

On Soft JP Continuous Mappings

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Abstract

In this paper, We introduce a new class of continuous functions called Soft JP continuous functions and discuss their relation with various forms of Soft continuous functions. Further We study the characterizations of Soft JP continuous functions and reveal the impact of Soft JP closure and Soft JP interior in those characterizations. Also We establish Soft JP irresolute and compare it with Soft JP continuous functions.

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INTRODUCTION

In 1999 Molodstove [5] initiated a new mathematical tool called Soft set Theory to eradicate the inadequacy in parametrization the uncertainty problems. Soft set Theory paved a new pathway to involve the parameters in the framework of the problems arisen with uncertainty. Muhammad Shabir and Naz [9] introduced Soft topological spaces. Meanwhile Aras and Sonmez [1] discussed the properties of Soft continuous mappings. In 2016, The authors of this paper [11] paved a new pathway by introducing a new class of generalized closed set called Soft JP closed sets in Soft

topological spaces. This paper is devoted to Soft JP continuous and Soft JP irresolute functions respectively. We can extend these theoretical bases to real world applications like information systems, medical diagnosis etc.

PRELIMINARIES

In this section, we present the basic definitions and results of Soft set theory which may be found in earlier studies. Throughout this work, X refers to an initial universe, E is a set of parameters, $P(X)$ is the power set of X and $A \subseteq E$. Throughout this work $(X, \tau, E), (Y, \sigma, K), (Z, \eta, R)$ are Soft topological spaces, $Cl(A, E), Int(A, E), SCl(A, E), \alpha Cl(A, E)$ means Soft closure, Soft interior, Soft semi closure, Soft α closure of the Soft set (A, E) respectively.

Definition 2.1[10]: Let τ be a collection of Soft sets over X with the fixed set of parameters E . Then τ is called a **Soft Topology** on X if

- i. $\tilde{\phi}, \tilde{X}$ belongs to τ .
- ii. The union of any number of Soft sets in τ belongs to τ .
- iii. The intersection of any two Soft sets in τ belongs to τ .

The triplet (X, τ, E) is called Soft Topological Spaces over X .

The members of X are called Soft Open sets (X, τ, E) in and complements of them are called Soft Closed sets in (X, τ, E) .

Definition 2.2: Let (X, τ, E) be a Soft Topological Space over X . Then the subset (A, E) of the Soft topological space (X, τ, E) is called,

Soft Semi-closed set[2] if $int(Cl(A, E)) \tilde{\subseteq} (A, E)$.

Soft α closed set[6] if $Cl(int(Cl(A, E))) \tilde{\subseteq} (A, E)$.

Soft generalized Closed set (briefly Soft g-Closed)[8] if $Cl(A, E) \tilde{\subseteq} (U, E)$ whenever $(A, E) \tilde{\subseteq} (U, E)$ and (U, E) is Soft Open in (X, τ, E) .

Soft generalized semi Closed set (briefly Soft gs-Closed)[7] if $S Cl(A, E) \tilde{\subseteq} (U, E)$ whenever $(A, E) \tilde{\subseteq} (U, E)$ and (U, E) is Soft Open in (X, τ, E) .

Soft \hat{g} closed set[14] if $Cl(A, E) \tilde{\subseteq} (U, E)$ whenever $(A, E) \tilde{\subseteq} (U, E)$ and (U, E) is Soft semi Open in (X, τ, E) .

Soft JP closed set[11]if $Scl(A,E) \cong Int(U,E)$ whenever $(A,E) \cong (U,E)$ and (U,E) is Soft \hat{g} open in (X,τ,E) .

The respective complements of the above sets are their open forms.

Definition2.3[7]:Let $x \in X$, then \tilde{X} is called the Soft singleton set if $x(\alpha) = x$ for all $\alpha \in E$.

Definition2.4[9]:

Let (A,E) be a Soft subset of the Soft set \tilde{X} . Then the Soft point e_x in \tilde{X} is said to be

- i) Soft interior point of (A,E) if (A,E) contains a Soft open set containing the Soft point \tilde{X} .
- ii) Soft limit point of (A,E) if every Soft open set containing the Soft point \tilde{X} intersects (A,E) in a Soft point different from \tilde{X} .

Definition2.5:[9]

Let (A,E) be a Soft subset of the Soft set \tilde{X} . The set of all limit points of (A,E) is called the derived set of (A,E) and it is denoted by $D[A,E]$.

Definition2.6:[9]Let (X,τ,E) be a Soft Topological Spaces over X . The Soft Interior of (F, E) denoted by $Int(F, E)$ is the union of all Soft open subsets contained in (F,E) . Clearly $Int(F, E)$ is the largest Soft open set over X which is contained in (F, E) .

$$i) SInt(F, E) = \tilde{U}\{(O,E): (O, E) \text{ is Soft open and } (O,E) \cong (F,E)\}.$$

ii) Soft Closure of (F, E) denoted by $Cl(F,E)$ is the intersection of Soft closed sets containing (F, E) . Clearly $Cl(F, E)$ is the smallest Soft closed set containing (F,E) .

$$Cl(F,E) = \{\tilde{\cap} (O, E): (O,E) \text{ is Soft closed and } (F, E) \cong (O,E)\}.$$

Definition2.7:[12]

Let (X,τ,E) be a Soft Topological Spaces over X . Then

- i) The **Soft JP Interior** of (F, E) denoted by $JPInt(F, E)$ is the union of all Soft JP open subsets contained in (F,E) .

$$JPInt(F, E) = \{\tilde{U}\{(O,E): (O, E) \text{ is Soft JP open and } (O,E) \cong (F,E)\}\}.$$

- ii) The **Soft JP Closure** of (F, E) denoted by $JPCl(F, E)$ is the intersection of Soft JP closed sets containing (F, E) .

$$JPCl(F, E) = \bigcap \{(O, E) : (O, E) \text{ is Soft JP closed and } (F, E) \subseteq (O, E)\}.$$

Definition 2.8:[13]

A Soft topological space (X, τ, E) is said to be a Soft T_{JP} space if every Soft JP closed set is Soft closed.

Definition 2.9:[13]

A Soft topological space (X, τ, E) is said to be a Soft sT_{JP} space if every Soft JP closed set is Soft semi closed.

Definition 2.10:[13]

A Soft topological space (X, τ, E) is said to be a Soft αT_{JP} space if every Soft JP closed set is Soft α closed.

Definition 2.11: A map $f: (X, \tau, E) \rightarrow (Y, \sigma, K)$ is said to be

1. Soft continuous [3] if inverse image of every Soft open set in (Y, σ, K) is Soft open in (X, τ, E)
2. Soft semi continuous [3] if inverse image of every Soft open set in (Y, σ, K) is Soft semi open in (X, τ, E)
3. Soft α continuous [4] if inverse image of every Soft open set in (Y, σ, K) is Soft α open in (X, τ, E)
4. Soft generalized semi (gs) continuous [7] if inverse image of every Soft open set in (Y, σ, K) is Soft gs open in (X, τ, E) .

III. SOFT JP CONTINUOUS FUNCTIONS:

Definition 3.1: A map $f: (X, \tau, E) \rightarrow (Y, \sigma, K)$ is said to be **Soft JP continuous** if inverse image of every Soft closed set in (Y, σ, K) is Soft JP closed in (X, τ, E) .

Proposition 3.2:

The map $f: (X, \tau, E) \rightarrow (Y, \sigma, K)$ is said to be Soft JP continuous if inverse image of every Soft open set in (Y, σ, K) is Soft JP open in (X, τ, E) .

Proof: Let $f: (X, \tau, E) \rightarrow (Y, \sigma, K)$ be Soft JP Continuous and (G, K) be a Soft open set in the Soft topological space (Y, σ, K) . Then $f^{-1}(G, K)^C$ is Soft JP closed in (X, τ, E) and so $f^{-1}(G, K)$ is Soft JP open set in (X, τ, E) .

Proposition 3.3:

If $f: (X, \tau, E) \rightarrow (Y, \sigma, K)$ is Soft continuous then it is Soft JP continuous .

Proof: The proof follows from the fact that every Soft open set is a Soft JP open set.

Remark 3.4:

The converse of the above Proposition is not true and it can be seen from the following example.

$X = \{x_1, x_2\}$, $Y = \{y_1, y_2\}$, $E = \{e_1, e_2\}$, $K = \{k_1, k_2\}$. Define $u: X \rightarrow Y$ and $p: E \rightarrow K$ as $u(x_1) = y_2$, $u(x_2) = y_1$, and $p(e_1) = k_2$, $p(e_2) = k_1$. Let us consider the Soft topology $\tau = \{\tilde{\phi}, \tilde{X}, (F_1, E), (F_2, E), (F_3, E)\}$ such that $F_1(e_1) = x_1$, $F_1(e_2) = x_2$, $F_2(e_1) = x_1$, $F_2(e_2) = X$, $F_3(e_1) = X$, $F_3(e_2) = x_2$. $\sigma = \{\tilde{\phi}, \tilde{Y}, (A, K)\}$ where $A(k_1) = \phi$, $A(k_2) = y_2$, let $f: (X, \tau, E) \rightarrow (Y, \sigma, K)$ is soft mapping then (A, K) is Soft open set in (Y, σ, K) Here $f^{-1}(A, K) = \{(e_1, x_1), (e_2, \phi)\}$ is Soft JP Open set but not Soft open set.

Proposition 3.5:

If $f: (X, \tau, E) \rightarrow (Y, \sigma, K)$ is Soft semi continuous then it is Soft JP continuous .

Proof: The proof follows from the fact that every Soft semi open set is a Soft JP open set.

Remark 3.6:

The converse of the above Proposition is not true and it can be seen from the following example.

$X = \{x_1, x_2\}$, $Y = \{y_1, y_2\}$, $E = \{e_1, e_2\}$, $K = \{k_1, k_2\}$. Define $u: X \rightarrow Y$ and $p: E \rightarrow K$ as $u(x_1) = y_1$, $u(x_2) = y_2$, and $p(e_1) = k_2$, $p(e_2) = k_1$. Let us consider the Soft topology $\tau = \{\tilde{\phi}, \tilde{X}, (F_1, E), (F_2, E)\}$ such that $F_1(e_1) = \phi$, $F_1(e_2) = x_1$, $F_2(e_1) = x_1$, $F_2(e_2) = x_1$. $\sigma = \{\tilde{\phi}, \tilde{Y}, (B, K)\}$ where $B(k_1) = \phi$, $B(k_2) = Y$, let $f: (X, \tau, E) \rightarrow (Y, \sigma, K)$ is soft mapping then (B, K) is Soft closed set in (Y, σ, K) Here $f^{-1}(B, K) = \{(e_1, \phi), (e_2, X)\}$ is Soft JP closed set but not Soft semi closed set.

Proposition 3.7:

If $f: (X, \tau, E) \rightarrow (Y, \sigma, K)$ is Soft α continuous then it is Soft JP continuous .

Proof: The proof follows from the fact that every Soft α open set is a Soft JP open set.

Remark3.8:

The converse of the above Proposition is not true and it can be seen from the following example.

$X=\{x_1,x_2\}$, $Y=\{y_1,y_2\}$, $E=\{e_1,e_2\}$, $K=\{k_1,k_2\}$. Define $u: X \rightarrow Y$ and $p: E \rightarrow K$ as $u(x_1) = y_2$, $u(x_2) = y_1$, and $p(e_1) = k_2$, $p(e_2) = k_1$. Let us consider the Soft topology $\tau = \{\tilde{\phi}, \tilde{X}, (F_1, E), (F_2, E), (F_3, E)\}$ such that $F_1(e_1)= x_1$, $F_1(e_2)= x_2$, $F_2(e_1)= x_1$, $F_2(e_2)= X$, $F_3(e_1)= X$, $F_3(e_2)= x_2$. $\sigma=\{\tilde{\phi}, \tilde{Y}, (C, K)\}$ where $C(k_1)=y_2$, $C(k_2)= Y$, let $f: (X, \tau, E) \rightarrow (Y, \sigma, K)$ is soft mapping then (C, K) is Soft closed set in (Y, σ, K) Here $f^{-1}(C, K) = \{(e_1, X), (e_2, x_1)\}$ is Soft JP closed set But not Soft α closed set.

Proposition3.9:

If $f: (X, \tau, E) \rightarrow (Y, \sigma, K)$ is Soft JP continuous then it is Soft gs continuous .

Proof: The proof follows from the fact that every Soft JP open set is a Soft gs open set.

Remark3.10:

The converse of the above Proposition is not true and it can be seen from the following example.

$X=\{x_1,x_2\}$, $Y=\{y_1,y_2\}$, $E=\{e_1,e_2\}$, $K=\{k_1,k_2\}$. Define $u: X \rightarrow Y$ and $p: E \rightarrow K$ as $u(x_1) = y_1$, $u(x_2) = y_2$, and $p(e_1) = k_1$, $p(e_2) = k_2$. Let us consider the Soft topology $\tau = \{\tilde{\phi}, \tilde{X}, (F_1, E), (F_2, E)\}$ such that $F_1(e_1)= x_2$, $F_1(e_2)= x_1$, $F_2(e_1)= x_2$, $F_2(e_2)= X$, $\sigma=\{\tilde{\phi}, \tilde{Y}, (D, K)\}$ where $D(k_1)= \phi$, $D(k_2)= y_2$, let $f: (X, \tau, E) \rightarrow (Y, \sigma, K)$ is soft mapping then (D, K) is Soft open set in (Y, σ, K) Here $f^{-1}(D, K) = \{(e_1, x_2), (e_2, \phi)\}$ is Soft JP Open set but not Soft open set.

Definition3.11:

The map $f: (X, \tau, E) \rightarrow (Y, \sigma, K)$ is said to be **Soft Semi JP continuous** if inverse image of every Soft semi closed set in (Y, σ, K) is Soft JP closed in (X, τ, E) .

Remark 3.12:

The Soft identity map is Soft semi JP continuous.

Proposition3.13:

For a subset (A,E) of a Soft topological space (X,τ,E) , the following are equivalent.

1. $SJPO(X,\tau,E)$ is closed under any union.
2. (A,E) is Soft JP closed if and only if $JPCl(A,E)=(A,E)$.
3. (A,E) is Soft JP open if and only if $JPInt(A,E)=(A,E)$.

Proof:

1→2: Let (A,E) be a Soft JP closed set in the Soft topological space (X,τ,E) Then by definition of Soft JP closure, $JPCl(A,E)=(A,E)$. Conversely assume that $JPCl(A,E)=(A,E)$, for each $x \in (A,E)^C$, $x \notin JPCl(A,E)$, therefore there exists a Soft JP open set $(G,E)_x$ such that $(G,E)_x \cap (A,E)^C \neq \emptyset$ and Hence $x \in (G,E)_x \subseteq (A,E)^C$ Therefore $(A,E)^C = \bigcup (G,E)_x$. Then by (1), $(A,E)^C$ is Soft JP open and Hence (A,E) is Soft JP closed.

2→3: Let (A,E) be a Soft JP open set. Then $(A,E)^C$ is a Soft JP closed set. Hence, $JPCl((A,E)^C) = (A,E)^C$, by hypothesis, $(JPCl((A,E)^C))^C = (A,E)$. That is $JPInt(A,E) = (A,E)$. converse part of (3) is obvious from converse part of (2).

3→1: let $\{(U,E)_\alpha : \alpha \in \Lambda\}$ be a family of Soft JP open sets of (X,τ,E) , Put $(U,E) = \bigcup (U,E)_\alpha$. For each $x \in (U,E)$ there exists $\alpha(x) \in \Lambda$ such that $x \in (U,E)_{\alpha(x)} \subseteq (U,E)$. Since $(U,E)_{\alpha(x)}$ is Soft JP open, $x \in JPInt(U,E)$ and so $(U,E) = JPInt(U,E)$. BY (3) (U,E) is Soft JP open. Then $SJPO(X,\tau,E)$ is closed under any union.

Theorem3.14:

Let $f: (X,\tau,E) \rightarrow (Y,\sigma,K)$ be a map. Assume that $SJPO(X,\tau,E)$ is closed under any union. then the following statements are equivalent.

1. The map f is Soft JP continuous.
2. The inverse image of each Soft open set in (Y,σ,K) is Soft JP open set in (X,τ,E) .
3. For each point $x \in X$ and each Soft open set (V,E) in (Y,σ,K) with $f(x) \in (V,E)$ there is Soft JP open set (U,E) in (X,τ,E) such that $x \in U, E, f(U, E) \subseteq (V, E)$.
4. For each subset (A,E) of (X,τ,E) , $f(JPCl(A,E)) \subseteq Cl(f(A,E))$.

5. For each subset (B,E) of (Y,σ,K) , $JPCl(f^{-1}(A,E)) \cong f^{-1} Cl(B,E)$.
6. For each subset (B,E) of (Y,σ,K) , $f^{-1} Int(A,E) \cong JPInt(f^{-1} B,E)$.

Proof:

1 \rightarrow 2: This is follows from Proposition 3.2.

1 \rightarrow 3: Suppose that (3) holds and let (G,K) be an Soft open set in (Y,σ,K) and let $x \in f^{-1}(G,K)$. Then $f(x) \in (G,K)$ and thus there exists a Soft JP open set $(U,E)_x$ such that $x \in (U,E)_x$ and $f((U,E)_x) \subseteq (G,K)$. Now, $x \in (U,E)_x \subseteq f^{-1}(G,K)$ and $f^{-1}(G,K) = \bigcup_{x \in f^{-1}(G,K)} (U,E)_x$. By Assumption, $f^{-1}(G,K)$ is Soft JP open in (X,τ,E) and therefore f is Soft JP continuous. Conversely suppose that (1) holds and let $f(x) \in (G,K)$. Then $x \in f^{-1}(G,K)$ in $SJPO(X,\tau,E)$, since f is Soft JP continuous, let $(U,E) = f^{-1}(G,K)$. then $x \in U,E$ and $f(U,E) \subseteq (G,K)$.

1 \rightarrow 4: Suppose that 1 holds and (A,E) be a subset of (X,τ,E) Now $(A,E) \subseteq f^{-1}(f(A,E))$ implies $(A,E) \subseteq f^{-1}(Cl(f(A,E)))$. Since $Cl(f(A,E))$ is a Soft closed set in (Y,σ,K) , by assumption,

$f^{-1}(Cl(f(A,E)))$ is a Soft JP closed set containing (A,E) . Consequently, $JPCl(A,E) \subseteq f^{-1}(Cl(f(A,E)))$. Thus $f(JPCl(A,E)) \subseteq Cl(f(A,E))$. Conversely suppose that (4) holds for any subset (A,E) of (X,τ,E) , Let (G,K) be a closed subset of (Y,σ,K) . Then by assumption, $f(SJPCl(f^{-1}(G,K))) \subseteq Cl(f(f^{-1}(G,K))) \subseteq Cl(G,K) = (G,K)$. That is , $JPCl(f^{-1}(G,K)) \subseteq f^{-1}(G,K)$ and so $f^{-1}(G,K)$ is Soft JP closed in (X,τ,E) .

(4) \rightarrow (5): Suppose that 4 holds (G,K) any Soft subset of (Y,σ,K) replacing (A,E) by $f^{-1}(G,K)$ in 4, then $f(JPCl(f^{-1}(B,E))) \subseteq f^{-1}(Cl(G,K))$. Conversely, suppose that (5) holds , let $(G,K) = f(A,E)$, where (A,E) is a Soft subset of (X,τ,E) Then , $JPCl(A,E) \subseteq JPCl(f^{-1}(G,K)) \subseteq f^{-1}(Cl(f(A,E)))$ and so $f(JPCl(A,E)) \subseteq Cl(f(A,E))$.

(5) \rightarrow (6): Let (G,K) be any subset of (Y,σ,K) . then by (5) $JPCl(f^{-1}(G,K))^C \subseteq f^{-1}(Cl(G,K)^C)$ and Hence $(JPInt(f^{-1}G,K)) \subseteq (f^{-1}(int(G,K)))$ Therefore, $f^{-1}(int(G,K)) \subseteq JPInt(f^{-1}(G,K))$.

(6) \rightarrow (1): suppose 6 holds. Let (G,K) be any closed subset of (Y,σ,K) . Now, $f^{-1}((G,K)^C) = f^{-1}(int(G,K)^C) \subseteq JPInt(f^{-1}(G,K)^C) = (JPCl(f^{-1}(F,E)))^C$ and hence $JPCl(f^{-1}(G,K)) \subseteq f^{-1}(G,K)$. By Proposition 3.13, $f^{-1}(G,K)$ is Soft JP closed . Hence f is Soft JP continuous.

Theorem3.15:

Let $f: (X, \tau, E) \rightarrow (Y, \sigma, K)$ be a Soft JP continuous map and (A, E) be Soft JP closed subset of (X, τ, E) Assume that $SJPC((Y, \sigma, K))$ is closed under finite intersection. Then the restriction $f_A: (A, \tau_{(A, E)}, E) \rightarrow (Y, \sigma, K)$ is Soft JP continuous.

Proof: Let (G, K) be a closed subset of (Y, σ, K) . Since f is Soft JP continuous $f^{-1}(G, K)$ is Soft JP closed in (X, τ, E) . By Assumption $f^{-1}(G, K) \tilde{\cap} (A, E) = (A, E)_1$ (say) is Soft JP closed in (X, τ, E) . Since $f_A^{-1}(G, K) = f^{-1}(G, K) \tilde{\cap} (A, E) = (A, E)_1$. It is sufficient to show that $(A, E)_1$ is Soft JP closed in $(A, \tau_{(A, E)}, E)$. Let $(G, E)_1$ be an Soft \hat{g} open set of $(A, \tau_{(A, E)}, E)$ such that $(A, E)_1 \tilde{\subset} (G, E)_1$. Then by hypothesis $(G, E)_1$ is Soft \hat{g} open in (X, τ, E) Since $(A, E)_1$ is Soft JP closed in (X, τ, E) , $Scl_{\{A, E\}}\{\tilde{X}\} \tilde{\subset} int((G, E)_1)$. Since (A, E) is Soft open, $Scl_{\{A, E\}}((A, E)_1) = Scl_{\{X, E\}}(A, E)_1 \tilde{\cap} (A, E) \tilde{\subset} Int(G, E)_1 \tilde{\cap} (A, E) = int(G, E)_1 \tilde{\cap} int(A, E) \tilde{\subset} int(G, E)_1$ and so $(A, E)_1 = (f_{(A, E)})^{-1}(G, K)$ is Soft JP closed set in (A, E) . Thus restriction $f_A: (A, \tau_{(A, E)}, E) \rightarrow (Y, \sigma, K)$ is Soft JP continuous.

Theorem 3.16: Pasting Lemma for Soft JP continuous map

Let (X, τ, E) be a Soft topological space with Soft topology τ and $\tilde{X} = (G, E) \tilde{\cup} (H, E)$. Let (Y, σ, K) be a Soft topological space with Soft topology σ . Let $f: (G, \tau_{(G, E)}, E) \rightarrow (Y, \sigma, K)$ and $g: (H, \tau_{(H, E)}, E) \rightarrow (Y, \sigma, K)$ be Soft JP continuous maps such that $f(x) = g(x)$ for every $x \in (G, E) \tilde{\cap} (H, E)$. Assume that $D[A, E] \tilde{\cap} D_S[A, E]$, for any $(A, E) \tilde{\subset} \tilde{X}$. Suppose that both (G, E) and (H, E) are open and closed in (X, τ, E) then their combination $f \nabla g: (X, \tau, E) \rightarrow (Y, \sigma, K)$ defined by

$(f \nabla g)(x) = f(x)$ if $x \in G$ and $(fg)(x) = g(x)$ if $x \in H$ is Soft JP continuous.

Proof: Let (F, E) be a Soft closed subset of (X, τ, E) then $f^{-1}(F, E)$ is Soft JP closed in $(G, \tau_{(G, E)}, E)$ and $g^{-1}(F, E)$ is Soft JP closed in $(H, \tau_{(H, E)}, E)$. Since (G, E) and (H, E) are both open and closed subsets of (X, τ, E) then $f^{-1}(F, E) \tilde{\cup} g^{-1}(F, E)$ is Soft JP closed set in (X, τ, E) Hence

$(f \nabla g)^{-1}(F, E) = [f^{-1}(F, E) \tilde{\cap} (G, E)] \tilde{\cup} [g^{-1}(F, E) \tilde{\cap} (H, E)] = (f \nabla g)^{-1}(G, E) \tilde{\cup} (f \nabla g)^{-1}(H, E)$ is Soft JP closed in (X, τ, E) Hence $f \nabla g$ is Soft JP continuous.

4. SOFT JP IRRESOLUTE:

Definition 4.1: A map $f: (X, \tau, E) \rightarrow (Y, \sigma, K)$ is said to be **Soft JP irresolute** if inverse image of every Soft JP closed set in (Y, σ, K) is Soft JP closed in (X, τ, E)

Proposition 4.2:

The map $f: (X, \tau, E) \rightarrow (Y, \sigma, K)$ is said to be Soft JP irresolute if inverse image of every Soft JP open set in (Y, σ, K) is Soft JP open in (X, τ, E)

Proof: Let $f: (X, \tau, E) \rightarrow (Y, \sigma, K)$ be Soft JP irresolute and (G, K) be an Soft JP open set in the Soft topological space (Y, σ, K) . Then $f^{-1}(G, K)^c$ is Soft JP closed in (X, τ, E) and so $f^{-1}(G, K)$ is Soft JP open set in (X, τ, E)

Proposition 4.3:

If a map $f: (X, \tau, E) \rightarrow (Y, \sigma, K)$ is Soft JP irresolute, then it is Soft JP continuous but not conversely.

Proof: Since every Soft open set is Soft JP open set, the proof follows.

Remark 4.4:

The converse of the above Proposition is not true and it can be seen from the following example.

$X = \{x_1, x_2\}$, $Y = \{y_1, y_2\}$, $E = \{e_1, e_2\}$, $K = \{k_1, k_2\}$. Define $u: X \rightarrow Y$ and $p: E \rightarrow K$ as $u(x_1) = y_1$, $u(x_2) = y_2$, and $p(e_1) = k_1$, $p(e_2) = k_2$. Let us consider the Soft topology $\tau = \{\tilde{\phi}, \tilde{X}, (F_1, E), (F_2, E)\}$ such that $F_1(e_1) = \phi$, $F_1(e_2) = x_1$, $F_2(e_1) = x_1$, $F_2(e_2) = x_1$. $\sigma = \{\tilde{\phi}, \tilde{Y}, (M, K)\}$ where $M(k_1) = y_1$, $M(k_2) = \phi$, let $f: (X, \tau, E) \rightarrow (Y, \sigma, K)$ is soft JP continuous mapping. Now consider the soft set (N, K) such that $N(k_1) = Y$, $N(k_2) = y_2$, (N, K) is Soft JP open set in (Y, σ, K) but $f^{-1}(N, K) = \{(e_1, X), (e_2, x_2)\}$ is not Soft JP open set in (X, τ, E) then f is not Soft JP irresolute.

Proposition 4.5:

Let (X, τ, E) be a Soft topological space and (Y, σ, K) be a Soft T_{JP} space and $f: (X, \tau, E) \rightarrow (Y, \sigma, K)$ be a map. then the following are equivalent.

1. f is Soft JP irresolute.
2. f is Soft JP continuous.

Proof:

1 \rightarrow 2: follows from Proposition

2 \rightarrow 1: Let (G, K) be a Soft JP closed set in (Y, σ, K) . Since (Y, σ, K) is a Soft T_{JP} space, (G, K) is a Soft closed set in (Y, σ, K) and by hypothesis, $f^{-1}(G, K)$ is Soft JP closed in (X, τ, E) . Therefore f is Soft JP irresolute.

Theorem 4.6:

If the bijective map $f: (X, \tau, E) \rightarrow (Y, \sigma, K)$ is a Soft \hat{g} irresolute and Soft α closed then the inverse map $f^{-1}: (Y, \sigma, K) \rightarrow (X, \tau, E)$ is Soft JP irresolute.

Proof: Let (A, E) be a Soft JP closed set in (X, τ, E) . Let $(f^{-1})^{-1}(A, E) = f(A, E) \cong (U, E)$, where (U, E) is a Soft \hat{g} open in (Y, σ, K) . Then $(A, E) \cong f^{-1}(U, E)$ holds. Since $f^{-1}(U, E)$ is Soft \hat{g} open in (X, τ, E) and (A, E) is Soft JP closed in (X, τ, E) , then $Scl(A, E) \cong \text{int}(f^{-1}(U, E))$ and hence $f(Scl(A, E)) \cong f(\text{int}(f^{-1}(U, E))) \cong \text{int}(f(f^{-1}(U, E))) = \text{int}(U, E)$. Since f is Soft α closed and we know that A map $f: (X, \tau, E) \rightarrow (Y, \sigma, K)$ is Soft α closed if and only if $Scl(f(A, E)) \cong f(Scl(A, E))$ for every subset $(A, E) \subseteq X$. Then, $Scl(f(A, E)) \cong f(Scl(A, E)) \cong \text{Int}(U, E)$. Thus $f(A, E)$ is Soft JP closed in (Y, σ, K) and so f^{-1} is Soft JP irresolute.

Theorem 4.7:

If the map $f: (X, \tau, E) \rightarrow (Y, \sigma, K)$ is a Soft \hat{g} irresolute, Soft Open, Soft α closed and (A, E) is Soft JP closed subset of (X, τ, E) then $f(A, E)$ is Soft JP closed in (Y, σ, K) .

Proof: Let (G, K) be an Soft \hat{g} open set in (Y, σ, K) with $f(A, E) \subseteq (G, K)$. Since f is Soft \hat{g} irresolute, $f^{-1}(G, K)$ is Soft \hat{g} open in (X, τ, E) containing (A, E) . Given that (A, E) is Soft JP closed therefore $Scl(A, E) \cong \text{int}(f^{-1}(G, K))$, that is $f(Scl(A, E)) \cong f(\text{int}(f^{-1}(G, K)))$

$\text{int}(f(f^{-1}(G, K))) = \text{int}(G, K)$. Since f is Soft α closed, $Scl(f(A, E)) \cong f(Scl(A, E)) \cong \text{Int}(A, E)$. Thus $f(A, E)$ is Soft JP closed.

Proposition 4.8:

If the map $f: (X, \tau, E) \rightarrow (Y, \sigma, K)$ is Soft JP irresolute, then f is an Soft irresolute if (X, τ, E) is a sT_{JP} space.

Proof: Let (G, K) be Soft semi closed subset of (Y, σ, K) . Then (G, K) is Soft JP closed in (Y, σ, K) . Since f is Soft JP irresolute, $f^{-1}(G, K)$ is Soft JP closed in (X, τ, E) . Also since (X, τ, E) is a sT_{JP} space, $f^{-1}(G, K)$ is Soft semi closed in (X, τ, E) . Then f is Soft irresolute.

Proposition 4.9:

If the map $f: (X, \tau, E) \rightarrow (Y, \sigma, K)$ is surjective Soft JP irresolute and Soft α closed map. if (X, τ, E) is a sT_{JP} space, then (Y, σ, K) is a sT_{JP} space.

Proof: Let (G, K) be Soft JP closed subset of (Y, σ, K) . Since f is Soft JP irresolute, f^{-1}

(G, K) is Soft JP closed in (X, τ, E) . Also since (X, τ, E) is a sT_{JP} space, $f^{-1}(G, K)$ is Soft semi closed in (X, τ, E) . Since f is Soft α closed map and surjective, (G, K) is Soft semi closed in (Y, σ, K) . Then (Y, σ, K) is a sT_{JP} space.

Proposition 4.10:

If the map $f: (X, \tau, E) \rightarrow (Y, \sigma, K)$ is Soft JP irresolute then $JPCl(f^{-1}(G, K)) \simeq f^{-1}(JPCl(G, K))$ for each subset (G, K) of (Y, K) . Assume that $SJPO(X, \tau, E)$ is closed under any union.

Proof: By assumption, $JPCl(G, K)$ is Soft JP closed in (Y, σ, K) and f is a Soft JP irresolute implies $f^{-1}(JPCl(G, K))$ is also Soft JP closed in (X, τ, E) . Also $f^{-1}(G, K) \simeq f^{-1}(JPCl(G, K))$ which implies $JPCl(f^{-1}(G, K)) \simeq f^{-1}(JPCl(G, K))$.

5. COMPOSITION THEOREMS:

Remark 5.1:

The composition of two Soft JP continuous maps need not to be Soft JP continuous and this is shown by the following example.

$X = \{x_1, x_2\}$, $Y = \{y_1, y_2\}$, $Z = \{z_1, z_2\}$, $E = \{e_1, e_2\}$, $K = \{k_1, k_2\}$, $R = \{r_1, r_2\}$. Define $u: X \rightarrow Y$, $V: Y \rightarrow Z$ and $p: E \rightarrow K$, $q: K \rightarrow R$ as $u(x_1) = y_1$, $u(x_2) = y_2$, $v(y_1) = z_1$, $v(y_2) = z_2$, and $p(e_1) = k_1$, $p(e_2) = k_2$, $q(k_1) = r_1$, $q(k_2) = r_2$. Let us consider the Soft topology $\tau = \{\tilde{\phi}, \tilde{X}, (F_1, E), (F_2, E)\}$ such that $F_1(e_1) = x_2$, $F_1(e_2) = x_1$, $F_2(e_1) = x_2$, $F_2(e_2) = x_1$, $\sigma = \{\tilde{\phi}, \tilde{Y}, (M, K)\}$ where $M(k_1) = Y$, $M(k_2) = y_1$ and $\eta = \{\tilde{\phi}, \tilde{Z}, (N, R)\}$ such that $N(r_1) = z_1$, $N(r_2) = z_2$, let $f: (X, \tau, E) \rightarrow (Y, \sigma, K)$ and $g: (Y, \sigma, K) \rightarrow (Z, \eta, R)$ are soft JP continuous mappings. but their composition is not Soft JP continuous because $(g \circ f)^{-1}(N, R) = \{(e_1, X), (e_2, X)\}$ is not Soft JP open in (X, τ, E) .

Proposition 5.2:

If $f: (X, \tau, E) \rightarrow (Y, \sigma, K)$ is Soft JP continuous and $g: (Y, \sigma, K) \rightarrow (Z, \eta, R)$ is Soft continuous $g \circ f: (X, \tau, E) \rightarrow (Z, \eta, R)$ is Soft JP Continuous.

Proof: Let (H, R) be a Soft closed set in (Z, η, R) . Since g is Soft continuous then $g^{-1}(H, R)$ is Soft closed in (Y, σ, K) . Since f is Soft JP continuous $f^{-1}(g^{-1}(H, R))$ is Soft JP closed set in (X, τ, E) . Thus $g \circ f$ is Soft JP continuous.

Proposition 5.3:

If $f: (X, \tau, E) \rightarrow (Y, \sigma, K)$ is Contra Soft semi continuous and $g: (Y, \sigma, K) \rightarrow (Z, \eta, R)$ is

Contra Soft continuous $\text{gof}: (X, \tau, E) \rightarrow (Z, \eta, R)$ is Soft JP Continuous.

Proof: Let (H, R) be a Soft closed set in (Z, η, R) . Since g is Contra Soft continuous then $g^{-1}(H, R)$ is Soft open in (Y, σ, K) . Since f is Contra Soft semi continuous $f^{-1}(g^{-1}(H, R))$ is Soft semi closed set in (X, τ, E) . Since every Soft semi closed set is Soft JP closed. $(\text{gof})^{-1}(H, R) = f^{-1}(g^{-1}(H, R))$ is Soft JP closed set in (X, τ, E) . Thus gof is Soft JP continuous.

Theorem 5.4:

If $f: (X, \tau, E) \rightarrow (Y, \sigma, K)$ and $g: (Y, \sigma, K) \rightarrow (Z, \eta, R)$ be any two maps then

1. $\text{gof}: (X, \tau, E) \rightarrow (Z, \eta, R)$ is Soft JP Irresolute if both f and g are Soft JP irresolute.
2. $\text{gof}: (X, \tau, E) \rightarrow (Z, \eta, R)$ is Soft JP Continuous if f is Soft JP irresolute and g is Soft JP continuous.

Proof:

1) Let (H, R) be a Soft JP closed set in (Z, η, R) . Since g is Soft JP irresolute then $g^{-1}(H, R)$ is Soft JPClosed in (Y, σ, K) . Since f is Soft JP irresolute $(\text{gof})^{-1}(H, R) = f^{-1}(g^{-1}(H, R))$ is Soft JP closed set in (X, τ, E) . Thus gof is Soft JP irresolute.

2) Let (H, R) be a JP closed set in (Z, η, R) . Since g is Soft JP continuous then $g^{-1}(H, R)$ is Soft JP closed in (Y, σ, K) . Since f is Soft JP irresolute $(\text{gof})^{-1}(H, R) = f^{-1}(g^{-1}(H, R))$ is Soft JP closed set in (X, τ, E) . Thus gof is Soft JP continuous.

6. CONCLUSION

In this paper, we introduced Soft JP continuous and Soft JP irresolute functions and studied their properties. By suitable Propositions and examples We established the relations between Soft JP continuous and other Soft continuous forms. We hope that these findings paved a new pathway to the researchers in this field. This study not only having the theoretical face but also applied in various scenario of real life.

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