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On Intuitionistic Fuzzy γ Generalized Closed Sets

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Abstract

In this paper, we have introduced the notion of intuitionistic fuzzy γ generalized closed sets, and investigated some of their properties and produced some characterization theorems.

Keywords: Intuitionistic fuzzy topology, intuitionistic fuzzyclosed sets, intuitionistic fuzzy γ generalized closed sets, intuitionistic fuzzy point.

1. INTRODUCTION

The concept of fuzzy sets was introduced by Zadeh[11] and later Atanassov[1] generalized this idea to intuitionistic fuzzy sets. On the other hand Coker [2] introduced intuitionistic fuzzy topological spaces using the notion of intuitionistic fuzzy sets. In this paper, we have introduced the notion of intuitionistic fuzzy γ generalized closed sets, and investigated some of their properties and obtained some interesting characterizations.

2. PRELIMINARIES

Definition 2.1: [1] An intuitionistic fuzzy set (IFS for short) A is an object having the form

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$$

where the functions $\mu_A : X \longrightarrow [0,1]$ and $\nu_A : X \longrightarrow [0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element x∈X to the set A respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each x ∈ X. Denote by IFS (X), the set of all intuitionistic fuzzy sets in X. An intuitionistic fuzzy set A in X is simply denoted by $A=\langle x, \mu_A, \nu_A\rangle$ instead of denoting $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$.

Definition 2.2: [1]Let A and B be two IFSs of the form $A=\{\langle x,\mu_A(x),\nu_A(x)\rangle: x \in X\}$ and $B = \{\langle x,\mu_B(x),\nu_B(x)\rangle : x \in X\}$. Then,

(a) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all x ∈ X,

(b) A = B if and only if $A \subseteq B$ and $A \supseteq B$,

(c) $A^c = \{\langle x,\nu_A(x),\mu_A(x)\rangle: x \in X\}$,

(d) $A\cup B = \{\langle x, \mu_A(x)\vee\mu_B(x),\nu_A(x)\wedge\nu_B(x)\rangle: x \in X\}$,

(e) $A\cap B = \{\langle x, \mu_A(x)\wedge\mu_B(x),\nu_A(x)\vee\nu_B(x)\rangle: x \in X\}$.

The intuitionistic fuzzy sets $0_\sim = \langle x, 0, 1\rangle$ and $1_\sim = \langle x, 1, 0\rangle$ are respectively the empty set and the whole set of X.

Definition 2.3: [2]An intuitionistic fuzzy topology(IFT in short) on X is a family τ of IFSs in X satisfying the following axioms:

(i) $0_\sim, 1_\sim \in \tau$,

(ii) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$,

(iii) $\cup G_i \in \tau$ for any family $\{G_i : i\in J\}\subseteq\tau$.

In this case the pair (X, τ) is called the intuitionistic fuzzy topological space (IFTS in short) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS in short) in X. The complement A^c of an IFOS A in an IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS in short) in X.

Definition 2.4: [5] An IFS $A=\{\langle x,\mu_A,\nu_A\rangle$ in an IFTS (X, τ) is said to be an

(i) intuitionistic fuzzy γ closed set (IFγCS in short) if $cl(int(A)) \cap int(cl(A)) \subseteq A$

(ii) intuitionistic fuzzy γ open set (IFγOS in short) if $A \subseteq int(cl(A)) \cup cl(int(A))$

Definition 2.5:[5] Let Abe an IFS in an IFTS (X, τ). Then the intuitionistic fuzzy γ-interior and intuitionistic fuzzy γ-closure of Aare defined by

γ-int(A) = ∪{ G/ G is an IFγOS in X and G ⊆ A },
γ-cl(A) = ∩{K/ K is an IFγCS in X and A⊆K}.

Note that for any IFS A in (X,τ), we have $\gamma cl(A^c) = (\gamma int(A))^c$ and $\gamma int(A^c) = (\gamma cl(A))^c$.

Definition 2.6: [6]Let A be an IFS in (X, τ), then

(i) γcl(A) ⊇ A∪(cl(int(A))∩int(cl(A)))

(ii) γint(A) ⊆ A∩(cl(int(A))∩int(cl(A)))

Definition 2.7:[3] An intuitionistic fuzzy point(IFP in short) written as $p_{(\alpha,\beta)}$, is defined to be an IFS of X given by

$$\begin{cases} p_{(\alpha,\beta)}(x) = (\alpha, \beta) & \text{if } x = p, \\ (0, 1) & \text{otherwise.} \end{cases}$$

An intuitionistic fuzzy point $p_{(\alpha,\beta)}$ is said to belong to a set A if $\alpha \leq \mu_A$ and $\beta \geq \nu_A$.

Definition 2.8:[9] Two IFSs are said to be q-coincident(A_qB in short) if and only if there exists an element x ∈ X such that $\mu_A(x) > \nu_B(x)$ or $\nu_A(x) < \mu_B(x)$.

Definition 2.9:[9]Two IFSs are said to be not q-coincident($A_{\bar{q}}B$ in short) if and only ifA ⊆ B^c.

Definition 2.10: [7]An IFS A in (X, τ) is an IFQ-set if int(cl(A)) = cl(int(A)).

Definition 2.11: [3]Let (X, τ) be an IFTS and A = {⟨x,μ_A,ν_A⟩ be an IFS in X. Then ***intuitionistic fuzzy kernel*** of A is the intersection of all IFOSs containing A.

Definition 2.12: [8]An A in (X, τ) is called an intuitionistic fuzzy nowhere dense set if there exists no IFOS U such that U ⊆ cl(A). That is int(cl(A)) = $0_\sim$.

Proposition 2.13: [8]Let A be an IFS in X. If A is an intuitionistic fuzzy nowhere dense set in X, then int(A) = $0_\sim$.

3. INTUITIONISTIC FUZZY Γ GENERALIZED CLOSED SETS

In this section we have introduced intuitionistic fuzzy γgeneralized closed sets and studied some of their properties.

Definition 3.1: An IFS A in an IFTS (X,τ) is said to be an intuitionistic fuzzy γ generalizedclosed set (IFγGCS for short) if γcl(A)⊆U whenever A⊆U and U is an IFγOS in (X,τ). The complement A^c of an IFγGCS A in an IFTS (X,τ) is called intuitionistic fuzzy γ generalizedopen set (IFγGOS in short) in X.

The family of all IFγGCSs of an IFTS (X,τ) is denoted by IFγGC(X).

Example 3.2: Let X = {a,b} and $G_1 = \langle x,(0.5_a,0.6_b), (0.5_a,0.4_b)\rangle$ and $G_2 = \langle x,(0.4_a,0.3_b), (0.6_a,0.7_b)\rangle$. Then τ = {$0_\sim$,$G_1$,$G_2$,$1_\sim$} is an IFT on X. Then

IFγO(X)={$0_\sim$, $1_\sim$, $\mu_a \in [0,1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$,$\nu_b \in [0,1]$/ either $\nu_a < 0.4$ or $\nu_b < 0.3$,$\nu_a \geq 0.5$ whenever $\nu_b < 0.6$, $0.4 \leq \nu_a \leq 0.5$ whenever $\nu_b \leq 0.4$, $\mu_a \geq 0.5$, $\mu_b \geq 0.6$, $0.5 \leq \nu_a < 0.6$ whenever $\nu_b \geq 0.6$, $\mu_a \geq 0.4$, $\mu_b \geq 0.3$ and $\nu_a \geq 0.6$ whenever $\nu_b \geq 0.4$, $0 \leq \mu_a + \nu_a \leq 1$ and $0 \leq \mu_b + \nu_b \leq 1$} and

IFγC(X)={$0_\sim$, $1_\sim$, $\mu_a \in [0,1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$,$\nu_b \in [0,1]$/ either $\mu_a < 0.4$ or $\mu_b < 0.3$, $\mu_a \geq 0.5$ whenever $\mu_b < 0.6$,$0.4 \leq \mu_a \leq 0.5$ whenever $\mu_b \leq 0.4$, $\nu_a \geq 0.5$, $\nu_b \geq 0.6$, $0.5 \leq \mu_a < 0.6$ whenever $\mu_b \geq 0.6$, $\nu_a \geq 0.4$, $\nu_b \geq 0.3$ and $\mu_a \geq 0.6$ whenever$\mu_b \geq 0.4$, $0 \leq \mu_a + \nu_a \leq 1$ and $0 \leq \mu_b + \nu_b \leq 1$}.

Let A=$\langle x,(0.3_a,0.2_b), (0.7_a,0.8_b)\rangle$ be an IFS in (X, τ).Then A is an IFγGCS in X.

Theorem 3.3: Every IFCS[2] is an IFγGCS in (X,τ) but not conversely in general.

Proof: Let A be an IFCS in X and let A⊆U where U is an IFγOS in X. Asγcl(A)⊆cl(A)=A⊆U, by hypothesis, we have γcl(A)⊆U. Hence A is an IFγGCS in (X,τ).

Example 3.4: In Example 3.2, the IFS A = $\langle x,(0.3_a,0.2_b), (0.7_a,0.8_b)\rangle$ is an IFγGCS but not an IFCS in (X, τ) as cl(A) = $G_1^c \neq A$.

Theorem 3.5: Every IFRCS is an IFγGCS in (X,τ) but not conversely in general.

Proof: Let A be an IFRCS[4]. Since every IFRCS is an IFCS [10], by theorem 3.3, A is an IFγGCS in (X,τ).

Example 3.6: In Example3.2, the IFS A = $\langle x,(0.3_a,0.2_b), (0.7_a,0.8_b)\rangle$ is an IFγGCS but not an IFRCS in X as cl(int(A)) = $0_\sim \neq$ A.

Theorem 3.7: Every IFSCS is an IFγGCS in (X,τ) but not conversely in general.

Proof: Let A be an IFSCS[4] in X and let A ⊆ U where U is an IFγOS in X. Since γcl(A) ⊆ scl(A)=A⊆ U, by hypothesis, we have γcl(A) ⊆ U. Hence A is an IFγGCS in (X,τ).

Example 3.8: In Example3.2, the IFS A = $\langle x,(0.3_a,0.2_b), (0.7_a,0.8_b)\rangle$ is an IFγGCS but not an IFSCS in X as int(cl(A)) = $G_2 \not\subseteq$ A.

Theorem 3.9: Every IFPCS is an IFγGCS in (X,τ) but not conversely in general.

Proof: Let A be an IFPCS[4] in X and let A ⊆ U where U is an IFγOS in X. Asγcl(A) ⊆ pcl(A)=A ⊆ U, by hypothesis, we have γcl(A) ⊆ U. Hence A is an IFγGCS in (X,τ).

Example 3.10: In Example3.2, the IFS A = $\langle x,(0.4_a,0.3_b), (0.6_a,0.7_b)\rangle$ is an IFγGCS but not an IFPCS in X, as cl(int(A)) = $G_1^c \not\subseteq$A.

Theorem 3.11: Every IFαCS is an IFγGCS in (X,τ) but not conversely in general.

Proof: Let A be an IFαCS[4] in X and letA⊆U where U is an IFγOS in (X,τ). Asγcl(A)⊆αcl(A)=A⊆U, by hypothesis, we have γcl(A) ⊆ U. Hence A is an IFγGCS in (X,τ).

Example 3.12: In Example3.2, the IFS A = $\langle x,(0.3_a,0.2_b), (0.7_a,0.8_b)\rangle$ is an IFγGCSbut not an IFαCS in X as cl(int(cl(A)))= $G_1^c \not\subseteq$A.

Theorem 3.13: Every IFγCSis an IFγGCS in (X,τ) but not conversely in general.

Proof: Let A be an IFγCS[6] andletA⊆ U where U is an IFγOS in (X,τ). Thenγcl(A)=A⊆U, by hypothesis, we have γcl(A)⊆U. Hence A is an IFγGCS in (X,τ).

Example 3.14: Let X= {a,b} and G_1 = $\langle x,(0.5_a,0.7_b) , (0.5_a,0.3_b)\rangle$ and G_2 = $\langle x,(0.5_a,0.6_b), (0.5_a,0.4_b)\rangle$. Then τ= {$0_\sim$,$G_1$,$G_2$, $1_\sim$} is an IFT on X.

IFγO(X)={$0_\sim$, $1_\sim$,$\mu_a\in [0,1]$, $\mu_b\in [0,1]$, $\nu_a\in [0,1]$,$\nu_b\in [0,1]$/ either $\nu_a < 0.5$or $\nu_b < 0.6$, $0\leq \mu_a+\nu_a\leq 1$ and $0\leq\mu_b+\nu_b\leq 1$}.

Then IFγC(X)={$0_\sim$, $1_\sim$, $\mu_a \in [0,1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$, $\nu_b \in [0,1]$/ either $\mu_a < 0.5$ or $\mu_b < 0.6$, $0 \leq \mu_a + \nu_a \leq 1$ and $0 \leq \mu_b + \nu_b \leq 1$}.

Let A=⟨x,(0.6_a,0.6_b), (0.4_a,0.4_b)⟩ be an IFS in X. Then A is an IFγGCS in X but not an IFγCS in X, as cl(int(A)) ∩ int(cl(A))= $1_\sim \nsubseteq$ A.

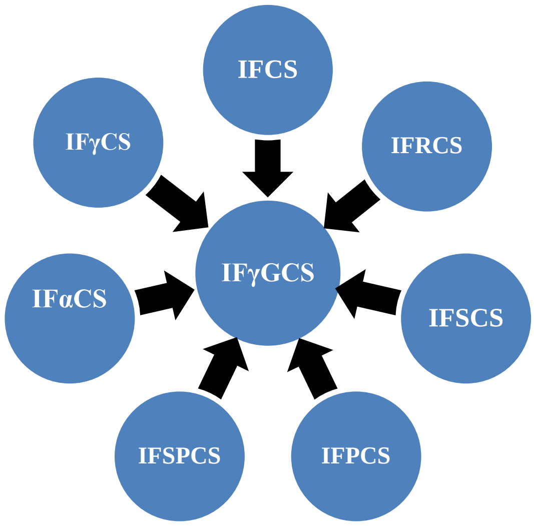
Theorem 3.15: Every IFSPCS is an IFγGCS in (X,τ) but not conversely in general.

Proof: Let A be an IFSPCS[10]. Let A⊆U and U be an IFγOS in X. Since A is an IFSPCS, spcl(A) = A. Then γcl(A)⊆ spcl(A) = A⊆ U, by hypothesis. Hence A is an IFγGCS in (X,τ).

Example 3.16: In Example 3.14, IFPC(X) = {$0_\sim$, $1_\sim$, $\mu_a \in [0, 1]$, $\mu_b \in [0, 1]$, $\nu_a \in [0, 1]$, $\nu_b \in [0, 1]$/ either $\mu_a < 0.5$ or $\mu_b < 0.6$, $0 \leq \mu_a + \nu_a \leq 1$ and $0 \leq \mu_b + \nu_b \leq 1$}.

Then the IFS A=⟨x,(0.6_a,0.6_b), (0.4_a,0.4_b)⟩is an IFγGCS in X. But A is not an IFSPCS in X, as we cannot find any IFPCS B such that int(B) ⊆ A ⊆ B in X.

In the following diagram, we have provided relations between various types of intuitionistic fuzzy closedness.



The reverse implications are not true in general in the above diagram.

Remark 3.17: The union of any two IFγGCS is not an IFγGCS in general as seen in following example.

Example 3.18: Let X = {a,b} and G_1 = ⟨x,(0.6_a,0.8_b), (0.4_a,0.2_b)⟩ and G_2 = ⟨x,(0.5_a,0.5_b), (0.4_a,0.4_b)⟩. Then τ = {$0_\sim$,G_1,G_2,$1_\sim$} is an IFT on X.

IFγO(X)={$0_\sim$, $1_\sim$, $\mu_a \in [0,1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$, $\nu_b \in [0,1]$/ either $\nu_a < 0.5$ or $\nu_b < 0.5$, $0 \leq \mu_a + \nu_a \leq 1$ and $0 \leq \mu_b + \nu_b \leq 1$}.

We have IFγC(X)={$0_\sim$, $1_\sim$, $\mu_a \in [0,1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$, $\nu_b \in [0,1]$/ either $\mu_a < 0.5$or $\mu_b < 0.5$, $0 \leq \mu_a + \nu_a \leq 1$ and $0 \leq \mu_b + \nu_b \leq 1$}.

Then the IFSs A=⟨x,(0.5_a,0.4_b), (0.4_a,0.5_b)⟩, B =⟨x,(0.4_a,0.6_b), (0.5_a,0.2_b)⟩ are IFγGCSs in (X,τ), as γcl(A)=A andγcl(B)=B. Now A∪B = ⟨x,(0.5_a,0.6_b) , (0.4_a,0.2_b)⟩⊆G_1 but γcl(A∪B)=$1_\sim$⊈G_1.

Remark 3.19: The intersection of any two IFγGCSs is not an IFγGCS in general as seen in following example.

Example 3.20: Let X = {a,b} and G_1 = ⟨x,(0.5_a,0.7_b) , (0.5_a,0.3_b)⟩ and G_2 = ⟨x,(0.5_a,0.6_b), (0.5_a,0.4_b)⟩. Then τ = {$0_\sim$,G_1,G_2,$1_\sim$} is an IFT on X.

IFγO(X)={$0_\sim$, $1_\sim$, $\mu_a \in [0,1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$, $\nu_b \in [0,1]$/ either $\nu_a < 0.5$ or $\nu_b < 0.6$, $0 \leq \mu_a + \nu_a \leq 1$ and $0 \leq \mu_b + \nu_b \leq 1$}.

We have IFγC(X)={$0_\sim$, $1_\sim$, $\mu_a \in [0,1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$, $\nu_b \in [0,1]$/ either $\mu_a < 0.5$ or $\mu_b < 0.6$, $0 \leq \mu_a + \nu_a \leq 1$ and $0 \leq \mu_b + \nu_b \leq 1$}.

Then the IFSs A=⟨x,(0.5_a,0.9_b), (0.5_a,0.1_b)⟩, B=⟨x,(0.5_a,0.7_b), (0.4_a,0.3_b)⟩ are IFγGCSs in (X,τ) but A∩B is not an IFγGCS in (X,τ).

Now A⊆$1_\sim$and γcl(A)=$1_\sim$⊆$1_\sim$, which implies A is an IFγGCS in X. we have B⊆$1_\sim$and γcl(B)=$1_\sim$⊆1. Therefore B is an IFγGCS in X. Now A∩B=⟨x,(0.5_a,0.7_b), (0.5_a,0.3_b)⟩⊆G_1 but γcl(A∩B)=$1_\sim$⊈G_1.

Theorem 3.21: Let (X,τ) be an IFTS.Then for every A∈IFγGC(X) and for every B∈IFS(X), A⊆B⊆γcl(A)⟹B∈IFγGC(X).

Proof: Let B⊆U and U be an IFγOS in X. Then since,A⊆B, A⊆U. By hypothesis B⊆γcl(A). Therefore γcl(B)⊆γcl(γcl(A))=γcl(A)⊆U, since A is an IFγGCS. HenceB∈IFγGC(X).

Theorem 3.22: An IFS A of an IFTS (X,τ) is an IFγGCS if and only if $A\, \bar{q}$F⟹γcl(A)$\bar{q}$F for every IFγCS F of X.

Proof: Necessity: Let F be an IFγCS and $A\ \bar{q}\ F$, then $A \subseteq F^c$ by definition 2.8, where F^c is an IFγOS. Then $\gamma cl(A) \subseteq F^c$, by hypothesis. Hence again by the same definition, $\gamma cl(A)\ \bar{q}\ F$.

Sufficiency: Let U be an IFγOS such that $A \subseteq U$. Then U^c is an IFγCS and $A \subseteq (U^c)^c$. By hypothesis, $A\ \bar{q}\ U^c \Longrightarrow \gamma cl(A)\ \bar{q}\ U^c$. Hence $\gamma cl(A) \subseteq (U^c)^c = U$. Therefore $\gamma cl(A) \subseteq U$. Hence A is an IFγGCS in X.

Theorem 3.23: If A is both an IFγOS and an IFγGCS in (X,τ) then A is an IFγCS in (X,τ).

Proof: Since $A \subseteq A$ and A is an IFγOS, by hypothesis $\gamma cl(A) \subseteq A$. But $A \subseteq \gamma cl(A)$. Therefore $\gamma cl(A) = A$. Hence A is an IFγCS in (X,τ).

Theorem 3.24: Let A be an IFγGCS in (X,τ) and $p_{(\alpha,\beta)}$ be an IFP in X such that $p_{(\alpha,\beta)q}\gamma cl(A)$ then $cl(p_{(\alpha,\beta)})_q A$.

Proof: Let A be an IFγGCS and let $p_{(\alpha,\beta)q}\gamma cl(A)$. If $cl(p(\alpha,\beta))\ \bar{q}\ A$, then by definition, $A \subseteq [cl(p_{(\alpha,\beta)})]^c$, where $[cl(p_{(\alpha,\beta)})]^c$ is an IFOS then it is an IFγOS. Then by hypothesis, $\gamma cl(A) \subseteq [cl(p_{(\alpha,\beta)})]^c = int(p_{(\alpha,\beta)})^c \subseteq [p_{(\alpha,\beta)}]^c$. This implies that $p_{(\alpha,\beta)}\ \bar{q}\ \gamma cl(A)$, which is a contradiction to the hypothesis. Hence $cl(p_{(\alpha,\beta)})_q A$.

Theorem 3.25: Let $F \subseteq A \subseteq X$ where A is an IFγOS and an IFγGCS in X. Then F is an IFγGCS in A if and only if F is an IFγGCS in X.

Proof: Necessity: Let U be an IFγOS in X and $F \subseteq U$. Also let F be an IFγGCS in A. Then $F \subseteq A \cap U$ and $A \cap U$ is an IFγOS in A. Hence gamma closure of F in A, $\gamma cl_A(F) \subseteq A \cap U$ and by Theorem 3.23, A is an IFγCS. Therefore $\gamma cl(A) = A$. Now gamma closure of F in X, $\gamma cl(F) \subseteq \gamma cl(F) \cap \gamma cl(A) = \gamma cl(F) \cap A = \gamma cl_A(F) \subseteq A \cap U \subseteq U$. That is $\gamma cl(F) \subseteq U$, whenever $F \subseteq U$. Hence F is an IFγGCS in X.

Sufficiency: Let V be an IFγOS in A such that $F \subseteq V$. Since A is an IFγOS in X, V is an IFγOS in X. Therefore $\gamma cl(F) \subseteq V$, since F is an IFγGCS in X. Thus, $\gamma cl_A(F) = \gamma cl(F) \cap A \subseteq V \cap A \subseteq V$. Hence F is an IFγGCS in A.

Theorem 3.26: For an IFS A, the following conditions are equivalent:

(i) A is an IFOS and an IFγGCS

(ii) A is an IFROS

Proof: (i)⟹(ii)

Let A be an IFOS and an IFγGCS. Then γcl(A)⊆A as A ⊆ A and A is an IFγOS in X, but A⊆γcl(A). This implies that γcl(A)=A. Therefore A is an IFγCS andintcl(A) = int(cl(A))∩cl(A))= int(cl(A)) ∩ cl(int(A)) ⊆A, by hypothesis. Henceint(cl(A))⊆A. Since A is an IFOS, it is an IFPOS. Hence A⊆int(cl(A)). Therefore A=int(cl(A)). Hence A is an IFROS.

(ii)⟹(i):

Let A be an IFROS. Therefore A=int(cl(A)). Since every IFROS is an IFOS we haveint(cl(A))∩cl(int(A))=A∩cl(int(A))=A∩cl(A)=A⊆A. Hence A is an IFγCS in X and thus A is an IFγGCS in X.

Theorem 3.27: For an IFOS A in (X,τ), the following conditions are equivalent:

(i) A is an IFCS

(ii) A is an IFγGCS and an IFQ-set

Proof: (i)⟹(ii) Since A is an IFCS, it is an IFγGCS by Theorem 3.3. Now int(cl(A))=int(A)=A=cl(A)=cl(int(A)), by hypothesis. Hence A is an IFQ-set.

(ii)⟹(i) Since A is an IFOS and an IFγGCS, by Theorem 3.26, A is an IFROS.

Therefore A=int(cl(A))=cl(int(A))=cl(A), by hypothesis. Hence A is an IFCS in X.

Theorem 3.28: If a subset A of an IFTS (X, τ) is nowhere dense, then it is anIFγGCS in (X, τ).

Proof: IfA is nowhere dense, then int(cl(A))=$0_\sim$. Let A ⊆ U where U is an IFγOS. Nowγcl(A)⊆scl(A) = A ∪ int(cl(A))= A ∪ $0_\sim$= A ⊆ U and hence A is an IFγGCS in (X, τ).

Theorem 3.29: Let (X,τ) be an IFTS. Then every IFS in (X,τ) is an IFγGCS if and only if IFγO(X)=IFγC(X).

Proof: Necessity: Suppose that every IFS in (X,τ) is an IFγGCS in X. Let U∈IFγO(X), and by hypothesis, γcl(U)⊆U⊆γcl(U). This implies γcl(U)=U. Therefore U∈IFγC(X). Hence IFγO(X)⊆IFγC(X)(i). Let A∈IFγC(X), then A^c∈IFγO(X)⊆IFγC(X). That is, A^c∈IFγC(X). Therefore A∈IFγO(X). Hence IFγC(X) ⊆IFγO(X)(ii). From (i) and (ii) IFγO(X)=IFγC(X).

Sufficiency: Suppose that IFγO(X)=IFγC(X). Let A⊆U and U be an IFγOS. Then U∈IFγO(X) and by hypothesis γcl(A)⊆γcl(U)= U, since U∈IFγC(X). Therefore A is an IFγGCS in X.

Theorem 3.30: If A is an IFROS and B is an IFαCS, then A ∩ B is an IFγGCS in (X, τ).

Proof: Let B be an IFαCS and A be an IFROS. Then cl(int(cl(B))) ⊆ B and int(cl(A)) = A. Therefore A ∩ B ⊇ A ∩ cl(int(cl(B))) = int(cl(A)) ∩ cl(int(cl(B))) ⊇ int(cl(A)) ∩ int(cl(B)) =int(cl(A ∩ B)). We have int(cl(A ∩ B) ⊆ A ∩ B. Hence A ∩ B is an IFSCS and by Theorem 3.7, A ∩ B is an IFγGCS in (X, τ).

Theorem 3.31: If A is both an IFROS and an IFγGCS in (X, τ) then A is an IFγ-clopen set in (X, τ).

Proof: Let A be an IFROS and an IFγGCS in (X, τ). Then A is an IFγOS and A ⊆ A, γcl(A) ⊆ A, by hypothesis. But A ⊆γcl(A). Therefore A = γcl(A). Hence A is an IFγCS in (X, τ). Hence A is an IFγ-clopen set in (X, τ).

Theorem 3.32: If A is both an IFαOS and an IFγGCS in (X, τ) then A is an IFβCS in (X, τ).

Proof: Let A be an IFαOS. Then A is an IFγOS. As A ⊆ A, by hypothesisγcl(A) ⊆ A. Since βcl(A) ⊆ γcl(A)⊆ A⊆ βcl(A), A is an IFβCS in (X, τ).

Theorem 3.33: An IFS A of X is an IFγGCS if γcl(A) ⊆ ker(A).

Proof: Let U be any IFγOS such that A ⊆ U. By hypothesisγcl(A) ⊆ ker(A) and since A ⊆ U, ker(A) ⊆ U. Therefore γcl(A) ⊆ U and hence A is an IFγGCS.

Theorem 3.34: If A is both an IFγOS and an IFγGCS in (X, τ) and suppose that F is an IFCS in X. Then A∩ F is an IFγGCS in (X, τ).

Proof: Since A is an IFγOS and an IFγGCS in (X, τ), then by Theorem 3.23 A is an IFγCS in X. But F is an IFCS in X. Therefore A∩ F is an IFγCS in X. Hence A∩ F is an IFγGCS in (X, τ).

Theorem 3.35: For an IFγGCS A in an IFTS (X,τ), the following conditions hold:

(i) If A is an IFROS then scl(A) is an IFγGCS

(ii) If A is an IFRCS then sint(A) is an IFγGCS

Proof:(i) Let A be an IFROS in (X, τ). Then int(cl(A)) = A. By definition we have scl(A) = A ∪int(cl(A)) = A. Since A is an IFγGCS in X, scl(A) is an IFγGCS in X.

(ii)Let A be an IFRCS in (X, τ). Then cl(int(A)) = A. By definition we have sint(A) = A ∩ cl(int(A)) = A. Since A is an IFγGCS in X, sint(A) is an IFγGCS in X.

Remark 3.36: Every IFOS, IFROS, IFSOS, IFPOS, IFαOS, IFγOS, IFSPOS in (X, τ) is an IFγGOS in (X, τ) but not conversely in general.

Proof: Straightforward.

Example 3.37: Obvious from examples 3.4, 3.6, 3.8, 3.10, 3.12, 3.14 and 3.16, by taking complement of A in the respective examples.

Theorem 3.38: Let (X, τ) be an IFTS. Then for every A ∈ IFγGO(X) and for every B ∈ IFS(X), γint(A) ⊆ B ⊆ A ⇒ B ∈ IFγGO(X).

Proof: Let A be any IFγGOS of X and B be any IFS of X. Let γint(A) ⊆ B ⊆ A. Then A^c is an IFγGCS and $A^c \subseteq B^c \subseteq \gamma cl(A^c)$. Therefore B^c is an IFγGCS which implies B is an IFγGOS in X. Hence B ∈ IFγGO(X).

Theorem 3.39: An IFS A of an IFTS (X,τ) is an IFγGOS if and only if F ⊆ γint(A) whenever F is an IFγCS and F ⊆ A.

Proof:Necessity: Suppose A is an IFγGOS in X. Let F be an IFγCS such that F ⊆ A. Then F^c is an IFγOS and $A^c \subseteq F^c$. By hypothesis A^c is an IFγGCS, we have $\gamma cl(A^c) \subseteq F^c$. Therefore F ⊆ γint(A).

Sufficiency: Let F be an IFγCS such that F ⊆ A and F ⊆ γint(A). Then $(\gamma int(A))^c \subseteq F^c$ and $A^c \subseteq F^c$. This implies that $\gamma cl(A^c) \subseteq F^c$, where F^c is an IFγOS. Therefore A^c is an IFγGCS. Hence A is an IFγGOS in X.

Theorem 3.40: Let (X, τ) be an IFTS. Then for every A ∈ IFS(X) and for every B ∈ IFβO(X), B ⊆ A ⊆ int(cl(int(B))) ⇒ A ∈ IFγGO(X).

Proof: Let B be an IFβOS. Then B ⊆cl(int(cl(B))). By hypothesis, A ⊆ int(cl(int(B))) ⊆ int(cl(int(cl(int(cl(B)))))) ⊆ int(cl(cl(int(cl(B))))) = int(cl(int(cl(B)))) ⊆ int(cl(cl(A))) ⊆ int(cl(A)) as B ⊆ A. Therefore A is an IFPOS and by Theorem 3.36, A is an IFγGOS. Hence A ∈ IFγGO(X).

Theorem 3.41: Let (X, τ) be an IFTS. Then for every A ∈ IFS(X) and for every B ∈ IFRC(X), B ⊆ A ⊆ int(cl(B)) ⇒ A ∈ IFγGO(X).

Proof: Let B be an IFRCS. Then B = cl(int(B)). By hypothesis, A ⊆ int(cl(B)) ⊆ int(cl(cl(int(B)))) = int(cl(int(B))) ⊆ int(cl(int(A))) as B ⊆ A. Therefore A is an IFαOS and by Theorem 3.36, A is an IFγGOS. Hence A ∈ IFγGO(X).

Theorem 3.42: Let (X, τ) be an IFTS then for every A ∈ IFSPO(X) and for every IFS B in X, A ⊆ B ⊆ cl(A) ⇒ B ∈ IFγGO(X).

Proof: Let A be an IFSPOSin X. Then there exists an IFPOS, (say) C such that C ⊆ A ⊆ cl(C). By hypothesis, A⊆ B. Therefore C ⊆ B. Since A ⊆ cl(C), cl(A) ⊆ cl(C) and B ⊆ cl(C), by hypothesis. Thus C ⊆ B ⊆ cl(C). This implies B is an IFSPOS. As every IFSPOS is an IFγGOS by Theorem 3.36, B ∈ IFγGO(X).

Theorem 3.43: If A is an IFγCS and an IFγGOS in (X, τ) then A is an IFγOS in (X, τ).

Proof: As A ⊇ A, by hypothesis γint(A) ⊇ A. But we have A ⊇ γint(A). This implies A = γint(A). Hence A is an IFγOS in X.

Theorem 3.44: Let (X, τ) be an IFTS. Then for every A ∈ IFS(X) and for every B ∈ IFSO(X), B ⊆ A ⊆ int(cl(B)) ⇒ A ∈ IFγGO(X).

Proof: Let B be an IFSOS in X. Then B ⊆ cl(int(B)). By hypothesis, A ⊆ int(cl(B)) ⊆ int(cl(cl(int(B)))) = int(cl(int(B))) ⊆ int(cl(int(A))) as B ⊆ A. Therefore A is an IFαOS and by Theorem 3.36, A is an IFγGOS. Hence A ∈ IFγGO(X).

Theorem 3.45: If A is an IFRCS and B is an IFαOS, then A ∪ B is an IFγGOS in (X, τ).

Proof: Let B be an IFαOS and A be an IFRCS. Then B ⊆ int(cl(int(B))) and cl(int(A)) = A. Therefore A ∪ B ⊆ A ∪int(cl(int(B))) = cl(int(A)) ∪int(cl(int(B))) ⊆ cl(int(A)) ∪cl(int(B)) ⊆ cl(int(A ∪ B)). We have A ∪ B ⊆ cl(int(A ∪ B)). Therefore A ∪ B is an IFSOS and by Theorem 3.36, A ∪ B is an IFγGOS in X.

Theorem 3.46: If an IFS A of an IFTS X is both an IFCS and an IFGOS, then A is an IFγGOS in (X, τ).

Proof: Suppose A is both an IFCS and an IFGOS. Then as A ⊆ A, by hypothesis A ⊆ int(A). But int(A) ⊆ A. Therefore int(A) = A. We have A is an IFOS, since every IFOS is an IFγGOS. Hence A is an IFγGOS in X.

4. APPLICATIONS OF INTUITIONISTIC FUZZY γ GENERALIZED CLOSED SET

In this section we have discussed some theoretical applications of intuitionistic fuzzy generalized γ closed sets.

Definition 4.1: An IFTS (X, τ) is an intuitionistic fuzzy γgeneralized c $T_{1/2}$ (IF$\gamma_c T_{1/2}$ in short) space if every IFγGCS in (X ,τ) is an IFCS in (X ,τ).

Definition 4.2: An IFTS (X, τ) is called an intuitionistic fuzzy γgeneralized $T_{1/2}$ (IF$\gamma_\gamma T_{1/2}$ in short) space if every IFγGCS in (X ,τ) is an IFγCS in (X ,τ).

Example 4.3: Let X = {a, b} and G = ⟨x,(0.5_a, 0.6_b), (0.5_a, 0.4_b)⟩. Then τ = {$0_\sim$, G, $1_\sim$} is an IFT on X.

IFγO(X) = {$0_\sim$, $1_\sim$, $\mu_a \in [0, 1]$, $\mu_b \in [0, 1]$, $\nu_a \in [0, 1]$, $\nu_b \in [0, 1]$/ either $\nu_a < 0.5$ or $\nu_b < 0.6$, $0 \leq \mu_a+\nu_a \leq 1$ and $0 \leq \mu_b+\nu_b \leq 1$}.

IFγC(X) = {$0_\sim$, $1_\sim$, $\mu_a \in [0, 1]$, $\mu_b \in [0, 1]$, $\nu_a \in [0, 1]$, $\nu_b \in [0, 1]$/ either $\mu_a < 0.5$ or$\mu_b < 0.6$, $0 \leq \mu_a+\nu_a \leq 1$ and $0 \leq \mu_b+\nu_b \leq 1$}.

Therefore the space (X, τ) is an intuitionistic fuzzy γgeneralized $T_{1/2}$, as every IFγGCS is an IFγCS in (X, τ).

Definition 4.4: An IFTS (X, τ) is an intuitionistic fuzzy γgeneralized pre $T_{1/2}$ (IF$\gamma_p T_{1/2}$ in short) space if every IFγGCS in (X ,τ) is an IFPCS in (X ,τ).

Example 4.5: Let X = {a, b} and and G = ⟨x,(0.5_a, 0.6_b), (0.5_a, 0.4_b)⟩. Then τ = {$0_\sim$, G, $1_\sim$} is an IFT on X.

IFγO(X) = {$0_\sim$, $1_\sim$, $\mu_a \in [0, 1]$, $\mu_b \in [0, 1]$, $\nu_a \in [0, 1]$, $\nu_b \in [0, 1]$/ either $\nu_a < 0.5$ or $\nu_b < 0.4$,, $0 \leq \mu_a+\nu_a \leq 1$ and $0 \leq \mu_b+\nu_b \leq 1$}.

IFγC(X) = {$0_\sim$, $1_\sim$, $\mu_a \in [0, 1]$, $\mu_b \in [0, 1]$, $\nu_a \in [0, 1]$, $\nu_b \in [0, 1]$/ either $\mu_a < 0.5$ or $\mu_b < 0.6$, , $0 \leq \mu_a+\nu_a \leq 1$ and $0 \leq \mu_b+\nu_b \leq 1$}.

IFPC(X) = {$0_\sim$, $1_\sim$, $\mu_a \in [0, 1]$, $\mu_b \in [0, 1]$, $\nu_a \in [0, 1]$, $\nu_b \in [0, 1]$/ either $\mu_a < 0.5$ or $\mu_b < 0.6$, $0 \leq \mu_a+\nu_a \leq 1$ and $0 \leq \mu_b+\nu_b \leq 1$}.

Therefore the space (X, τ) is an $IF\gamma_pT_{1/2}$, as every IFγGCS is an IFPCS in (X, τ).

Theorem 4.6: Every $IF\gamma_pT_{1/2}$space is an $IF\gamma_\gamma T_{1/2}$space but not conversely in general.

Proof: Let (X, τ) be an $IF\gamma_pT_{1/2}$ space and let A be an IFγGCS in X. By hypothesis A is an IFPCS in X. Since every IFPCS is an IFγCS, A is an IFγCS in X. Hence (X ,τ) is an $IF\gamma_\gamma T_{1/2}$space.

Example 4.7: Let X = {a, b} and $G_1 = \langle x,(0.5_a, 0.6_b) , (0.5_a, 0.4_b)\rangle$ and $G_2 = \langle x,(0.4_a, 0.3_b) , (0.6_a, 0.7_b)\rangle$. Then τ = {$0_\sim$, G_1, G_2, $1_\sim$} is an IFT on X.

IFγO(X)= {$0_\sim$, $1_\sim$, $\mu_a \in [0,1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$,$\nu_b \in [0,1]$/ either $\nu_a < 0.4$ or $\nu_b < 0.3$, $\nu_a \geq 0.5$ whenever $\nu_b < 0.6$, $0.4 \leq \nu_a \leq 0.5$ whenever $\nu_b \leq 0.4$, $\mu_a \geq 0.5$, $\mu_b \geq 0.6$, $0.5 \leq \nu_a < 0.6$ whenever $\nu_b \geq 0.6$, $\mu_a \geq 0.4$, $\mu_b \geq 0.3$ and $\nu_a \geq 0.6$ whenever $\nu_b \geq 0.4$, $0 \leq \mu_a+\nu_a \leq 1$ and $0 \leq \mu_b+\nu_b \leq 1$}.

Then, IFγC(X) = {$0_\sim$, $1_\sim$, $\mu_a \in [0,1]$, $\mu_b \in [0, 1]$, $\nu_a \in [0,1]$, $\nu_b \in [0,1]$/ either $\mu_a < 0.4$ or $\mu_b < 0.3$, $\mu_a \geq 0.5$ whenever $\mu_b < 0.6$, $0.4 \leq \mu_a \leq 0.5$ whenever $\mu_b \leq 0.4$, $\nu_a \geq 0.5$, $\nu_b \geq 0.6$, $0.5 \leq \mu_a < 0.6$ whenever $\mu_b \geq 0.6$, $\nu_a \geq 0.4$, $\nu_b \geq 0.3$ and $\mu_a \geq 0.6$ whenever $\mu_b \geq 0.4$, $0 \leq \mu_a+\nu_a \leq 1$ and $0 \leq \mu_b+\nu_b \leq 1$}.

The space (X, τ) is an $IF\gamma_\gamma T_{1/2}$ space, as every IFγGCS is an IFγCS in (X, τ), but (X, τ) is not an $IF\gamma_pT_{1/2}$ space, since $A = \langle x,(0.4_a, 0.3_b), (0.6_a, 0.7_b)\rangle$ is an IFγGCS in (X, τ), but not an IFPCS as $cl(int(A)) = cl(G_2) = G_1^c \neq A$.

Theorem 4.8: Every $IF\gamma_cT_{1/2}$space is an $IF\gamma_\gamma T_{1/2}$space but not conversely in general.

Proof: Let (X, τ) be an $IF\gamma_cT_{1/2}$ space and let A be an IFγGCS in X. By hypothesis A is an IFCS in X. Since every IFCS is an IFγCS, A is an IFγCS in X. Hence (X ,τ) is an $IF\gamma_\gamma T_{1/2}$space.

Example 4.9: Let X = {a, b} and $G_1 = \langle x,(0.5_a, 0.6_b) , (0.5_a, 0.4_b)\rangle$ and $G_2 = \langle x,(0.4_a, 0.3_b) , (0.6_a, 0.7_b)\rangle$. Then τ = {$0_\sim$, G_1, G_2, $1_\sim$} is an IFT on X.

IFγO(X)= {$0_\sim$, $1_\sim$, $\mu_a \in [0,1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$,$\nu_b \in [0,1]$/ either $\nu_a < 0.4$ or $\nu_b < 0.3$, $\nu_a \geq 0.5$ whenever $\nu_b < 0.6$, $0.4 \leq \nu_a \leq 0.5$ whenever $\nu_b \leq 0.4$, $\mu_a \geq 0.5$, $\mu_b \geq 0.6$, $0.5 \leq \nu_a < 0.6$ whenever $\nu_b \geq 0.6$, $\mu_a \geq 0.4$, $\mu_b \geq 0.3$ and $\nu_a \geq 0.6$ whenever $\nu_b \geq 0.4$, $0 \leq \mu_a+\nu_a \leq 1$ and $0 \leq \mu_b+\nu_b \leq 1$}.

Then, IFγC(X) = {0~, 1~, $\mu_a \in [0,1]$, $\mu_b \in [0, 1]$, $\nu_a \in [0,1]$, $\nu_b \in [0,1]$/ either $\mu_a < 0.4$ or $\mu_b < 0.3$, $\mu_a \geq 0.5$ whenever $\mu_b < 0.6$, $0.4 \leq \mu_a \leq 0.5$ whenever $\mu_b \leq 0.4$, $\nu_a \geq 0.5$, $\nu_b \geq 0.6$, $0.5 \leq \mu_a < 0.6$ whenever $\mu_b \geq 0.6$, $\nu_a \geq 0.4$, $\nu_b \geq 0.3$ and $\mu_a \geq 0.6$ whenever $\mu_b \geq 0.4$, $0 \leq \mu_a + \nu_a \leq 1$ and $0 \leq \mu_b + \nu_b \leq 1$}.

The space (X, τ) is an IFγ_γ $T_{1/2}$ space, as every IFγGCS is an IFγCS in (X, τ), but not an IF$\gamma_c T_{1/2}$ space, since A = ⟨x,(0.3_a, 0.2_b), (0.7_a, 0.8_b)⟩ is an IFγGCS in (X, τ), but not an IFCS as cl(A) = $G_1^c \neq$ A.

Theorem 4.10: An IFTS (X,τ) is an IF$\gamma_\gamma T_{1/2}$space if and only if IFγO(X) = IFγGO(X).

Proof: Obvious.

Theorem 4.11: An IFTS (X, τ) is an IF$\gamma_c T_{1/2}$space iff IFγGO(X) = IFO(X).

Proof: Obvious.

Theorem 4.12: For any IFS A in (X,τ) where X is an IF$\gamma_\gamma T_{1/2}$space, A∈IFγGO(X) if and only if for every IFP $p_{(\alpha,\beta)} \in$ A, there exist an IFγGOS in X such that $p_{(\alpha,\beta)} \in B \subseteq$ A.

Proof: Necessity: If A∈IFγGO(X), then we can take B = A so that $p_{(\alpha,\beta)} \in B \subseteq A$ for every IFP $p_{(\alpha,\beta)} \in$ A.

Sufficiency: Let A be an IFS in(X, τ) and assume that there exists B∈IFγGO(X) such that $p_{(\alpha,\beta)} \in B \subseteq A$. Since X is an IF$\gamma_\gamma T_{1/2}$space, B is an IFγOS. Then $A = \bigcup_{p_{(\alpha,\beta)} \in A} \{p_{(\alpha,\beta)}\} \subseteq \bigcup_{p_{(\alpha,\beta)} \in A} B \subseteq A$. Therefore $A = \bigcup_{p_{(\alpha,\beta)} \in A} B$, which is an IFγOS. Hence A is an IFγGOS in X.

Remark4.13: In an IF$\gamma_\gamma T_{1/2}$ space,

(i) Any union of IFγGCS is an IFγGCS

(ii) Any intersection of IFγGOS is an IFγGOS

Proof: Since every IFγGCS is an IFγCS in an IF$\gamma_\gamma T_{1/2}$ space, the proof is obvious.

Remark 4.14: Not every IF$\gamma_\gamma T_{1/2}$ space is an IF$T_{1/2}$ space.

Example 4.15: Let X = {a, b} and G = ⟨x,(0.5_a, 0.6_b), (0.5_a, 0.4_b)⟩. Then τ = {$0_\sim$, G, $1_\sim$} is an IFT on X.

IFγO(X) = {$0_\sim$, $1_\sim$, $\mu_a \in [0, 1]$, $\mu_b \in [0, 1]$, $\nu_a \in [0, 1]$, $\nu_b \in [0, 1]$/ either $\nu_a < 0.5$ or $\nu_b < 0.6$, $0 \leq \mu_a+\nu_a \leq 1$ and $0 \leq \mu_b+\nu_b \leq 1$}.

IFγC(X) = {$0_\sim$, $1_\sim$, $\mu_a \in [0, 1]$, $\mu_b \in [0, 1]$, $\nu_a \in [0, 1]$, $\nu_b \in [0, 1]$/ either $\mu_a < 0.5$ or $\mu_b < 0.6$, $0 \leq \mu_a+\nu_a \leq 1$ and $0 \leq \mu_b+\nu_b \leq 1$}.

Since all IFγGCS in X are IFγCS in (X, τ),it is an IFγ$\gamma_\gamma T_{1/2}$ space. But it is not an IF$T_{1/2}$ space, since A = ⟨ x, (0.6_a, 0.7_b) (0.4_a, 0.3_b)⟩ is an IFGCS in X but not an IFCS as cl(A) = $1_\sim \neq$ A.

Theorem 4.16: If A is both an IFOS and an IFγGCS in X and if an IF$\gamma_c T_{1/2}$space in X, then

(i) A is an IFROS (ii) A is an IFRCS (iii) A is an IFQ- set

Proof: Let A be an IFγGCS in X, then by hypothesis, A is an IFCS in X. (i) Since int(cl(A)) = int(A) = A, A is an IFROS in X and

(ii) cl(int(A)) = cl(A) = A and therefore A is IFRCS in X.

(iii) Now int(cl(A)) = int(A) = A = cl(A) = cl(int(A)). Then A is an IFQ-set.

REFERENCES

[1] **Atanassov, K.,** Intuitionistic fuzzy sets,Fuzzy Sets and Systems,1986, 87-96.

[2] **Coker, D.,**An introduction to intuitionistic fuzzy topological space,Fuzzy Sets and Systems,1997,81-89.

[3] **Coker, D. and Demirci, M.,** On intuitionistic fuzzy points,Notes on Intuitionistic Fuzzy Sets, 1995, 79-84.

[4] **Gurcay, H. , Coker, D. and Es. Haydar, A.,** On fuzzy continuity in intuitionistic fuzzy topological spaces, The J. Fuzzy Mathematics, 1997,365-378.

[5] **Hanafy, I.M.,** Intuitionistic fuzzy γ-continuity, Canad. Math. Bull., 2009, 1-11.

[6] **Kanimozhi, R. and Jayanthi, D.,** On intuitionistic fuzzy generalized γ closed sets, International Journal of Scientific Engineering and Applied Science, 2016, 23-27.

[7] **Santhi, R. and Jayanthi, D.,** Generalized semi- pre connectedness in

intuitionistic fuzzy topological spaces,Annals of Fuzzy Mathematics and Informatics, 2012, 243-253.

[8] **ThakurS.Sand Dhavaseelan, R.,** Nowhere dense sets in intuitionistic fuzzy topological spaces, Proceedings of National Seminar on Recent Developments in Topology, 2015, 17-21.

[9] **Thakur S. S and Rekha Chaturvedi,** Regular generalized closed sets in intuitionistic fuzzy topological spaces, Universitatea Din Bacau, Studii SiCercetariStiintifice,Seria: Mathematica, 2006, 257-272.

[10] **Thakur, S.S. and Singh, S.,** On fuzzy semi-pre open sets and fuzzy semi-pre continuity, Fuzzy sets and systems, 1998, 383-391.

[11] **Zadeh, L.A.,**Fuzzy Sets, Information and control,1965, 338-353.