omega_1T_0\right)+\left[\frac{f_1}{2(\omega_1^2-\Omega^2)}\right]\exp\left(i\Omega T_0\right)+\left[\frac{f_1}{2i(\omega_1^2-\Omega^2)}\right]\exp\left(i\Omega T_0\right)$$
$$+\left[\frac{f_2A_1}{2(\omega_1^2-(\Omega_1+\omega_1)^2)}\right]\exp\left(i(\Omega_1+\omega_1)T_0\right)+\left[\frac{f_2A_1}{2i(\omega_1^2-(\Omega_1+\omega_1)^2)}\right]\exp\left(i(\Omega_1+\omega_1)T_0\right)$$
$$+\left[\frac{f_2A_1}{2(\omega_1^2-(\Omega_1-\omega_1)^2)}\right]\exp\left(i(\Omega_1-\omega_1)T_0\right)+\left[\frac{f_2A_1}{2i(\omega_1^2-(\Omega_1-\omega_1)^2)}\right]\exp\left(i(\Omega_1-\omega_1)T_0\right)+c.c \tag{9a}$$

$$z_{21}=\left[\frac{\alpha_2A_2^3}{8\omega_2^2}\right]\exp\left(3i\omega_2T_0\right)+\left[\frac{f_1}{2(\omega_2^2-\Omega^2)}\right]\exp\left(i\Omega T_0\right)+\left[\frac{f_1}{2i(\omega_2^2-\Omega^2)}\right]\exp\left(i\Omega T_0\right)$$
$$+\left[\frac{f_2A_2}{2(\omega_2^2-(\Omega_1+\omega_2)^2)}\right]\exp\left(i(\Omega_1+\omega_2)T_0\right)+\left[\frac{f_2A_2}{2i(\omega_2^2-(\Omega_1+\omega_2)^2)}\right]\exp\left(i(\Omega_1+\omega_2)T_0\right)$$
$$+\left[\frac{f_2A_2}{2(\omega_2^2-(\Omega_1-\omega_2)^2)}\right]\exp\left(i(\Omega_1-\omega_2)T_0\right)+\left[\frac{f_2A_2}{2i(\omega_2^2-(\Omega_1-\omega_2)^2)}\right]\exp\left(i(\Omega_1-\omega_2)T_0\right)+c.c \tag{9b}$$

We can rewrite Eqs. (9) in the form

$$z_{11}=K_1\exp(3i\omega_1T_0)+K_2\exp\left(i\Omega T_0\right)+K_3\exp\left(i(\Omega_1+\omega_1)T_0\right)$$
$$+K_4\exp\left(i(\Omega_1-\omega_1)T_0\right)+cc \tag{10a}$$

$$z_{21}=H_1\exp(3i\omega_2T_0)+H_2\exp\left(i\Omega T_0\right)+H_3\exp\left(i(\Omega_1+\omega_2)T_0\right)$$
$$+H_4\exp\left(i(\Omega_1-\omega_2)T_0\right)+cc \tag{10b}$$

where $K_i, (i=1,...,4)$ and $H_i, (i=1,...,4)$ are a complex function in T_1, T_2 and cc represents the complex conjugates. From the above-derived solutions, many resonance cases can be deduced. The reported resonance cases are classified into:

(A) Primary Resonance: $\Omega \cong \omega_1$, $\Omega \cong \omega_2$.

(B) Sub-Harmonic Resonance: $\Omega_1 \cong 2\omega_1$, $\Omega_1 \cong 2\omega_2$.

(C) Simultaneous or Incident Resonance: Any combination of the above resonance cases is considered as simultaneous or incident resonance.

4. STABILITY OF MOTION

4.1. For the first mode of vertical shaking conveyor system

Stability of the considered system is investigated at the simultaneous primary $\Omega \cong \omega_1$ and principle parametric $\Omega_1 \cong 2\omega_1$ are considered. Two detuning parameters σ_1 and σ_2 such that

$$\Omega = \omega_1 + \varepsilon\sigma_1 \text{ and } \Omega_1 - \omega_1 = \omega_1 + \varepsilon\sigma_3 \tag{11}$$

This case represents the system worst case. Substituting Eq. (11) into Eq. (7a) and eliminating the secular terms, leads to the solvability conditions for the first order approximation, we get

$$\begin{aligned}2i\omega_1 D_1 A_1 = -2i\,\omega_1\mu_1 A_1 - 3\alpha_1 A_1^2\bar{A}_1 - i\,\omega_1 G_1 A_1 + \left(\frac{1-i}{2}\right) f_1 \exp(i\sigma_1 T_1)\\ + \left(\frac{1-i}{2}\right) f_2 \bar{A}_1 \exp(i\sigma_3 T_1)\end{aligned} \tag{12}$$

To analyze the solutions of Eq. (12), we express $A_1(T_1,T_2)$ and $A_2(T_1,T_2)$ in the polar form

$$A_1(T_1,T_2) = \frac{a_1}{2}e^{i\varphi_1} \quad , \ \bar{A}_1(T_1,T_2) = \frac{a_1}{2}e^{-i\varphi_1} \tag{13}$$

where a_1 and φ_1 are the steady state amplitude and phase of the motion of the first mode. Substituting Eq. (13) into Eq. (12) and equating the real and imaginary parts we obtain the following equations describing the modulation of the amplitude and phase of the first modes of vertical shaking conveyor response:

$$\dot{a}_1 = \mu_1 a_1 + \frac{G_1 a_1}{2} - \frac{f_1}{2\omega_1}\cos\theta_1 + \frac{f_1}{2\omega_1}\sin\theta_1 - \frac{f_2 a_1}{4\omega_1}\cos\theta_2 + \frac{f_1 a_1}{4\omega_1}\sin\theta_2 \quad (14a)$$

$$a_1\dot{\varphi}_1 = \frac{3\alpha_1 a_1^3}{8\omega_1} - \frac{f_1}{2\omega_1}\cos\theta_1 - \frac{f_1}{2\omega_1}\sin\theta_1 - \frac{f_2 a_1}{4\omega_1}\cos\theta_2 - \frac{f_1 a_1}{4\omega_1}\sin\theta_2 \quad (14b)$$

Where

$$\theta_1 = \sigma_1 T_1 - \varphi_1 \text{ and } \theta_2 = \sigma_3 T_1 - 2\varphi_1 \quad (15)$$

Form the system of Eqs. (14) to have stationary solutions, the following conditions must be satisfied:

$$\dot{a}_1 = \dot{\theta}_1 = \dot{\theta}_2 = 0 \quad (16)$$

It follows from Eq. (15) that $\dot{\varphi}_1 = \sigma_1 = \frac{\sigma_3}{2} = \sigma$

Hence, the steady state solutions of Eqs. (14) are given by

Hence, the fixed points of Eqs. (22)- (23) are given by

$$\mu_1 a_1 + \frac{G_1 a_1}{2} - \frac{f_1}{2\omega_1}\cos\theta_1 + \frac{f_1}{2\omega_1}\sin\theta_1 - \frac{f_2 a_1}{4\omega_1}\cos\theta_2 + \frac{f_1 a_1}{4\omega_1}\sin\theta_2 = 0 \quad (17a)$$

$$a_1\sigma_1 - \frac{3\alpha_1 a_1^3}{8\omega_1} + \frac{f_1}{2\omega_1}\cos\theta_1 + \frac{f_1}{2\omega_1}\sin\theta_1 + \frac{f_2 a_1}{4\omega_1}\cos\theta_2 + \frac{f_1 a_1}{4\omega_1}\sin\theta_2 = 0 \quad (17b)$$

Solving the resulting algebraic equations for the fixed points, we obtained

$$\sigma_1^2 + \left(-\frac{3\alpha_1 a_1^2}{4\omega_1}\right)\sigma_1 + \left(\mu_1^2 + \mu_1 G_1 + \frac{G_1^2}{4} + \frac{9\alpha_1^2 a_1^4}{64\omega_1^2} - \frac{f_1^{\,2}}{2a_1^2\omega_1^2} - \frac{f_2^{\,2}}{8\omega_1^2} - \frac{f_1 f_2}{2a_1\omega_1^2}\right) = 0 \quad (18)$$

4.2. For the second mode of vertical shaking conveyor system

The stability is investigated at the simultaneous primary $\Omega \cong \omega_2$ and sub-harmonic $\Omega_1 \cong 2\omega_2$ are considered. We introduce detuning parameters σ_3 and σ_4 such that

$$\Omega = \omega_2 + \varepsilon\sigma_2 \quad \text{and} \quad \Omega_1 - \omega_2 = \omega_2 + \varepsilon\sigma_4 \tag{19}$$

This case represents the system worst case. Substituting Eq. (19) into Eq. (7b) and eliminating the secular terms, leads to the solvability conditions for the first order approximation, we get

$$\begin{aligned} 2i\omega_2 D_1 A_2 = &-2i\,\omega_2\mu_2 A_2 - 3\alpha_2 A_2^2\bar{A}_2 - i\,\omega_2 G_2 A_2 + \left(\frac{1-i}{2}\right) f_1 \exp(i\sigma_2 T_1) \\ &+ \left(\frac{1-i}{2}\right) f_2 \bar{A}_1 \exp(i\sigma_4 T_1) \end{aligned} \tag{20}$$

Appling the same process as the stability of the first mode to Eq. (20), the frequency equations for angular vibration can be obtained as

$$\sigma_2^2 + \left(-\frac{3\alpha_2 a_2^2}{4\omega_2}\right)\sigma_2 + \left(\mu_2^2 + \mu_2 G_2 + \frac{G_2^2}{4} + \frac{9\alpha_2^2 a_2^4}{64\omega_2^2} - \frac{f_1^{\,2}}{2a_2^2\omega_2^2} - \frac{f_2^{\,2}}{8\omega_2^2} - \frac{f_1 f_2}{2a_2\omega_2^2}\right) = 0 \tag{21}$$

5. RESULTS AND DISCUSSIONS

To determine the numerical solution and response of the given system of equations (1) and (2), the Runge-Kutta of fourth order method was applied. Within this section, the results are presented in graphical forms as steady state amplitudes (a_1, a_2) against detuning parameters (σ_1, σ_2) and the time response for both the modes of the vertical shaking conveyor system and controller. The system original equations (1) and (2) have been solved numerically using **ODE45** Matlab solver. The numerical solution of the mathematical modeling and its stability is studied here and the solutions of the frequency response function regarding the stability of the electromechanical system and the controller are examined. The effects of various parameters on the steady state solution are obtained and studied also different resonance cases are reported and discussed.

5.1 Numerical results

Fig. 1 shows that the time response and phase-plane before control at the simultaneous primary and sub-harmonic resonance $\Omega \cong \omega_n$ and $\Omega_1 \cong 2\omega_n$, n= (1, 2) which is one of the worst resonance cases. From this figure we have that the amplitude of the first mode of the vertical shaking conveyor system is increased to about **3000%** of the greatest excitation force amplitude f_1, while the amplitude of the second mode is increased to about **1600%** and becomes stable and the phase plane shows limit cycle.

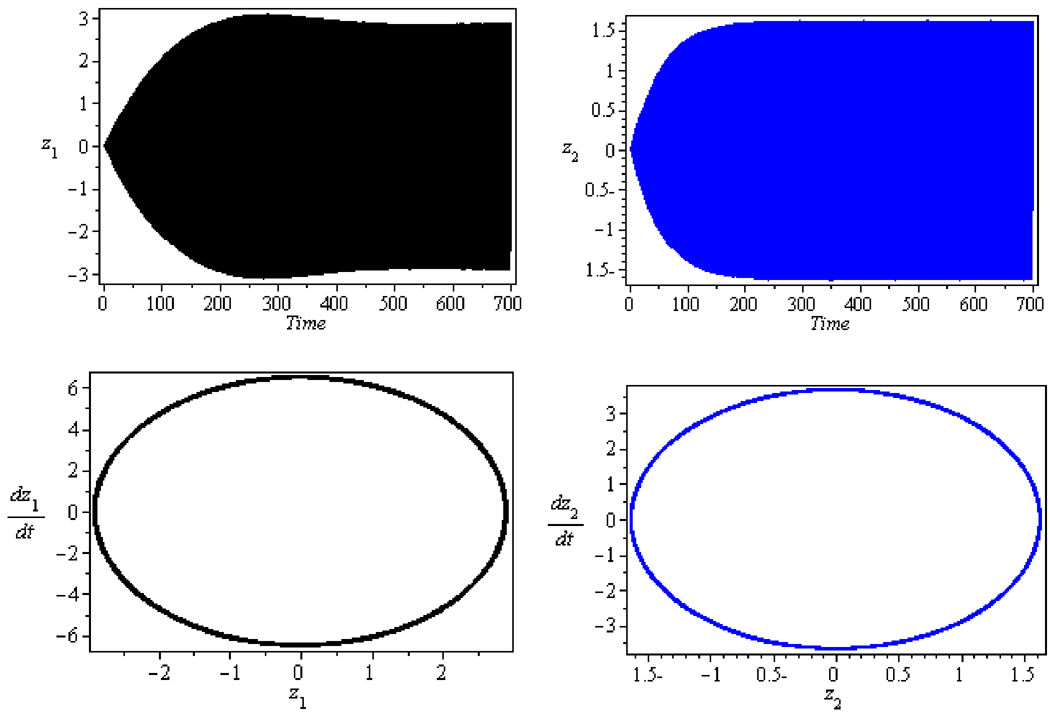


Fig. 1. The response amplitude of the vertical shaking conveyor system before control at the case $(\Omega \cong \omega_n$ and $\Omega_1 \cong 2\omega_n$, n= (1, 2)).

$$\mu_1 = 0.00825,\ \mu_2 = 0.01875, \alpha_1 = 0.005,\ \alpha_2 = 0.0083,\ f_1 = 0.1,\ f_2 = 0.002,$$
$$\Omega = 2.25,\ \Omega_1 = 4.5,\ \omega_1 = 2.25,\ \omega_2 = 2.25,\ G_1 = 0,\ G_2 = 0$$

Fig. 2 simulates the system time histories for the vertical shaking conveyor system after adding the control at simultaneous primary and sub-harmonic resonance case, where $\Omega \cong \omega_n$ and $\Omega_1 \cong 2\omega_n$, n= (1, 2). According to this figure, the steady state amplitude for the vertical shaking conveyor is **5%**, but the steady state amplitude of the controller is about **4%** of maximum excitation amplitude f_1. In addition, the effectiveness of the controller E_a (E_a= the steady state amplitude for system before control/the steady state amplitude for the system after control) is about **600** for the first mode of the vertical shaking conveyor system, while E_a of the second mode is about **400**.

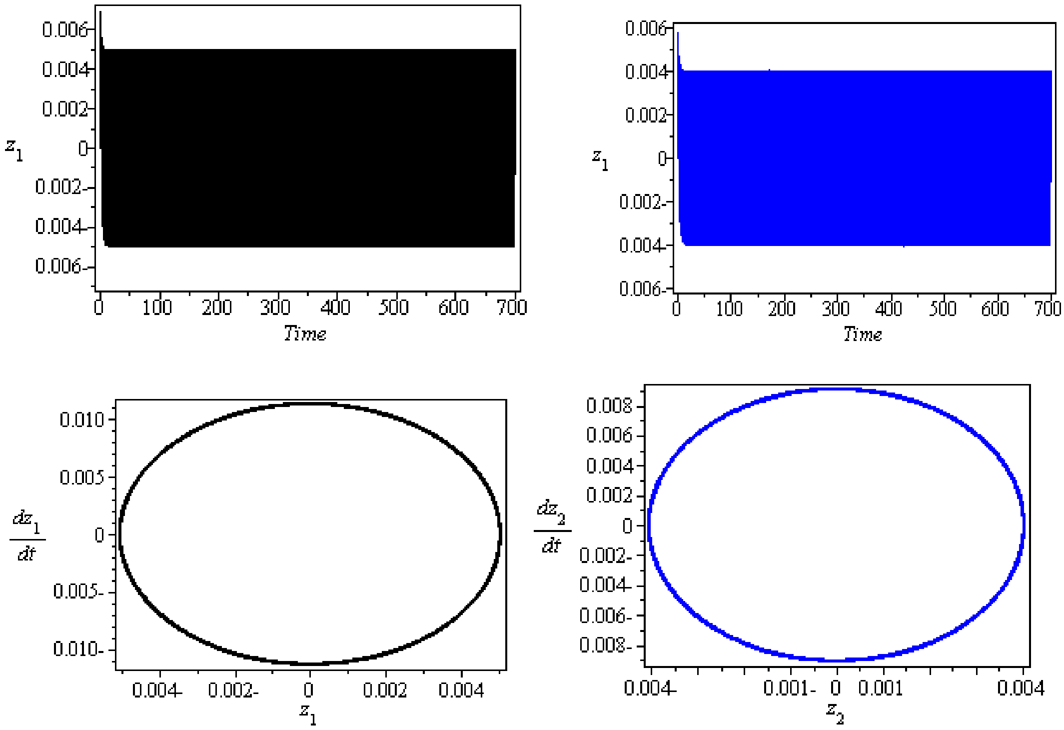


Fig. 2. The response amplitude of the vertical shaking conveyor system after control at the case $(\Omega \cong \omega_n$ and $\Omega_1 \cong 2\omega_n$, n= (1, 2)).

$$\mu_1 = 0.00825,\ \mu_2 = 0.01875, \alpha_1 = 0.005,\ \alpha_2 = 0.0083,\ f_1 = 0.1,\ f_2 = 0.002,$$
$$\Omega = 2.25,\ \Omega_1 = 4.5,\ \omega_1 = 2.25,\ \omega_2 = 2.25,\ G_1 = 12.5,\ G_2 = 15.5$$

5.2 Response curves and effects of different parameters

In this section, the frequency response equations given by Eqs. (18) and (21) are solved numerically at the same values of the parameters shown in Fig. 2.

Fig. 3a shows the steady state amplitudes of the first mode of the vertical shaking conveyor system against the detuning parameter σ_1 at the practical case, where $a_1 \neq 0,\ a_2 \neq 0$.

Figs. 3(b, c) show that the steady state amplitude of the first mode is a monotonic decreasing function in the natural frequency ω_1 and the coefficient control G_1. The steady state amplitude of the first mode is a monotonic increasing function in the excitation amplitudes f_1 and f_2 as shown in Figs. 3 (d, e).

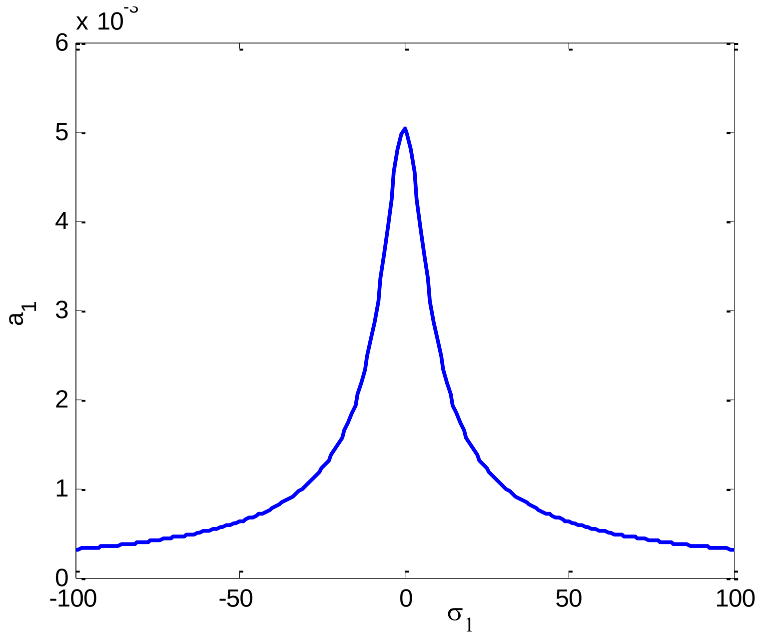


Fig.3a. Effects of the detuning parameter σ_1

$$\mu_1 = 0.00825,\ \alpha_1 = 0.005, f_1 = 0.1,\ f_2 = 0.002,\ \omega_1 = 2.25,\ G_1 = 12.5$$

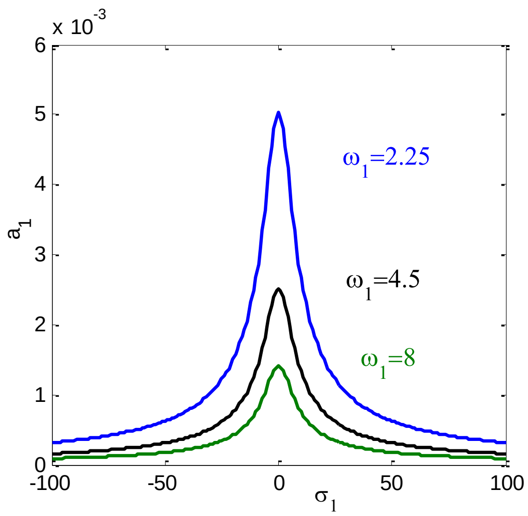


Fig. 3b. Effect of the natural frequency ω_1

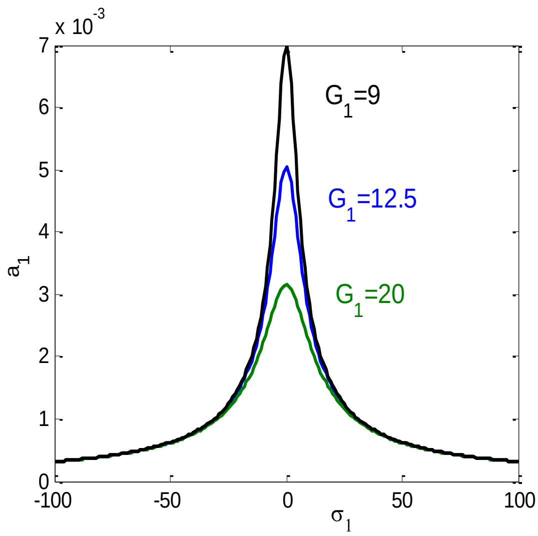
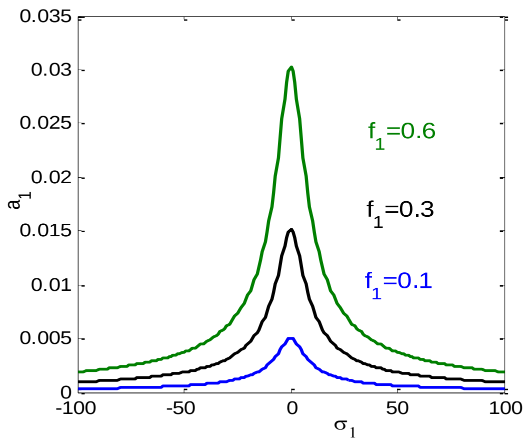


Fig. 3c. Effects of the excitation amplitude f_1



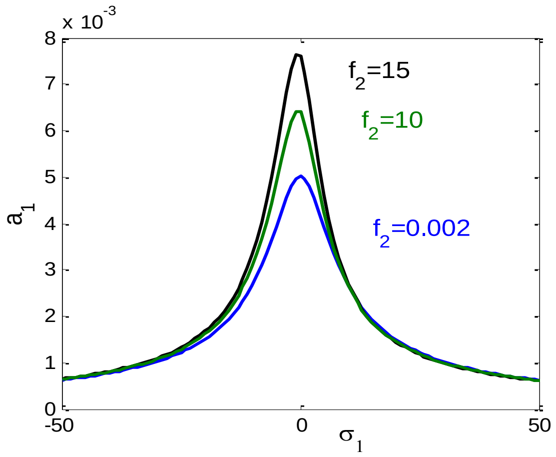


Fig. 3d. Effects of the excitation amplitude f_2. **Fig. 3e.** Effects of the gain G_1

Fig. 3. Effect of system parameters on the frequency response curves of the first mode at ($a_1 \neq 0$ and $a_2 \neq 0$)

Fig. 4a shows the steady state amplitudes of the second mode of the vertical shaking conveyor system against the detuning parameter σ_2 at the practical case, where $a_1 \neq 0$, $a_2 \neq 0$.

Figs. 4(b, c) show that the steady state amplitude of the second mode of the vertical shaking conveyor system is a monotonic decreasing function in the natural frequency ω_2 and the coefficient control G_2.The steady state amplitude of the second mode is a monotonic increasing function in the excitation amplitudes f_1 and f_2 as shown in Figs. 4 (d, e).

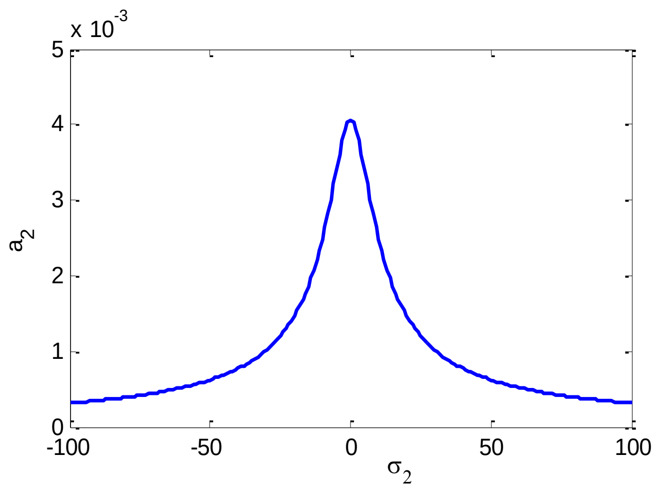


Fig.4a. Effects of the detuning parameter σ_2

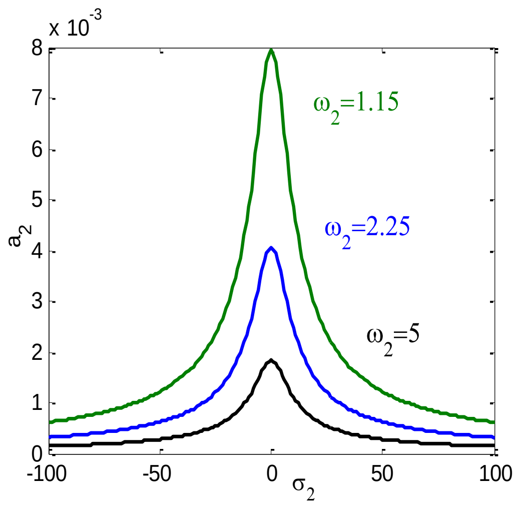


Fig. 4b. Effect of the natural frequency ω_2

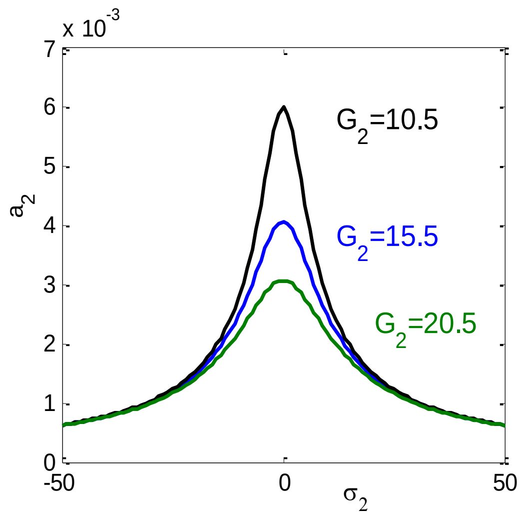


Fig. 4c. Effects of the gain G_2

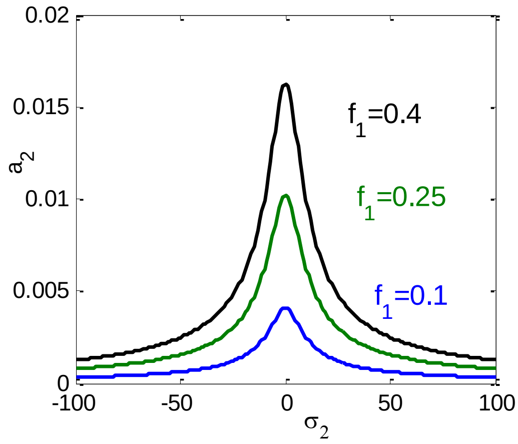


Fig.4d. Effects of the excitation amplitude f_1

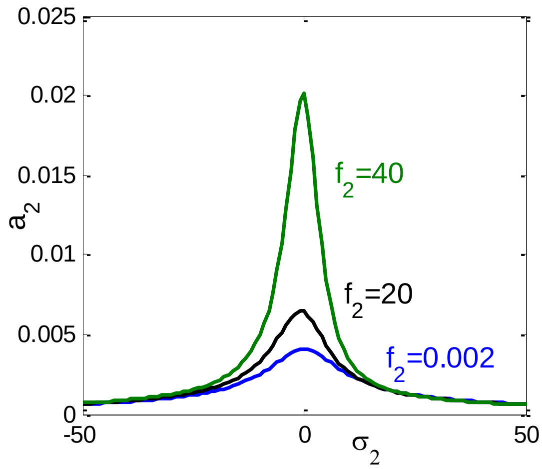


Fig.4e. Effects of the excitation amplitude f_2.

Fig. 4. Effect of system parameters on the frequency response curves of the c second mode at $(a_1 \neq 0$ and $a_2 \neq 0)$

6. CONCLUSIONS

A mathematical and numerical study on controlling the nonlinear response of vibrational vertical conveyor under mixed excitation have been studied. The problem is described by a two-degree-of-freedom system of nonlinear ordinary differential equations. The case of simultaneous primary and principle parametric resonance is studied by applying multiple time scale perturbation method. Both the frequency response equations and the phase-plane technique are applied to study the stability of the system. The effect of the different parameters of the system is studied numerically. From the above study the following may be concluded:

1. The oscillation of the two modes becomes stable and the steady state amplitudes z_1 and z_2 are about **0.1** and **0.08** at the non-resonant case ($\Omega \neq \Omega_1 \neq \omega_1 \neq \omega_2$).
2. the oscillation of the first mode of the vertical shaking conveyor system is increased to about **3000%,** while the amplitude of the second mode is increased to about **1600%** at the worst resonance case (the simultaneous primary and sub-harmonic resonance ($\Omega \cong \omega_n$ and $\Omega_1 \cong 2\omega_n$).
3. The steady state amplitude for the vertical shaking conveyor is **5%**, but the steady state amplitude of the controller is about **4%** of maximum excitation amplitude f_1.
4. The effectiveness of the controller E_a is about **600** for the first mode of the vertical shaking conveyor system, while E_a of the second mode is about **400**.
5. The steady state amplitude of the first and second modes of the vertical shaking conveyor system are monotonic decreasing functions in the natural frequencies ω_1 and ω_2, the coefficients of control G_1 and G_2.
6. The steady state amplitudes of the first and second modes are monotonic increasing functions in the excitation amplitudes f_1 and f_2.

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