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Fuzzy type-2 in Shortest Path and Maximal Flow Problems

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Abstract

In this paper, we introduce general conceptions of fuzzy set type-2, and implementing it in the field of networks, firstly with shortest path and secondly, with maximal flow problems, when the arc (route) between two nodes is fuzzy type-2 where the intersection point of two sets is fuzzy, to illustrate the core of this paper provide it by two examples, with shortest path problem determine the optimal route from node source to sink through the fuzzy indices, on the other side, the second example is maximal flow problem.

Keywords: Fuzzy set type-2, linear and mathematical programming, shortest path, maximal flow.

1-INTRODUCTION

The basic concept of fuzzy type-2 presented by scientist Zadeh in 1975 [1], as an extension of the concept of ordinary fuzzy set, it characterized by membership function, which is a degree of belong each element of this set in interval $[0,1]$ in contrast to fuzzy set type-1, where the degree of membership numbers are not fuzzy in the period $[0,1]$, these sets can be used in cases where the uncertainty related to the degree of membership itself, This does not mean we need to cause the highly fuzzy in order to use this type of fuzzy, the fuzzy type-2 relationship is a new method to increase fuzzing or ambiguity of the relationship, the fuzzy relationship of the second type can be as E. Hisdal [2] described as "increasing uncertainty in the description, it means increased capability to deal with accurate information correctly and logically ", in 2012, Dinagar and Anbalagan [3] Using the principle of extension in the study of arithmetic of fuzzy numbers.

2- THE GENERAL CONCEPTS OF FUZZY SET TYPE-2

It is possible to denote the fuzzy set type-2 by $\{\tilde{A}\}$, and membership function $\{\mu_{\tilde{A}}(x,w)\}$, where $\{x \in X\}$ and $\{w \in K_x \subseteq [0,1]\}$, that is:

$$\tilde{A} = \{((x, w), \ \mu_{\tilde{A}}(x, w)) \mid \forall x \in X, \forall w \in K_x \subseteq [0,1]\}, \text{ as } \{0 \le \mu_{\tilde{A}}(x, w) \le 1\} \tag{1}$$

Also the following mathematical expression

$$\tilde{A} = \left\{ \int_{x \in X} \int_{w \in K_x} \left(\mu_{\tilde{A}}(x,w) / (x,w) \right), \ K_x \subseteq [0,1] \right\} \tag{2}$$

Where $\{\int\int\}$ represented the union of all feasible variables $\{w, x\}$**.** The uncertainty of firstly membership of fuzzy type-2 $\{\tilde{A}\}$ is consisting of a restricted area which we call it the fingerprint of uncertainty (FOU), it is a union of all primary membership that is $\left\{ FOU(\tilde{A}) = \int_{x \in X} K_x \right\}$ it can also be described (FOU) as the upper limits and minimum of membership function where,

$$\left\{ FOU(\tilde{A}) = \int_{x \in X} \left[\mu_{\tilde{A}}(x)^l, \ \mu_{\tilde{A}}(x)^u \right] \right\} \tag{3}$$

3. FUZZY SET TYPE-2 OF THE INDICES OF NETWORKS

Suppose that on some set X are defined crisp sets $F_i, i \in M$, where $M = \{1,...,m\}$ - set of indices. Also, let on the set of indices M given a fuzzy set $\tilde{M}$ with a membership function $\mu(i), i \in M$. Define the concept $\tilde{F} = \bigcap_{i \in \tilde{M}} F_i$ is the intersection of fuzzy sets $\tilde{M}$

crisp sets $F_i, i \in M$ for this, on the set X for $i \in M$ we define the membership function (characteristic function) of a crisp set F_i which we denote by [9]:

$$\phi_i(x) = \begin{cases} 1, & x \in F_i, \\ 0, & x \notin F_i. \end{cases} \tag{4}$$

For any $x \in X$ we introduce the dominance relation on the set of indices M. We say that a set with index $i \in M$ dominates the set with the index $j \in M$ for $x \in X$ and denoted by $i \overset{x}{\succ} j$, if such inequalities are valid $\phi_i(x) \le \phi_j(x), \mu(i) \ge \mu(j)$

And; at least one of them is strict, we denoted by:

$$\tilde{\mu}(x,i) = \begin{cases} \mu(i), & i \in PO(x), \\ 0, & i \notin PO(x), \end{cases} \quad where \quad PO(x) = \{i \in M \mid j \overset{x}{\nsucc} i \quad \forall j \in M\}$$

(5)

Intersection $\tilde{F} = \bigcap_{i \in \tilde{M}} F_i$ fuzzy set $\tilde{M}$, crisp sets $F_i, i \in M$, we shall call a fuzzy set type-2, this is given by triples $(x, \psi(x,y))$ where $\psi : X \times Y \to \{0,1\}$ - fuzzy mapping, performed as a fuzzy membership function and defined as :

$$\psi(x,y) = \max_{i \in M}\{\tilde{\mu}(x,i) \mid \phi_i(x) = y\} \tag{6}$$

If $\exists i \in M \;\; \phi_i(x) = y$;

If $\forall i \in M \;\; \phi_i(x) \ne y$

x - element of set X;

$y -$ element of universal set,

Value of membership mapping $\psi(x,y)$ fuzzy set type-2 $\tilde{F}$. $Y = \{0,1\}$

The problems of mathematical programming with a fuzzy set of connection indexes with properties which are illustrated in the following:

Select number $\xi \in (0,1)$, which characterizes the minimum reliability of membership of the arc network plan for decision maker.

Construct a set of communication routes indices $S^{\xi} = \{(i,j) \in S \mid \xi \le s(i,j)\}$ which have the reliability of membership of arc network plan at least a given number $\xi \in (0,1)$ but not equal to one;

- For each path an index $(k,l) \in S^{\xi}$ minimizes the cost of transportation (either with shortest path, or with maximal flow) $g(x)$ on the set of allowable arc network plans D with additional constraints $x_{ij} > 0,\ (i,j) \in S^*$ it is necessary to share the paths connecting of the network and add $x_{kl} \le 0$

Another constraint which prohibits to the chosen route with an index (k,l) , while solves the task.

$$\min_{x \in X} g(x),\ x_{kl} \le 0,\ x_{ij} > 0,\ \ (i,j) \in S^* \tag{7}$$

its solution is denoted by $x^{(kl)}$, from the solutions $x^{(kl)}, (k,l) \in S^{\xi}$, choose a record $\hat{x}$ by the value of the objective function.

$$\hat{x} = \arg \min_{(k,l) \in S^{\xi}} g(x^{(kl)}) \tag{8}$$

4. SHORTEST PATH PROBLEM

The shortest path problem is a mathematical form that derived from transportation model. The purpose is to find the shortest path between a single source and a single destination. We need only to recognize which to supply one at the origin node while the other as demand at the destination node.

All the intermediate nodes have zero demand and supply. The lengths of the arcs are holds as costs. The shortest path from the source to the sink is then determined by the arcs that carry a nonzero flow in the optimal solution of the transshipment model [5].

5- A MATHEMATICAL FORMULATION OF THE SHORTEST PATH PROBLEM

We hope to send a one unit of flow commodity from node (1) to node (m) with minimum cost, depending on the fuzzy set of indices of arcs [6], the corresponding mathematical formulation becomes:

$$\left.\begin{array}{l} Min\ z = \sum_{i=1}^{m}\sum_{j=1}^{m} c_{ij}x_{ij} \\ s.t. \\ \sum_{j=1}^{m} x_{ij} - \sum_{k=1}^{m} x_{ki} = \begin{cases} 1 & if\ i=1 \\ 0 & if\ i \neq 1\ or\ m \\ -1 & if\ i = m \end{cases} \\ x_{ij} \geq 0 \quad , i, j = 1,\dots,m. \end{array}\right\} \tag{9}$$

6. Finding the shortest path problem of fuzzy type-2 (proposal technique)

To show the proposal technique let's take the following example.

Example (6-1): Consider the problem of finding the shortest path of fuzzy type 2 by using the proposed network algorithm shown below [7].

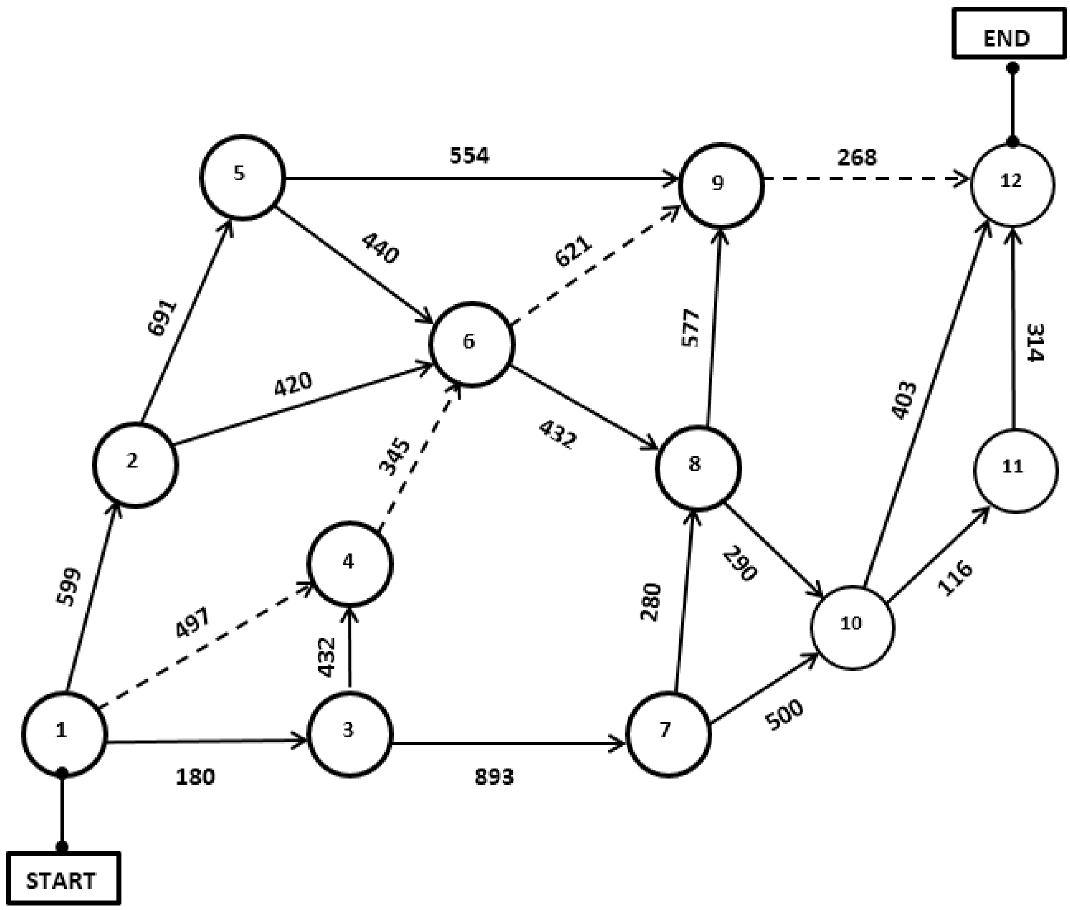

Figure (6-1) Represented network of lines travel from node 1 to node 12.

Firstly, to solve the network above by using the fuzzy type-2 and stay away from the traditional techniques, and to find the shortest path, we formulate a mathematical model for the network as follows:

$$\begin{aligned} Min\ z(x) = {} & 599x_{1,2} + 180x_{1,3} + 497x_{1,4} + 691x_{2,5} + 420x_{2,6} + 432x_{3,4} + 893x_{3,7} + 345x_{4,6} + \\ & 440x_{5,6} + 554x_{5,9} + 432x_{6,8} + 621x_{6,9} + 280x_{7,8} + 500x_{7,10} + 577x_{8,9} + 290x_{8,10} + \\ & 268x_{9,12} + 116x_{10,11} + 403x_{10,12} + 314x_{11,12} \end{aligned}$$

$S.To:$

$$\left.\begin{array}{l} x_{1,2} + x_{1,3} + x_{1,4} = 1,\ x_{2,5} + x_{2,6} - x_{1,2} = 0,\ x_{3,4} + x_{3,7} - x_{1,3} = 0,\ x_{4,6} - x_{1,4} - x_{3,4} = 0; \\ x_{5,6} + x_{5,9} - x_{2,5} = 0,\ x_{6,8} + x_{6,9} - x_{2,6} - x_{4,6} - x_{5,6} = 0,\ \ x_{7,8} + x_{7,10} - x_{3,7} = 0; \\ x_{8,9} + x_{8,10} - x_{6,8} - x_{7,8} = 0,\ x_{9,12} - x_{5,9} - x_{6,9} - x_{8,9} = 0,\ x_{10,11} + x_{10,12} - x_{7,10} - x_{8,10} = 0; \\ x_{11,12} - x_{10,11} = 0,\ x_{9,12} + x_{10,12} + x_{11,12} = 1;\ x_{ij} \ge 0;\ i = \overline{1,11};\ j = \overline{2,12}\,. \end{array}\right\} \quad (10)$$

The set $I = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$ represented the start of path for nodes of network, the set $J = \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$ represented the end paths for nodes of network, the set $\{arc\}$ represent all paths of network $arc = \{(i, j) \mid i \in I,\ j \in J\}$.

Suppose the decision maker cannot be clearly determine which paths will be effective, but can only give a set of fuzzy indicators or indexes $\{arc\}$ with membership function $\{arc(i, j) = 1\}$ for all $\{I, J\}$, except $\{arc(1,4) = 0.2,\ \ arc(4,6) = 0.1,\ arc(6,9) = 0.8,\ arc(9,12) = 0.7,\}$

Now we will apply the proposed algorithm to find the shortest path of fuzzy type-2 as shown below:

- We choose the greatest reliability for the unacceptable solution of the above network, let it be $(\mu = 0.8)$.
- Determine the set of indicators for paths that have a fuzzy membership degree $\{arc\}$ no more than $(\mu = 0.8)$, which take the form $\{arc^{\mu} = \{(i, j) \in arc \mid\ arc(i, j) \le \mu\} = \{(1,4), (4,6), (6,9), (9,12)\}\}$.

  For $\{(i,\ j) = (1,4)\}$, we solve the following model:

$$\left.\begin{array}{l} Min\ z(x) = 599x_{1,2} + 180x_{1,3} + 497x_{1,4} + 691x_{2,5} + 420x_{2,6} + 432x_{3,4} + 893x_{3,7} + 345x_{4,6} + \\ \qquad 440x_{5,6} + 554x_{5,9} + 432x_{6,8} + 621x_{6,9} + 280x_{7,8} + 500x_{7,10} + 577x_{8,9} + 290x_{8,10} + \\ \qquad 268x_{9,12} + 116x_{10,11} + 403x_{10,12} + 314x_{11,12} \\ S.To: \\ \quad x_{1,2} + x_{1,3} + x_{1,4} = 1,\ x_{2,5} + x_{2,6} - x_{1,2} = 0,\ x_{3,4} + x_{3,7} - x_{1,3} = 0,\ x_{4,6} - x_{1,4} - x_{3,4} = 0; \\ \quad x_{5,6} + x_{5,9} - x_{2,5} = 0,\ x_{6,8} + x_{6,9} - x_{2,6} - x_{4,6} - x_{5,6} = 0,\ \ x_{7,8} + x_{7,10} - x_{3,7} = 0; \\ \quad x_{8,9} + x_{8,10} - x_{6,8} - x_{7,8} = 0,\ x_{9,12} - x_{5,9} - x_{6,9} - x_{8,9} = 0,\ x_{10,11} + x_{10,12} - x_{7,10} - x_{8,10} = 0; \\ \quad x_{11,12} - x_{10,11} = 0,\ x_{9,12} + x_{10,12} + x_{11,12} = 1,\ x_{1,4} \le 0,\ x_{ij} > 0,\ (i,j) \in \left\{arc \setminus arc^{\mu}\right\}; \\ \quad x_{ij} \ge 0;\ i = \overline{1,11};\ j = \overline{2,12}. \end{array}\right\} \quad (11)$$

The optimal path of network above with objective function $z(x^{(1,4)}) = 1846$ is: $\{1 \to 3 \to 4 \to 6 \to 9 \to 12\}$.

Note, that when finding the optimal solution of the constraint $x_{ij} > 0,\ (i,j) \in \left\{arc \setminus arc^{\mu}\right\}$, were given as $x_{ij} \ge \rho,\ (i,j) \in arc \setminus arc^{\mu}$, where $\rho > 0$, some "small" number which was selected empirically, Therefore here and further, the values obtained in the optimal solution of the order ρ were written as zeros.

For $\{(i,\ j) = (4,6)\}$, we solve the following model:

$$\left.\begin{array}{l} Min\ z(x) = 599x_{1,2} + 180x_{1,3} + 497x_{1,4} + 691x_{2,5} + 420x_{2,6} + 432x_{3,4} + 893x_{3,7} + 345x_{4,6} + \\ \qquad 440x_{5,6} + 554x_{5,9} + 432x_{6,8} + 621x_{6,9} + 280x_{7,8} + 500x_{7,10} + 577x_{8,9} + 290x_{8,10} + \\ \qquad 268x_{9,12} + 116x_{10,11} + 403x_{10,12} + 314x_{11,12} \\ S.To: \\ \quad x_{1,2} + x_{1,3} + x_{1,4} = 1,\ x_{2,5} + x_{2,6} - x_{1,2} = 0,\ x_{3,4} + x_{3,7} - x_{1,3} = 0,\ x_{4,6} - x_{1,4} - x_{3,4} = 0; \\ \quad x_{5,6} + x_{5,9} - x_{2,5} = 0,\ x_{6,8} + x_{6,9} - x_{2,6} - x_{4,6} - x_{5,6} = 0,\ \ x_{7,8} + x_{7,10} - x_{3,7} = 0; \\ \quad x_{8,9} + x_{8,10} - x_{6,8} - x_{7,8} = 0,\ x_{9,12} - x_{5,9} - x_{6,9} - x_{8,9} = 0,\ x_{10,11} + x_{10,12} - x_{7,10} - x_{8,10} = 0; \\ \quad x_{11,12} - x_{10,11} = 0,\ x_{9,12} + x_{10,12} + x_{11,12} = 1,\ x_{4,6} \le 0,\ x_{ij} > 0,\ (i,j) \in \left\{arc \setminus arc^{\mu}\right\}; \\ \quad x_{ij} \ge 0;\ i = \overline{1,11};\ j = \overline{2,12}. \end{array}\right\} \quad (12)$$

The optimal path of network above with objective function $z\,(x^{\,(4,6)}) = 1908$ is: $\{1 \to 2 \to 6 \to 9 \to 12\}$.

For $\{(i,\ j) = (6,9)\}$, we solve the following model:

$$\left.\begin{array}{l} Min\ z(x) = 599x_{1,2} + 180x_{1,3} + 497x_{1,4} + 691x_{2,5} + 420x_{2,6} + 432x_{3,4} + 893x_{3,7} + 345x_{4,6} + \\ \qquad 440x_{5,6} + 554x_{5,9} + 432x_{6,8} + 621x_{6,9} + 280x_{7,8} + 500x_{7,10} + 577x_{8,9} + 290x_{8,10} + \\ \qquad 268x_{9,12} + 116x_{10,11} + 403x_{10,12} + 314x_{11,12} \\ S.To: \\ \quad x_{1,2} + x_{1,3} + x_{1,4} = 1,\ x_{2,5} + x_{2,6} - x_{1,2} = 0,\ x_{3,4} + x_{3,7} - x_{1,3} = 0,\ x_{4,6} - x_{1,4} - x_{3,4} = 0; \\ \quad x_{5,6} + x_{5,9} - x_{2,5} = 0,\ x_{6,8} + x_{6,9} - x_{2,6} - x_{4,6} - x_{5,6} = 0,\ \ x_{7,8} + x_{7,10} - x_{3,7} = 0; \\ \quad x_{8,9} + x_{8,10} - x_{6,8} - x_{7,8} = 0,\ x_{9,12} - x_{5,9} - x_{6,9} - x_{8,9} = 0,\ x_{10,11} + x_{10,12} - x_{7,10} - x_{8,10} = 0; \\ \quad x_{11,12} - x_{10,11} = 0,\ x_{9,12} + x_{10,12} + x_{11,12} = 1,\ x_{69} \le 0,\ x_{ij} > 0,\ (i,j) \in \left\{arc \setminus arc^{\mu}\right\}; \\ \quad x_{ij} \ge 0;\ i = \overline{1,11};\ j = \overline{2,12}. \end{array}\right\} \quad (13)$$

The optimal path of network above with objective function $z(x^{(6,9)}) = 1967$ is: $\{1 \to 4 \to 6 \to 8 \to 10 \to 12\}$

By the optimal solutions above note that the optimal path is $\{1 \to 4 \to 6 \to 8 \to 10 \to 12\}$,For $\{(i,\ j) = (9,12)\}$, we solve the following model:

$$\left.\begin{array}{l} Min\ z(x) = 599x_{1,2} + 180x_{1,3} + 497x_{1,4} + 691x_{2,5} + 420x_{2,6} + 432x_{3,4} + 893x_{3,7} + 345x_{4,6} + \\ \qquad 440x_{5,6} + 554x_{5,9} + 432x_{6,8} + 621x_{6,9} + 280x_{7,8} + 500x_{7,10} + 577x_{8,9} + 290x_{8,10} + \\ \qquad 268x_{9,12} + 116x_{10,11} + 403x_{10,12} + 314x_{11,12} \\ S.To: \\ \quad x_{1,2} + x_{1,3} + x_{1,4} = 1,\ x_{2,5} + x_{2,6} - x_{1,2} = 0,\ x_{3,4} + x_{3,7} - x_{1,3} = 0,\ x_{4,6} - x_{1,4} - x_{3,4} = 0; \\ \quad x_{5,6} + x_{5,9} - x_{2,5} = 0,\ x_{6,8} + x_{6,9} - x_{2,6} - x_{4,6} - x_{5,6} = 0,\ \ x_{7,8} + x_{7,10} - x_{3,7} = 0; \\ \quad x_{8,9} + x_{8,10} - x_{6,8} - x_{7,8} = 0,\ x_{9,12} - x_{5,9} - x_{6,9} - x_{8,9} = 0,\ x_{10,11} + x_{10,12} - x_{7,10} - x_{8,10} = 0; \\ \quad x_{11,12} - x_{10,11} = 0,\ x_{9,12} + x_{10,12} + x_{11,12} = 1,\ x_{9,12} \le 0,\ x_{ij} > 0,\ (i,j) \in \left\{arc \setminus arc^{\mu}\right\}; \\ \quad x_{ij} \ge 0;\ i = \overline{1,11};\ j = \overline{2,12}. \end{array}\right\} \quad (14)$$

The optimal path of network above with objective function $z(x^{(9,12)}) = 2082$ is: $\{1 \to 3 \to 4 \to 6 \to 8 \to 10 \to 12\}$,The minimum objective function $\left\{z(x^{(1,4)}) = 1846\right\}$, with the reliability of acceptable solution is equal to one, and the reliability of unacceptable solution is not greater than (0.8).

7. THE MAXIMAL FLOW PROBLEM

To give a description to the maximal flow problem, consider a network consist of m nodes and n arcs, through which article will flow, conjugation of each arc (i, j) with lower bound of flow $\ell_{ij} = 0$, and upper bound of flow v_{ij}. In such a network, the aim to find the maximum quantity of flow from node 1 to node m.The maximal flow problem can be mathematically formulated as below:

$$\left.\begin{array}{l} \max\ f \\ s.to \\ \sum_{j=1}^{m} x_{ij} - \sum_{k=1}^{m} x_{ki} = \begin{cases} f & if\ i=1 \\ 0 & if\ i \neq 1\ or\ m \\ -f & if\ i=m \end{cases} \\ x_{ij} \geq 0, x_{ij} \leq v_{ij} \quad , i, j = 1, \ldots, m. \end{array}\right\} \qquad (15)$$

8. FINDING THE MAXIMAL FLOW PROBLEM OF FUZZY TYPE-2 (PROPOSAL TECHNIQUE)

We apply the same proposal technique that shows above on the maximal flow problem, by the following example:

Example (8-2): Consider the problem of finding the maximal flow of fuzzy type 2 by using the proposed network algorithm shown below [7].

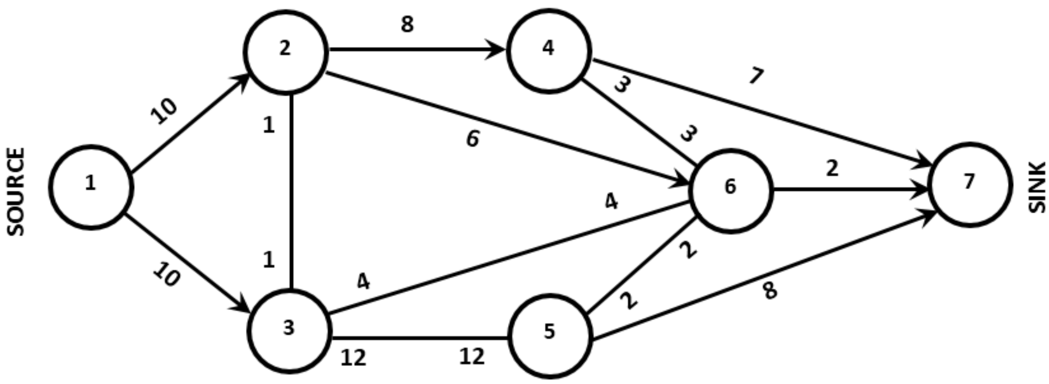

Figure (8-1) Represented network of provide deposit of water

Firstly, to solve the network above by using the fuzzy type-2 and stay away from the traditional techniques, to find the maximal flow, we formulate a mathematical model for the network as follows:

$$\left.\begin{array}{l} Max\ z(x) = x_{1,2} + x_{1,3} \\ S.To: \\ x_{2,3} + x_{2,4} + x_{2,6} - x_{1,2} - x_{3,2} = 0, \\ x_{3,4} + x_{3,7} - x_{1,3} = 0, \\ x_{3,2} + x_{3,5} + x_{3,6} - x_{1,3} - x_{2,3} - x_{6,3} = 0, \\ x_{4,6} + x_{4,7} - x_{2,4} - x_{6,4} = 0, \\ x_{5,6} + x_{5,7} - x_{3,5} - x_{6,5} = 0, \\ x_{6,3} + x_{6,4} + x_{6,5} + x_{6,7} - x_{2,6} - x_{3,6} + x_{4,6} + x_{5,6} = 0, \\ x_{1,2} \le 10,\ x_{1,3} \le 10,\ x_{2,3} \le 1,\ x_{2,4} \le 8,\ x_{2,6} \le 6, \\ x_{3,2} \le 1,\ x_{3,5} \le 12,\ x_{3,6} \le 4,\ x_{4,6} \le 3,\ x_{4,7} \le 7, \\ x_{5,6} \le 2,\ x_{5,7} \le 8,\ x_{6,3} \le 4,\ x_{6,4} \le 3,\ x_{6,5} \le 2,\ x_{6,7} \le 2; \\ x_{ij} \ge 0;\ i = \overline{1,6};\ \ j = \overline{2,7}\ . \end{array}\right\} \tag{16}$$

The set $I = \{1,2,3,4,5,6\}$ represented the start of path for nodes of network, the set $J = \{2,3,4,5,6,7\}$ represented the end paths for nodes of network, the set $\vec{\Omega}$ represented all paths of network $\vec{\Omega} = \{(i,j) \mid i \in I,\ j \in J\}$.

Suppose the decision maker cannot be clearly determined which path will be effective, but can only give a set of fuzzy indicators or indexes $\vec{\Omega}$ with membership function $\vec{\Omega}(i,j) = 1$,for all $\{I,\ J\}$, except:

$\left\{\vec{\Omega}(1,2) = 0.3,\ \vec{\Omega}(2,4) = 0.8,\ \vec{\Omega}(2,6) = 0.6,\ \vec{\Omega}(3,5) = 0.1,\ \vec{\Omega}(5,7) = 0.9,\ \vec{\Omega}(6,7) = 0.4\right\}$. We choose the greatest reliability for unacceptable solution of the above network, let it be $(\delta = 0.9)$.

Determine the set of indicators for paths that have a fuzzy membership degree $\vec{\Omega}$ no more than $(\delta = 0.9)$, which takes the form

$$\vec{\Omega}^{\delta} = \{(i,j) \in \vec{\Omega} \mid \ \vec{\Omega}(i,j) \le \delta\} = \{(1,2),(2,4),(2,6),(3,5),(5,7),(6,7)\}.$$

For $\{(i,\ j) = (1,2)\}$, we solve the following model:

$$\left.\begin{array}{l} Max\ z(x) = x_{1,2} + x_{1,3} \\ S.To:\ x \in X,\ \ x_{1,2} \le 0,\ x_{ij} > 0,\ (i,\ j) \in \left\{ \vec{\Omega} \setminus \vec{\Omega}^{\delta} \right\}. \end{array}\right\} \quad (17)$$

The maximal flow is $z(x^{(1,2)}) = 10$, Note, that when finding the optimal solution of the constraint $x_{ij} > 0,\ (i,\ j) \in \left\{ \vec{\Omega} \setminus \vec{\Omega}^{\delta} \right\}$, were given in the form $x_{ij} \ge \varepsilon,\ (i,\ j) \in \left\{ \vec{\Omega} \setminus \vec{\Omega}^{\delta} \right\}$, where $\varepsilon > 0$, some "small" number which was selected empirically, Therefore here and further, the values obtained in the optimal solution of the order ε were written as zeros.

For $\{(i,\ j) = (2,4)\}$, we solve the following model:

$$\left.\begin{array}{l} Max\ z(x) = x_{12} + x_{13} \\ S.To:\ x \in X,\ \ x_{24} \le 0,\ x_{ij} > 0,\ (i,\ j) \in \left\{ \vec{\Omega} \setminus \vec{\Omega}^{\delta} \right\}. \end{array}\right\} \quad (18)$$

The maximal flow is $z(x^{(2,4)}) = 13$

For $\{(i,\ j) = (2,6)\}$, we solve the following model:

$$\left.\begin{array}{l} Max\ z(x) = x_{12} + x_{13} \\ S.To:\ x \in X,\ \ x_{26} \le 0,\ x_{ij} > 0,\ (i,\ j) \in \left\{ \vec{\Omega} \setminus \vec{\Omega}^{\delta} \right\}. \end{array}\right\} \quad (19)$$

The maximal flow is $z(x^{(2,6)}) = 17$

For $\{(i,\ j) = (3,5)\}$, we solve the following model:

$$\left.\begin{array}{l} Max\ z(x) = x_{12} + x_{13} \\ S.To:\ x \in X,\ \ x_{35} \le 0,\ x_{ij} > 0,\ (i,\ j) \in \left\{ \vec{\Omega} \setminus \vec{\Omega}^{\delta} \right\}. \end{array}\right\} \quad (20)$$

The maximal flow is $z(x^{(3,5)}) = 11$

For $\{(i,\ j) = (5,7)\}$, we solve the following model:

$$\left.\begin{array}{l} Max\ z(x) = x_{12} + x_{13} \\ S.To:\ x \in X,\ \ x_{57} \le 0,\, x_{ij} > 0,\ (i,\, j) \in \left\{ \overrightarrow{\Omega} \setminus \overrightarrow{\Omega}^{\delta} \right\}. \end{array}\right\} \qquad (21)$$

The maximal flow is $z(x^{(5,7)}) = 9$

For $\{(i,\ j) = (6,7)\}$, we solve the following model:

$$\left.\begin{array}{l} Max\ z(x) = x_{12} + x_{13} \\ S.To:\ x \in X,\ \ x_{67} \le 0,\, x_{ij} > 0,\ (i,\, j) \in \left\{ \overrightarrow{\Omega} \setminus \overrightarrow{\Omega}^{\delta} \right\}. \end{array}\right\} \qquad (22)$$

The maximal flow is $z(x^{(6,7)}) = 15$

By the optimal solutions above note that the maximal flow is $z(x^{(2,6)}) = 17$, with the reliability of the acceptable solution equal to one, and the reliability of the unacceptable solution is not greater than (0.9), we depend on package Win-QSB to obtain the optimum solution[8].

9. CONCLUSIONS

In this paper, we introduce an algorithm to find shortest path length and maximal flow of fuzzy type-2 from source node to sink node on a network, In conclusion, it should be noted that the proposed method not only expands the field of application of fuzzy mathematical programming in the case of a network with a fuzzy set of communication paths between indices, but it can also give a new approach to solve other sets of fuzzy optimization problems such as transportation problem when paths among suppliers and consumers are fuzzy set .

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