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A Study on Vertex Degree of Cartesian Product of Intuitionistic Triple Layered Simple Fuzzy Graph

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Abstract

In this paper, we discussed Triple Layered Fuzzy graph and we introduced Intuitionistic Triple Layered Simple Fuzzy graph (ITLFG). We established vertex degree of Cartesian product of Intuitionistic Triple Layered Simple Fuzzy graph.

Keywords: Fuzzy graph, Order and size of the fuzzy graph, Intuitionistic fuzzy graph, Triple layered fuzzy graph, Vertex degree of ITLFG, Cartesian Product of ITLFG.

I. INTRODUCTION

The concept of a fuzzy relation was defined by Zadeh in 1965[10] and it has many applications in the analysis of cluster patterns. In 1975 Rosenfeld considered fuzzy relations on fuzzy sets and find the structure of fuzzy graphs[12]. In 1983 Atanassov[1] described the concept of intuitionistic fuzzy sets as a generalization of fuzzy sets[10]. Fuzzy set gives the degree of membership of an element in given set while intuitionistic fuzzy set gives both the degree of membership and non-membership which are more or less independent from each other. The condition is the sum of these two degrees should not exceed one. In [3] Karunambigai M. G. and Parvathi R, introduced intuitionistic fuzzy graph as a special case of Atanassov's Intuitionistic Fuzzy graph.

The operations on Intuitionistic fuzzy graph was introduced by R. Parvathi, M. G. Karunambigai and K. Atanassov [4]. Degree, Order and Size of Intuitionistic Fuzzy Graph was introduced by A. NaggorGani and S. ShajithaBegum [5]. The degree of a vertex in fuzzy graphs was introduced by A. NagooorGani and K. Radha [2]. The Double Layered Fuzzy graph was introduced by T. Pathinathan and J. Jesintha Rosline. They described some of the properties of triple layered fuzzy graph [9]. The vertex degree of Cartesian product of intuitionistic double layered fuzzy graph was introduced by T. Pathinathan and J. Jesintha Rosline[14]. In this paper, L. Jethurth Emelda Mary and R.Rajalakshmi introduced Intuitionistic Triple layered fuzzy graph (ITLFG). The Cartesian product of vertex degree of ITLFG is defined under certain condition. This relationship is illustrated with examples.

II. PRELIMINARIES

In this section we collect some of the basic definitions and notions.

2.1 Fuzzy graph

A fuzzy graph G is a pair of functions G: (σ,μ) where σ is a fuzzy subset of a non-empty vertex set S and μ is a symmetric fuzzy relation on σ. The underlying crisp graph of G: (σ,μ) is denoted by G*: (σ*,μ*)

2.2 Intuitionistic Fuzzy Graph

An intuitionistic fuzzy graph is of the form G: (V,E) .Where ,

(i)$V=\{v_1,v_2,v_3,\dots,v_n\}$ such that $\mu_1:V\to[0,1]$ and $\nu_1:V\to[0,1]$ denote the degree of membership and non-membership of the element $v_i \in V$ respectively and $0 \le \mu_1(v_i)+\nu_1(v_i) \le 1$ for every $v_i \in V$, $\forall$ i=1,2,….,n

(ii) $E \subseteq V \times V$ where $\mu_2 : E \to [0,1]$and $\nu_2: E \to [0,1]$ are such that

$\mu_2(v_i,v_j) \le min(\mu_1(v_i),\mu_1(v_j))$ and $\nu_2(v_i,v_j) \le max(\nu_1(v_i),\nu_1(v_j))$

and $0 \le \mu_2(v_i,v_j) + \nu_2(v_i,v_j) \le 1$ for every $(v_i,v_j) \in E$ $\forall\, i,j = 1,2,3,\dots,n$

2.3 Order of Intuitionistic Fuzzy Graph

Let G: (V,E) be an IFG. Then the order of G is defined to be $O(G) = (O\mu(G), O\nu(G))$ where $O\mu(G) = \sum_{v\in V} \mu_1(v)$ and $O\nu(G) = \sum_{v\in V} \nu_1(v)$

2.4 Size of Intuitionistic Fuzzy Graph

Let G: (V,E) be an IFG. Then the size of G is defined as $S(G) = (S\mu(G), S\nu(G))$ where $S\mu(G) = \sum_{v\in V} \mu_2(u,v)$ and $S\nu(G) = \sum_{v\in V} \nu_2(u,v)$

2.5 Degree of Vertex of Intuitionistic Fuzzy graph

Let G = (V, E) be an Intuitionistic Fuzzy graph. The degree of a vertex v is defined by $d(v) = (d_\mu(v), d_\nu(v))$ Where $d_\mu(v) = \sum_{u\neq v} \mu_2(u,v)$ and $d_\nu(v) = \sum_{u\neq v} \nu_2(u,v)$

2.6 Cartesian product of two Intuitionistic Fuzzy Graph

The Cartesian product of two IFG G_1 and G_2 is defined as a IFG, $G:G_1\times G_2 = (V', E')$ where $V'=V_1\times V_2$ and $E' =\{ (u_1,u_2)(v_1,v_2)/\ u_1 = v_1$ and $u_2v_2 \in E_2$ or $u_2 = v_2$ and $u_1v_1 \in E_1\}$ with $\langle(\mu_1 \times \mu_1'), (\nu_1 \times \nu_1')\rangle(u_1, u_2) = \langle \min(\mu_1(u_1), (\mu_1'(u_2)), \max(\nu_1(u_1), \nu_1'(u_2))\rangle$,

for every $(u_1, u_2) \in V$ and

$$\langle(\mu_2 \times \mu_2'), (\nu_2 \times \nu_2')\rangle(u_1,u_2)(v_1,v_2) = \begin{cases} \langle \min(\mu_1(u_1), \mu_2'(u_2,v_2)), \max(\nu_1(u_1), (\nu_2'(u_2,v_2))\rangle \\ \qquad if\ u_1 = v_1 and\ (u_2,v_2) \in E_2 \\ \langle \min(\mu_1'(u_2), \mu_2(u_1,v_1)), \max(\nu_1'(u_2), \nu_2(u_1,v_1))\rangle \\ \qquad if\ u_2 = v_2\ and\ (u_1,v_1) \in E_1 \\ \langle 0,0\rangle, \qquad otherwise. \end{cases}$$

2.7 Double Layered Fuzzy graph

Let $G: (\sigma, \mu)$ be a fuzzy graph with the underlying crisp graph $G^* : (\sigma^*, \mu^*)$. The pair DL(G): (σ_{DL}, μ_{DL}) is defined as follows.

The vertex set of DL(G) be $\sigma^* \cup \mu^*$,The fuzzy subset σ_{DL} is defined as

$$\sigma_{DL} = \begin{cases} \sigma(u) & if\ u \in \sigma^* \\ \mu(uv) & if\ uv\ \in \mu^* \end{cases}$$

The Fuzzy relation μ_{DL} on $\sigma^* \cup \mu^*$ is defined as

$$\mu_{DL} = \begin{cases} \mu(uv) & \text{if } u, v \in \sigma^* \\ \mu(e_i)\wedge\mu(e_j) & \text{if the edge } e_i \text{ and } e_j \text{ have a node in common between them} \\ \sigma(e_i)\wedge\mu(e_j) & \text{if the edges } e_i \text{ and } e_j \text{ have a node in common between them either clockwise or anticlockwise.} \\ 0 & \text{otherwise} \end{cases}$$

By definition, $\mu_{DL} = \sigma_{DL}(u)\wedge\sigma_{DL}(v)$ for all u,v in $\sigma^* \cup \mu^*$. Here μ_{DL} is the fuzzy relation on the fuzzy subset σ_{DL} . Hence the pair $DL(G): (\sigma_{DL}, \mu_{DL})$ is defined as Double Layered Fuzzy Graph (DLFG).

2.8 Triple Layered Fuzzy Graph

Let $G: (\sigma, \mu)$ be a fuzzy graph with the underlying crisp graph $G^* : (\sigma^*, \mu^*)$. The pair $TL(G): (\sigma_{TL}, \mu_{TL})$ is defined as follows. The vertex set of TL(G) be $\sigma^* \cup \mu^* \cup \mu^*$. The fuzzy subset σ_{TL} is defined as $\sigma_{TL} = \begin{cases} \sigma(u) & if\ u \in \sigma^* \\ 2\mu(uv) & if\ uv \in \mu^* \end{cases}$

The Fuzzy relation μ_{TL} on $\sigma^* \cup \mu^*$ is defined as

$$\mu_{TL} = \begin{cases} \mu(uv) \text{ if } u, v \in \sigma^* \\ \mu(e_i)\wedge\mu(e_j) \text{ if the edge } e_i \text{ and } e_j \text{ have a vertex in common between them} \\ \sigma(u_i)\wedge\mu(e_j) \text{if } u_i \in \sigma^* \text{ and } e_i \in \mu^* \text{ and each } e_i \text{ is incident with } u_i \text{ in clockwise direction} \\ \sigma(u_i)\wedge\mu(e_j) \text{if } u_i \in \sigma^* \text{and } e_i \in \mu^* \text{ and each } e_i \text{ is incident with } u_i \text{ in anticlockwise direction} \\ 0 \text{ Otherwise} \end{cases}$$

By definition, $\mu_{TL}(u, v) \leq \sigma_{TL}(u)\wedge\sigma_{TL}(v)$ for all u,v in $\sigma^* \cup \mu^*$. Here μ_{TL} is the fuzzy relation on the fuzzy subset σ_{TL} . Hence the pair $TL(G): (\sigma_{TL}, \mu_{TL})$ is defined as Triple Layered Fuzzy Graph (TLFG).

Symbols and its meanings:

(u_i, μ_1, ν_1) – Degree of membership and non-membership of the vertex u_i of G_1.

(e_{ij}, μ_2, ν_2) – Degree of membership and non-membership of the edge relation

$$e_{ij} = (u_i, u_j) \text{ on V of } G_1$$

(v_i, μ_1', ν_1') – Degree of membership and non-membership of the vertex v_i of G_1.

(e_{ij}, μ_2', ν_2') – Degree of membership and non-membership of the edge relation

$$e_{ij} = (v_i, v_j) \text{ on V of } G_2$$

III. INTUITIONISTIC TRIPLE LAYERED FUZZY GRAPH (ITLFG)

Definition 3.1

Let $G:\langle(v_i,\mu_1,\nu_1),(e_{ij},\mu_2,\nu_2)\rangle$ be an intuitionistic fuzzy graph with the underlying crisp graph$G^*:(\sigma^*,\mu^*)$. The pair $TL(G):\langle(v_i,\mu_{TL_1},\nu_{TL_1}),(e_{ij},\mu_{TL_2},\nu_{TL_2})\rangle$ is called the Intuitionistic Triple Layered Fuzzy graph and is defined as follows. The Vertex set of ITL(G) be $\langle\mu_{TL_1,}\nu_{TL_1}\rangle$. The fuzzy subset $\langle\mu_{TL_1,}\nu_{TL_1}\rangle$ is defined as

$$\langle\mu_{TL_1,}\nu_{TL_1}\rangle=\begin{cases}\langle\mu_1(u),\nu_1(u)\rangle & if\ u\in\sigma^*\\ \langle\mu_2(uv),\nu_2(uv)\rangle & if\ uv\in\mu^*\end{cases}$$

where $0\le\mu_{TL_1}+\nu_{TL_1}\le 1$.

The fuzzy relation $\langle\mu_{TL_2,}\nu_{TL_2}\rangle\ on\ \sigma^*\cup\mu^*\cup\mu^*$ is defined as

$$\langle\mu_{TL_2,}\nu_{TL_2}\rangle=\begin{cases}\langle\mu_2(uv),\gamma_2(uv)\rangle & if\ u,v\ \in\sigma^*\\ \langle\mu_2(e_i)\wedge\mu_2(e_j),\nu_2(e_i)\vee\nu_2(e_j)\rangle & if\ the\ edge\ e_i and\ e_j\ have\ a\ vertex\ in\ common\ between\ them\\ \langle\mu_1(u_i)\wedge\mu_2(e_j),\nu_1(u_i)\vee\nu_2(e_i)\rangle & if\ u_i\in\sigma^*\ and\ e_i\in\mu^*\ and\ each\ e_i\ is\ incident\ with\ sigle\ u_i\\ & either\ clockwise\ or\ anticlockwise\\ 0 & otherwise\end{cases}$$

By definition $0\le\mu_2(uv)+\nu_2(uv)\le 1$ for all (u,v) in $\sigma^*\cup\mu^*\cup\mu^*$. Here $\langle\mu_{TL_2,}\nu_{TL_2}\rangle$ is a fuzzy relation on the fuzzy subset $\langle\mu_{TL_1,}\nu_{TL_1}\rangle$

Example 3.1

Consider the Intuitionistic fuzzy graph $G:(\sigma,\mu)$ with n=3 vertices whose crisp graph is a cycle.

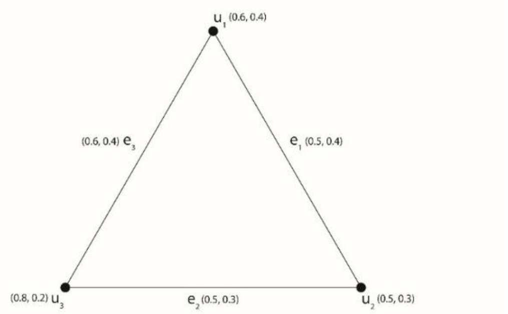


Fig.1. Intuitionstic Fuzzy Graph $G:(\sigma,\mu)$

Then the Intuitionistic Triple Layered Fuzzy graph is given by

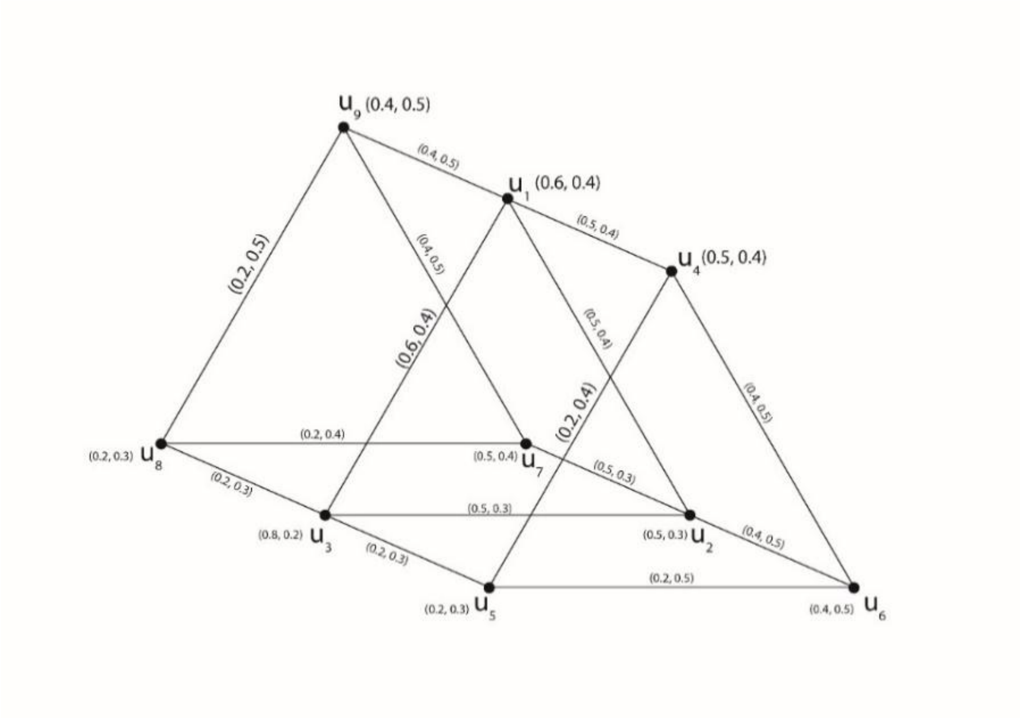


Fig.2. ITLFG $TL(G): (\sigma_{TL}, \mu_{TL})$

Remark:3.1. For each value of n we can get different ITLFG.

IV. DEGREE OF VERTEX IN CARTESIAN PRODUCT OF ITLFGS

In this section we introduced Cartesian product of ITLFG.

Definition:4.1

Let $G_1: \langle (v_i, \mu_1, \nu_1), (e_{ij}, \mu_2, \nu_2) \rangle$ and $G_2: \langle (v_i, \mu_1', \nu_1'), (e_{ij}, \mu_2', \nu_2') \rangle$ be two ITLFGs. Then the vertex degree of Cartesian product of G1 and G2 is defined by

$$d_{G_1 \times G_2}(u_1, v_1) = \left\{ \begin{matrix} \sum_{(u_1,u_2)(v_1,v_2) \in E} (\mu_2 \times \mu_2')(u_1, u_2)(v_1, v_2), \\ \sum_{(u_1,u_2)(v_1,v_2) \in E} (\nu_2 \times \nu_2')(u_1, u_2)(v_1, v_2) \end{matrix} \right\}$$

$$= \begin{Bmatrix} \sum_{u_1=v_1,(u_2,v_2)\in E_1} \mu_1(u_1)\wedge\mu_2'(u_2,v_2), \\ \sum_{u_1=v_1,(u_2,v_2)\in E_1} \nu_1(u_1)\vee\nu_2'(u_2,v_2) \end{Bmatrix} + \begin{Bmatrix} \sum_{u_2=v_2,(u_1,v_1)\in E_1} \mu_1{}'(u_2)\wedge\mu_2(u_1,v_1), \\ \sum_{u_2=v_2,(u_1,v_1)\in E_1} \nu_1{}'(u_2)\vee\nu_2(u_1,v_1) \end{Bmatrix}$$

$$d_{G_1\times G_2}(u_1,v_1) = \begin{Bmatrix} \sum_{u_1=v_1,(u_2,v_2)\in E_1} \mu_1(u_1)\wedge\mu_2'(u_2,v_2), \\ \sum_{u_1=v_1,(u_2,v_2)\in E_1} \nu_1(u_1)\vee\nu_2'(u_2,v_2) \end{Bmatrix} + \begin{Bmatrix} \sum_{u_2=v_2,(u_1,v_1)\in E_1} \mu_1{}'(u_2)\wedge\mu_2(u_1,v_1), \\ \sum_{u_2=v_2,(u_1,v_1)\in E_1} \nu_1{}'(u_2)\vee\nu_2(u_1,v_1) \end{Bmatrix}$$

THEOREM: 4.1

The order of Intuitionistic Triple Layered Fuzzy graph is equal to the sum of the order and size of the intuitionistic simple graph.

Proof :

The vertex set of ITL(G) is $(\sigma^* \cup \mu^* \cup \mu^*)$ and the fuzzy subset $\langle \mu_{TL_1}, \nu_{TL_1} \rangle$ of $\sigma^* \cup \mu^* \cup \mu^*$ is defined as,

$$\langle \mu_{TL_1}, \nu_{TL_1} \rangle = \begin{cases} \langle \mu_1(u), \nu_1(u) \rangle & if\ u \in \sigma^* \\ \langle \mu_2(uv), \nu_2(uv) \rangle & if\ uv \in \mu^* \end{cases}$$

$$Order(G) = \sum_{u\,\in\, V\cup E\cup E} \langle \mu_{TL_1}(u), \nu_{TL_1}(u) \rangle$$

$$= \sum_{u\in v} \sigma_{TL}(u) + \sum_{u\in E} \sigma_{TL}(u) + \sum_{u\in E} \sigma_{TL}(u)$$

$$= \sum_{u\in V} \langle \mu_{TL_1}(u), \nu_{TL_1}(u) \rangle + \sum_{u\in E} \langle \mu_{TL_2}(u), \nu_{TL_2}(u) \rangle$$

$$+ \sum_{u\in E} \langle \mu_{TL_2}(u), \nu_{TL_2}(u) \rangle$$

$$= Order(G) + Size(G) + Size(G)$$

$$Order\ ITL(G) = Order(G) + 2Size(G)$$

THEOREM: 4.2

The vertex degree of Cartesian product of Intuitionistic Triple Layered Fuzzy graph is equal to the sum of the vertex degree of two Intuitionistic Triple Layered Fuzzy graph.

(i.e) Let $G_1: \langle (v_i, \mu_1, \nu_1), (e_{ij}, \mu_2, \nu_2) \rangle$ and $G_2: \langle (v_i, {\mu_1}', {\nu_1}'), (e_{ij}, {\mu_2}', {\nu_2}') \rangle$ be two ITLFGs. If $\mu_1 \geq \mu_2', \nu_1 \leq \nu_2'\ and\ \mu_1' \geq \mu_2, \nu_1' \leq \nu_2$, then $d_{G_1 \times G_2}(u_1, u_2) = d_{G_1}(u_1) + d_{G_2}(u_2)$.

Proof:

Let $G_1: \langle (v_i, \mu_1, \nu_1), (e_{ij}, \mu_2, \nu_2) \rangle$ be an ITLFG.

Then the vertex degree of G_1 is $d_{G_1}(v) = \begin{Bmatrix} \sum_{v \in E_1} \mu_2(v), \\ \sum_{v \in E_1} \nu_2(v) \end{Bmatrix}$

Let $G_2: \langle (v_i, {\mu_1}', {\nu_1}'), (e_{ij}, {\mu_2}', {\nu_2}') \rangle\ be\ an$ ITLFG.

And the vertex degree of G_2 is $d_{G_2}(v) = \begin{Bmatrix} \sum_{v \in E_1} {\mu_2}'(v), \\ \sum_{v \in E_1} {\nu_2}'(v) \end{Bmatrix}$

The Vertex degree of Cartesian product of two ITLFG is,

$$d_{G_1 \times G_2} = \begin{cases} \displaystyle\sum_{(u_1,u_2)(v_1,v_2) \in E} (\mu_2 \times \mu_2')(u_1, u_2)(v_1, v_2), \\ \displaystyle\sum_{(u_1,u_2)(v_1,v_2) \in E} (\nu_2 \times \nu_2')(u_1, u_2)(v_1, v_2) \end{cases}$$

$$= \begin{Bmatrix} \sum_{u_1 = v_1, (u_2, v_2) \in E_1} \mu_1(u_1) \wedge \mu_2'(u_2, v_2), \\ \sum_{u_1 = v_1, (u_2, v_2) \in E_1} \nu_1(u_1) \vee \nu_2'(u_2, v_2) \end{Bmatrix}$$

$$+ \begin{Bmatrix} \displaystyle\sum_{u_2 = v_2, (u_1, v_1) \in E_1} {\mu_1}'(u_2) \wedge \mu_2(u_1, v_1), \\ \displaystyle\sum_{u_2 = v_2, (u_1, v_1) \in E_1} {\nu_1}'(u_1) \vee \nu_2(u_1, v_1) \end{Bmatrix}$$

$$= \begin{Bmatrix} \sum_{(u_2,v_2)\in E_1} \mu_2'(u_2, v_2), \\ \sum_{(u_2,v_2)\in E_1} \nu_2'(u_2, v_2) \end{Bmatrix} + \begin{Bmatrix} \sum_{(u_1,v_1)\in E_1} \mu_2(u_1, v_1), \\ \sum_{(u_1,v_1)\in E_1} \nu_2(u_1, v_1) \end{Bmatrix}$$

$$= d_{G_2}(u_2) + d_{G_1}(u_1)$$

$$d_{G_1\times G_2} = d_{G_2}(u_2) + d_{G_1}(u_1).$$

We can illustrate the proof of this theorem with the following example.

Example:4.1

Consider the Intuitionistic fuzzy graph G_1 with vertices u_1 = (0.6,0.4), u_2 = (0.8,0.2), u_3 =(0.5,0.3); and edges u_1u_2 = (0.5,0.4), u_2u_3 = (0.2,0.3) and u_3u_1 = (0.4,0.3).

Also G_2 with vertices v_1 = (0.8,0.2), v_2 = (0.6,0.3), v_3 = (0.5,0.4) v_4 =(0.7,0.1); and edges v_1v_2 = (0.5,0.3), v_2v_3 = (0.2,0.4), v_3v_4 = (0.3,0.3) and v_4v_1 = (0.4,0.2).

The ITLFG of G_1 is given by,

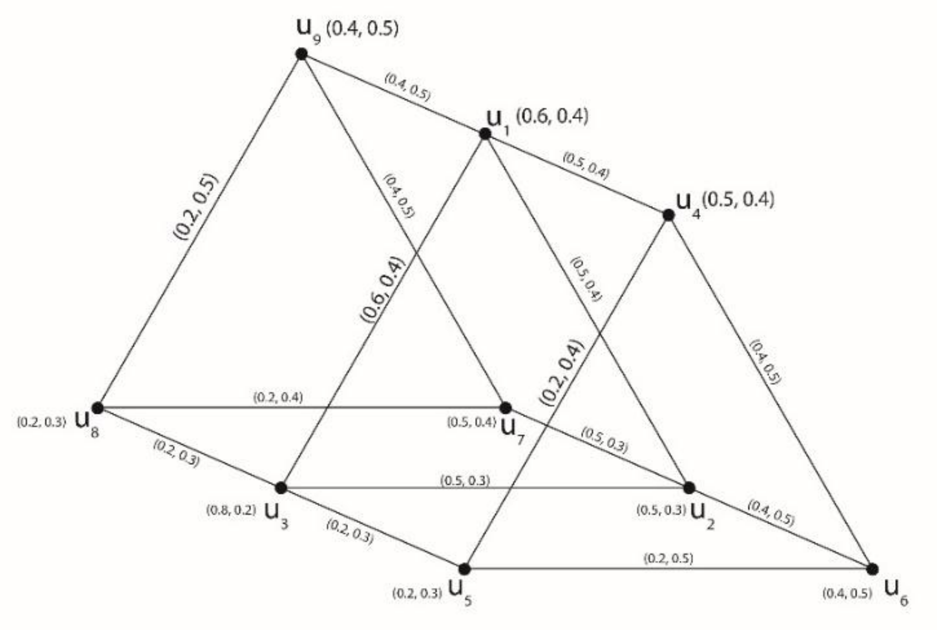


Fig 1. ITL(G_1)

The ITLFG of G_2 is given by,

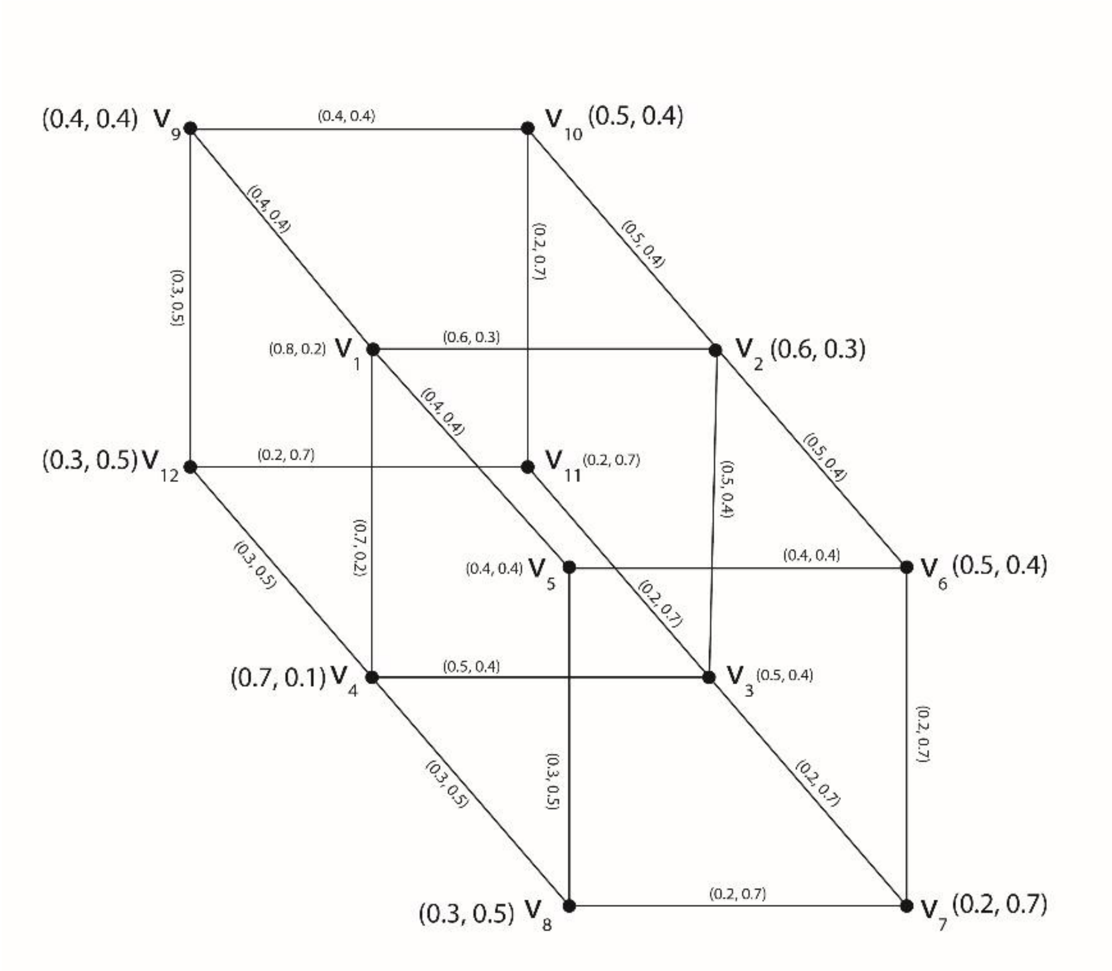


Fig. 2. ITL(G_2)

The Cartesian product of two triple layered fuzzy graph is given by

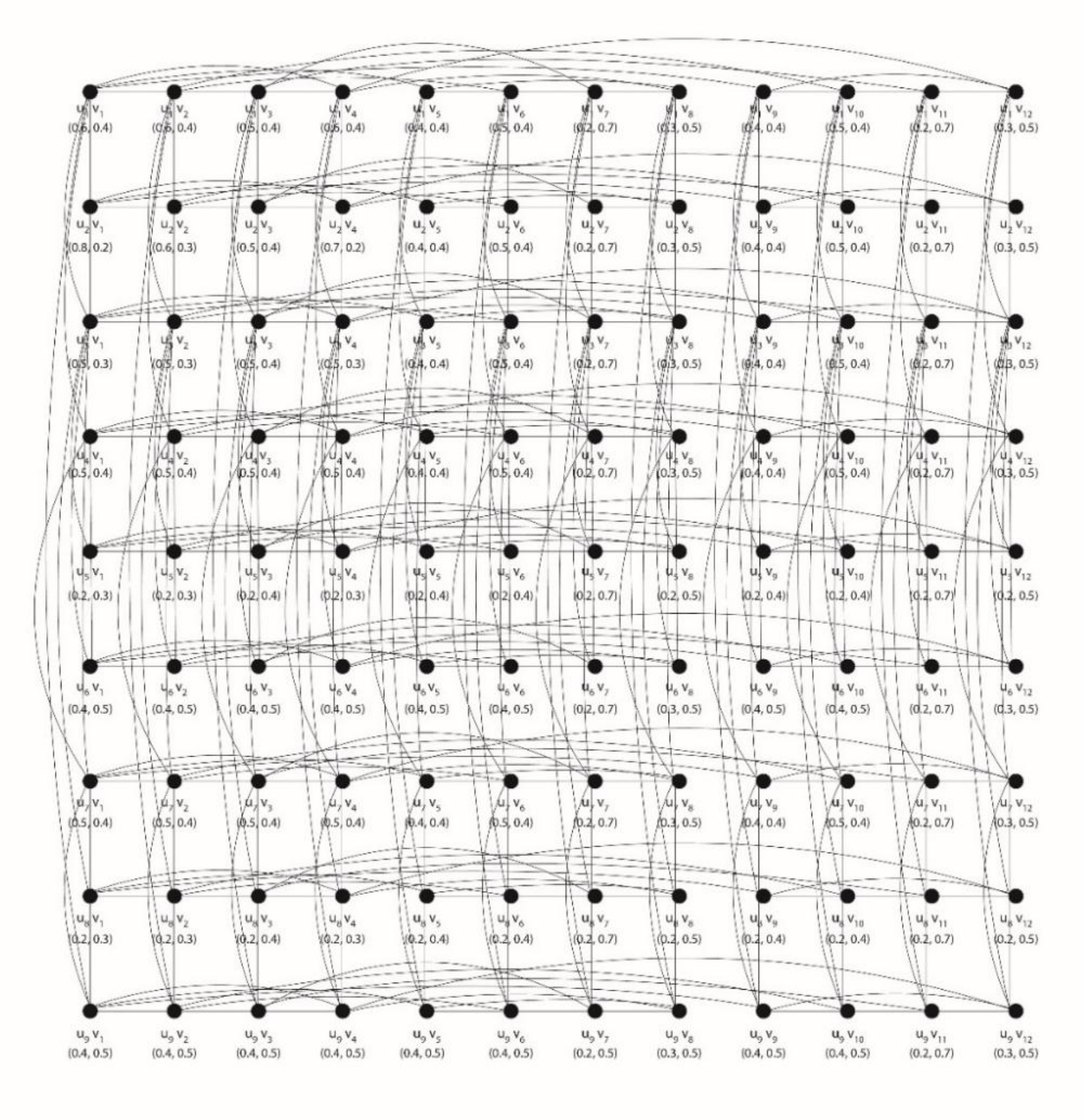
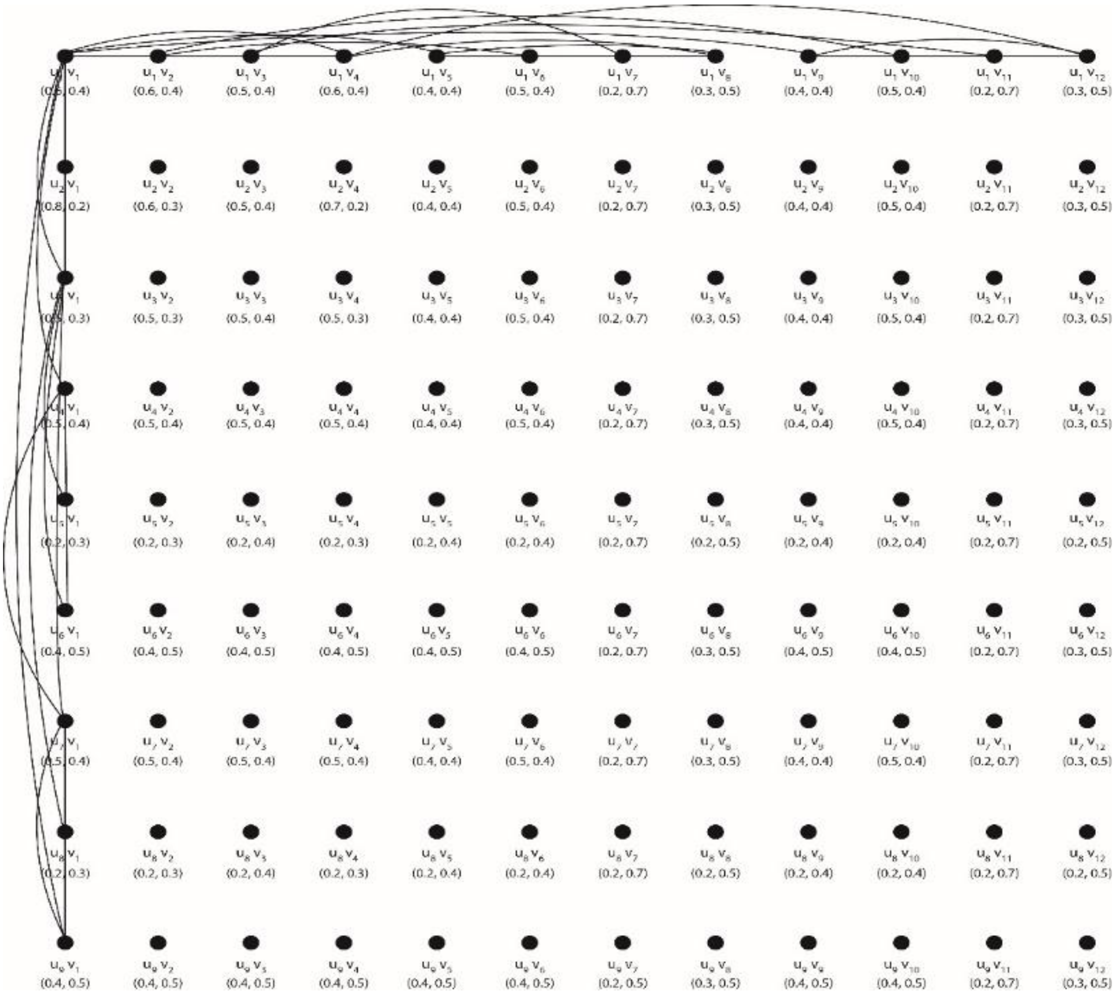


Fig. 3 ITL(G_1) × ITL(G_2)

Enlargement of u_1v_1 and its associated vertices and edges are given in following figure for more clarity.



Here $d_{G_1 \times G_2}(u_1, V_2) = (4.1,3.0)$

It is verified from the above graph that finding the vertex degree of Cartesian product of two ITLFG is a complicated one. Since the two ITLFG satisfies the conditions$\mu_1 \geq \mu_2', \nu_1 \leq \nu_2'$ $and\ \mu_1' \geq \mu_2, \nu_1' \leq \nu_2$ by theorem 4.2, it is verified that $d_{G_1 \times G_2}(u_1, v_2) = (4.1,3.0)$ which is equal to $d_{G_1}(u_1) + d_{G_2}(v_2) = (2.0,1.7) + (2.1,1.3)$

= (4.1, 3.0)

$$d_{G_1 \times G_2}(u_1, v_2) = d_{G_1}(u_1) + d_{G_2}(v_2)$$

Remark:

The Intuitionistic Triple Layered Fuzzy graph is applicable for only connected cyclic graph except tree.

V. CONCLUSION

In this paper the Intuitionistic Triple Layered Fuzzy Graph is defined and its Cartesian product of vertex degree is found under certain conditions and illustrate with some example. This work can be extended to any other simple Intuitionistic Triple Layered Fuzzy graph.

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