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Deformation of Viscothermoelastic Semi Infinite Cylinder with Mechanical Forces and Heat Sources

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Abstract

This paper is based on the study of variation of displacements, temperature change and stresses, in case of mechanical forces and heat sources of viscothermoelastic semi infinite cylinder. The Kelvin – Voigt model is used to investigate the problem. The solid is assumed to be homogenous and isotropic semi infinite solid cylinder subjected to coupled theory of viscothermoelasticity. The partial differential equations have been converted into ordinary differential equations by applying Laplace and Hankel transformations. The Hankel transformations are to be converted analytically into inverse Laplace transforms and then we apply numerical methods. Numerical results have been obtained by the use of MATLAB software tools. The displacements, temperature and stresses so obtained in physical domain and are computed numerically for Copper material. Numerical simulated results have been presented graphically for coupled theory of visothermoelastcity.

Keywords: Mechanical loads; Thermal loads; Kelvin–Voigt model; Viscothermoelasticity; Laplace transformation; Hankel transformation.

1. INTRODUCTION

The study of viscothermoelastic materials have many applications in the field of science and technology such as thermal power plants, pressure vessel technology, aerospace engineering, chemical pipes etc. The coupled theory of thermoelasticity was obtained by coupling between thermal and strain fields. The propagation of surface waves and effect of thermal properties in a thermoelastic half space have been studied by Lockett [1] in the context of coupled thermoelasticity. Boley and Tolins [2] have used transformation with respect to time to obtain the transient temperature and strain in a coupled thermoelastic half space. Chadwick and Windle [3] investigated the effects of heat conduction on Rayleigh waves in semi-infinite elastic solid for thermal boundary conditions. Liangh and Scarton [4] had studied viscothermoelastic and thermoelastic wave propagation in fluid filled steel tube for the vibrations in circumferential mode in the context of coupled thermoelasticity. Ning et al. [5] calculated temperature field by using finite element method in a plane subjected to a pulsed heat input in the frame work of non-Fourier heat conduction theory. The study of ultrasound by heat deposition due to laser irradiation had been studied by Achenbach [6]. Sherif and El-Maghraby [7] solved a dynamic problem for an infinite thermoelastic solid with penny shaped crack. Dhaliwal and Singh [8] have taken such type of problems for classical and non classical theories of thermoelasticity. Love [9] and Graff [10] have given more attention to such type of problems.

Linear Viscoelasticity theory was well established by Bland [11]. Several mathematical problems based on viscoelastic models have been used by Boit [12], Ewing et al. [13], Hunter [14] and Flugge [15] to manage the energy dissipation in vibrating solids, where as it is observed that internal friction produces reduction of energy and dispersion. Cooper [16] and Ilioushin and Pobedria [17] investigated homogeneous plane wave problems of plane boundary between viscoelastic materials. Othman [19] proved the uniqueness reciprocity theorem for generalized viscothermoelasticity. Othman [20] studied the problem of half-space whose surface is subjected to a thermal shock under the effect of rotation in the context of generalized viscothermoelasticity. Sharma [21] investigated three dimensional vibration analysis of thermoelastic panel. Sherief and El-Maghraby [22] solved the dynamic problem of infinite thermoelastic solid with an internal penny shaped crack. Abdel-Halim and Elfalaky [23] solved an infinite thermoelastic solid weakened by an internal penny shaped crack. Tripathi et al. [24] discussed temperature and thermal stress distribution in a semi infinite cylinder.

The main aim of this paper is to study the homogenous and isotropic semi infinite cylinder subjected to coupled theory of viscothermoelasticity to present the variation of displacements, temperature change and stresses in case of mechanical forces and heat sources. Partial differential equations have been converted into ordinary differential equations by applying combination of Laplace and Hankel transforms. By inverting these dual transforms we obtain displacements, temperature change and stresses and solved numerically and presented graphically for Copper material.

2. FORMULATION OF PROBLEM

We consider a dynamical problem for semi infinite homogenous isotropic thick plate of height $2h$ and radius r defined as $-h \le z \le h$, $0 \le r \le a$. The nature of material is taken as viscoelastic, described by the Kelvin–Voigt model of linear viscoelasticity. The problem has been investigated using the cylindrical polar co–ordinates (r , ϑ , z). We consider axis of symmetry about z – axis at the origin in the middle plane between the upper and lower faces of plate and all quantities are independent of the co–ordinate ϑ . The displacement vector $\mathbf{u} = (u , 0 , w)$ and temperature change $T(r , z , t)$ is taken. All the considered functions will depend on r , z and t only. The governing basic equations of motion and heat conduction in the absence of body forces along with heat sources can be written as [22].

$$\mu^* \nabla^2 u - \frac{\mu^*}{r^2} u + (\lambda^* + \mu^*) \frac{\partial}{\partial r}\left(\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} \right) - \beta^* \frac{\partial T}{\partial r} = \rho \frac{\partial^2 u}{\partial t^2} \tag{1}$$

$$\mu^* \nabla^2 w + (\lambda^* + \mu^*) \frac{\partial}{\partial z}\left(\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} \right) - \beta^* \frac{\partial T}{\partial z} = \rho \frac{\partial^2 w}{\partial t^2} \tag{2}$$

$$k \nabla^2 T - \rho C_e T = \beta^* T_0 \left(\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} \right) \tag{3}$$

where

$$\lambda^* = \lambda\left(1 + \alpha_0 \frac{\partial}{\partial t}\right) \quad , \quad \mu^* = \mu\left(1 + \alpha_1 \frac{\partial}{\partial t}\right) \quad , \quad \beta^* = \beta_e\left(1 + \beta_0 \frac{\partial}{\partial t}\right) \quad ,$$

$$\beta_e = (3\lambda + 2\mu)\alpha_T \quad , \beta_0 = \left(\frac{(3\lambda\alpha_0 + 2\mu\alpha_1)\alpha_T}{\beta_e} \right) \tag{4}$$

The Laplace's operator and constitutive relations are

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2} \tag{5}$$

$$\sigma_{rr} = 2\mu^* \frac{\partial u}{\partial r} + \lambda^* \left(\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} \right) - \beta^* T \tag{6}$$

$$\sigma_{zz} = 2\mu^* \frac{\partial w}{\partial z} + \lambda^* \left(\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} \right) - \beta^* T \tag{7}$$

$$\sigma_{rz} = \mu^* \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial r} \right) \tag{8}$$

Here σ_{ij} ; $(i\,,\ j = r\,,\ z\,)$ are the stress components; β_e and β^* are the thermoelastic and viscothermoelastic coupling constant, α_0, α_1 are the viscoelastic relaxation times; λ^* , μ^* are Lame's viscoelastic parameters; λ , μ are Lame's parameters; α_T is the coefficient of linear thermal expansion; ρ is mass density; C_e is the specific heat at constant strain; k is the thermal conductivity.

3. INITIAL AND REGULAR BOUNDARY CONDITIONS

The medium is considered to be at rest, both mechanically and thermally, so that initial conditions are given by

$$u(r\,, z\,, t) = 0 = \frac{\partial u(r\,, z\,, t)}{\partial t} \quad , \qquad w(r\,, z\,, t) = 0 = \frac{\partial w(r\,, z\,, t)}{\partial t}, \quad \text{for} \quad t = 0\,,$$

$$T(r\,, z\,, t) = 0 = \frac{\partial T(r\,, z\,, t)}{\partial t} \qquad \text{for} \quad t = 0\,, \tag{9.1}$$

Mechanical loading boundary conditions

$$\sigma_{zz}(r\,, z\,, t) = f(r\,, t) \quad , \quad \text{for} \quad 0 \le r \le a \quad \text{at} \quad z = \pm h \tag{9.2}$$

$$\sigma_{rz}(r\,, z\,, t) = 0 \quad , \quad \text{for} \quad 0 \le r < \infty \quad \text{at} \quad z = \pm h$$

$$T(r\,, z\,, t) = 0 \quad , \quad \text{for} \quad 0 \le r < \infty \quad \text{at} \quad z = \pm h$$

Thermal loading boundary conditions

$$\sigma_{zz}(r\,, z\,, t) = 0 \quad , \quad \text{for} \quad 0 \le r < \infty \quad \text{at} \quad z = \pm h \tag{9.3}$$

$$\sigma_{rz}(r\,, z\,, t) = 0 \quad , \quad \text{for} \quad 0 \le r < \infty \quad \text{at} \quad z = \pm h$$

$$\frac{\partial T(r\,, z\,, t)}{\partial z} = f(r\,, t)\,, \quad \text{for} \quad 0 \le r \le a \quad \text{at} \quad z = \pm h$$

4. SOLUTION OF THE PROBLEM

For convenience, we introduce following non – dimensional quantities to remove complexity

$$R=\frac{r}{c_1}\quad,\quad Z=\frac{z}{c_1}\quad,\quad \theta=\frac{T}{T_0}\quad,\quad U=\frac{\rho c_1 u}{\beta_e T_0}\quad,\quad W=\frac{\rho c_1 w}{\beta_e T_0}\quad,\quad \tau=\frac{t}{c_1}\ ,$$

$$\tau_{ij}=\frac{\sigma_{ij}}{\beta_e T_0}\quad,\quad \varepsilon=\frac{T_0\beta_e{}^2}{\rho C_e(\lambda+2\mu)}\quad,\quad \delta^2=\frac{c_2^2}{c_1^2}\quad,\quad \omega^*=\frac{C_e(\lambda+2\mu)}{k}\quad,\quad \hat{\alpha}_0=\frac{\alpha_0}{c_1}\ , \tag{10}$$

$$\hat{\alpha}_1=\frac{\alpha_1}{c_1}\quad,\quad \hat{\beta}_0=\frac{\hat{\beta}_0}{c_1}\quad,\quad \delta_0=\hat{\alpha}_0+2\delta^2(\hat{\alpha}_1-\hat{\alpha}_0)\ ,\quad c_1^2=\frac{(\lambda+2\mu)}{\rho}\quad,\quad c_2^2=\frac{\mu}{\rho}\ ,$$

Using non – dimensional quantities of equation (10) in equations (1) to (9) we get

$$\left(1+\delta_0\frac{\partial}{\partial\tau}\right)\left(\frac{\partial^2U}{\partial R^2}+\frac{1}{R}\frac{\partial U}{\partial R}-\frac{U}{R^2}\right)+\delta^2\left(1+\hat{\alpha}_1\frac{\partial}{\partial\tau}\right)\frac{\partial^2U}{\partial Z^2}+\left(1-\delta^2+(\delta_0-\hat{\alpha}_1\delta^2)\frac{\partial}{\partial\tau}\right)\frac{\partial^2W}{\partial R\,\partial Z}-\left(1+\hat{\beta}_0\frac{\partial}{\partial\tau}\right)\frac{\partial\theta}{\partial R}=\frac{\partial^2U}{\partial\tau^2} \tag{11}$$

$$\left(1-\delta^2+(\delta_0-\hat{\alpha}_1\delta^2)\frac{\partial}{\partial\tau}\right)\left(\frac{\partial^2U}{\partial R\,\partial Z}+\frac{1}{R}\frac{\partial U}{\partial Z}\right)+\left(1+\delta_0\frac{\partial}{\partial\tau}\right)\frac{\partial^2W}{\partial Z^2}+\delta^2\left(1+\hat{\alpha}_1\frac{\partial}{\partial\tau}\right)\left(\frac{\partial^2W}{\partial R^2}+\frac{1}{R}\frac{\partial W}{\partial R}\right)-\left(1+\hat{\beta}_0\frac{\partial}{\partial\tau}\right)\frac{\partial\theta}{\partial Z}=\frac{\partial^2W}{\partial\tau^2} \tag{12}$$

$$\left(\frac{\partial^2\theta}{\partial R^2}+\frac{1}{R}\frac{\partial\theta}{\partial R}+\frac{\partial^2\theta}{\partial Z^2}\right)-\omega^*\theta=\varepsilon\,\omega^*\left(1+\hat{\beta}_0\frac{\partial}{\partial\tau}\right)\left(\frac{\partial U}{\partial R}+\frac{U}{R}+\frac{\partial W}{\partial Z}\right) \tag{13}$$

$$\tau_{RR}=\left(1+\delta_0\frac{\partial}{\partial\tau}\right)\frac{\partial U}{\partial R}+(1-2\delta^2)\left(1+\hat{\alpha}_0\frac{\partial}{\partial\tau}\right)\left(\frac{U}{R}+\frac{\partial W}{\partial Z}\right)-\left(1+\hat{\beta}_0\frac{\partial}{\partial\tau}\right)\theta \tag{14}$$

$$\tau_{ZZ}=\left(1+\delta_0\frac{\partial}{\partial\tau}\right)\frac{\partial W}{\partial Z}+(1-2\delta^2)\left(1+\hat{\alpha}_0\frac{\partial}{\partial\tau}\right)\left(\frac{\partial U}{\partial R}+\frac{U}{R}\right)-\left(1+\hat{\beta}_0\frac{\partial}{\partial\tau}\right)\theta \tag{15}$$

$$\tau_{RZ}=\delta^2\left(1+\hat{\alpha}_1\frac{\partial}{\partial\tau}\right)\left(\frac{\partial U}{\partial Z}+\frac{\partial W}{\partial R}\right) \tag{16}$$

5. SOLUTION OF THE PROBLEM IN LAPLACE TRANSFORM DOMAIN

To solve the system of equations (11) to (16) we use the Laplace transform defined by

$$L\{f(R\,,Z\,,\tau)\}=\int_0^\infty e^{-p\tau}f(R,Z,\tau)d\tau=\bar{f}(R,Z,p) \tag{17}$$

$$\frac{\partial^2\overline{U}}{\partial R^2}+\frac{1}{R}\frac{\partial\overline{U}}{\partial R}-\frac{\overline{U}}{R^2}-p\frac{1}{\delta_0^*}\overline{U}+\frac{\delta^2\hat{\alpha}_0^*}{\delta_0^*}\frac{\partial^2\overline{U}}{\partial Z^2}+\frac{(1-\delta^2)\delta^*}{\delta_0^*}\frac{\partial^2\overline{W}}{\partial R\,\partial Z}-\frac{\hat{\beta}_0^*}{\delta_0^*}\frac{\partial\overline{\theta}}{\partial R}=0 \tag{18}$$

$$\frac{(1-\delta^2)\delta^*}{\delta_0^*}\left(\frac{\partial^2\overline{U}}{\partial R\,\partial Z}+\frac{1}{R}\frac{\partial\overline{U}}{\partial Z}\right)+\frac{\partial^2\overline{W}}{\partial Z^2}+\frac{\delta^2\hat{\alpha}_0^*}{\delta_0^*}\left(\frac{\partial^2\overline{W}}{\partial R^2}+\frac{1}{R}\frac{\partial\overline{W}}{\partial R}\right)-p\frac{1}{\delta_0^*}\overline{W}-\frac{\hat{\beta}_0^*}{\delta_0^*}\frac{\partial\overline{\theta}}{\partial Z}=0 \tag{19}$$

$$\frac{\partial^2\overline{\theta}}{\partial R^2}+\frac{1}{R}\frac{\partial\overline{\theta}}{\partial R}+\frac{\partial^2\overline{\theta}}{\partial Z^2}-\omega^*\overline{\theta}=\varepsilon\,\omega^* p\,\hat{\beta}_0^*\left(\frac{\partial\overline{U}}{\partial R}+\frac{\overline{U}}{R}+\frac{\partial\overline{W}}{\partial Z}\right) \tag{20}$$

$$\bar{\tau}_{RR}=p\delta_0^*\frac{\partial\overline{U}}{\partial R}+(1-2\delta^2)p\alpha_0^*\left(\frac{\overline{U}}{R}+\frac{\partial\overline{W}}{\partial Z}\right)-p\beta^*\overline{\theta} \tag{21}$$

$$\bar{\tau}_{ZZ}=p\delta_0^*\frac{\partial\overline{W}}{\partial Z}+(1-2\delta^2)p\alpha_0^*\left(\frac{\partial\overline{U}}{\partial R}+\frac{\overline{U}}{R}\right)-p\beta^*\overline{\theta} \tag{22}$$

$$\bar{\tau}_{RZ}=p\delta^2\alpha_1^*\left(\frac{\partial\overline{U}}{\partial Z}+\frac{\partial\overline{W}}{\partial R}\right) \tag{23}$$

where $p^{-1}+\delta_0=\delta_0^*$, $p^{-1}+\alpha_0=\alpha_0^*$, $p^{-1}+\alpha_1=\alpha_1^*$,
$p^{-1}+\dfrac{\delta_0-\delta^2\alpha_1}{1-\delta^2}=\delta^*$, $p^{-1}+\beta_0=\beta_0^*$,

6. SOLUTION OF THE PROBLEM IN HANKEL TRANSFORM DOMAIN

We define Hankel transform with parameter α by

$$H\{\bar{f}(R\,,Z\,,\tau)\}=\int_0^\infty \bar{f}(R,Z,\tau)RJ_n(\alpha R)dR=\tilde{f}(\alpha,Z,p) \tag{24}$$

where $n=0$ for $\overline{W}$ & $\overline{\theta}$ and $n=1$ for $\overline{U}$ only. Upon applying Hankel Transform in equations (17) to (23) with parameter α of the function and the resulting equations and stresses are given by

$$(D^2 - a_{11})\tilde{U} - a_{12}D\tilde{W} + a_{13}\tilde{\theta} = 0 \tag{25}$$

$$a_{21}D\tilde{U} + (D^2 - a_{22})\tilde{W} - a_{23}D\tilde{\theta} = 0 \tag{26}$$

$$a_{31}\tilde{U} + a_{32}D\tilde{W} - (D^2 - a_{33})\tilde{\theta} = 0 \tag{27}$$

$$\tilde{\tau}_{RR} = -p\delta_0^*\alpha\tilde{U} + (1-2\delta^2)p\alpha_0^*\left(\frac{1}{\alpha}\tilde{U} + D\tilde{W}\right) - p\beta^*\tilde{\theta} \tag{28}$$

$$\tilde{\tau}_{ZZ} = (1-2\delta^2)p\alpha_0^*\alpha\tilde{U} + p\delta_0^*D\tilde{W} - p\beta^*\tilde{\theta} \tag{29}$$

$$\tilde{\tau}_{RZ} = p\delta^2\alpha_1^*\left(D\tilde{U} - \alpha\tilde{W}\right) \tag{30}$$

where

$$D = \frac{d}{dZ}\ , \quad c_1^* = \frac{\delta^2\hat{\alpha}_0^*}{\delta_0^*}\ , \quad c_2^* = \frac{(1-\delta^2)\delta^*}{\delta_0^*}\ , \quad c_3^* = \frac{\hat{\beta}_0^*}{\delta_0^*}\ , \quad c_4^* = \varepsilon\omega^*\hat{\beta}_0^*\ ,$$

$$a_{11} = \frac{p}{\delta_0^* c_1^*} + \frac{\alpha^2}{c_1^*}\ , \quad a_{12} = \frac{c_2^*\alpha}{c_1^*}\ , \quad a_{13} = \frac{c_3^*\alpha}{c_1^*}\ ,$$

$$a_{21} = c_2^*\alpha\ , \quad a_{22} = c_1^*\alpha^2 + \frac{p}{\delta^*}\ , \quad a_{23} = c_3^*$$

$$a_{31} = pc_4^*\alpha\ , \quad a_{32} = pc_4^*\ , \quad a_{33} = \alpha^2 - \omega^*$$

7. SOLUTION TO OBTAIN FIELD FUNCTIONS

Eliminating $\tilde{W}$ and $\tilde{\theta}$ between equations (25) to (27) we obtain the ordinary differential equation that satisfied with $\tilde{U}$

$$\left\langle D^6 - LD^4 + MD^2 - N\right\rangle\tilde{U} = 0 \tag{31}$$

where $L = a_{11} + a_{22} + a_{33} - a_{23}a_{32} - a_{12}a_{21}$

$$M = a_{22}a_{33} + a_{11}a_{22} + a_{11}a_{33} + a_{31}a_{13} - a_{11}a_{23}a_{32} + a_{12}a_{23}a_{31} - a_{12}a_{21}a_{33} - a_{13}a_{21}a_{32}$$

$$N = a_{11}a_{23}a_{33} + a_{13}a_{31}a_{22}$$

In a similar manner, we can show that $\tilde{W}$ and $\tilde{\theta}$ satisfy the equation

$$\left\langle D^6 - LD^4 + MD^2 - N \right\rangle \left\{ \tilde{W}, \tilde{\theta} \right\} = 0 \qquad (32)$$

The equation (31) can be factorized as

$$\left\{ (D^2 - m_1^2)(D^2 - m_2^2)(D^2 - m_3^2) \right\} \tilde{U} = 0 \qquad (33)$$

Solution of the equation (31) can be assumed as

$$\tilde{U} = \sum_{i=1}^{3} L_i \cosh m_i Z \qquad (34)$$

Similarly we can find

$$\tilde{W} = \sum_{i=1}^{3} L_i V_i \sinh m_i Z \qquad (35)$$

$$\tilde{\theta} = \sum_{i=1}^{3} L_i S_i \cosh m_i Z \qquad (36)$$

where L_i are arbitrary constant parameters and m_i^2 are roots of characteristic equation of equation (33).

$$V_i = \frac{\Delta'}{\Delta} \quad ; \quad S_i = \frac{\Delta''}{\Delta} \quad ; \quad \Delta = m_i^4 - (a_{22} + a_{33} - a_{23}a_{32}) m_i^2 + a_{22}a_{33}$$

$$\Delta' = -a_{21} m_i^2 + (a_{21}a_{33} - a_{23}a_{31}) \quad ; \quad \Delta'' = -(a_{33} - a_{21}a_{32}) m_i^2 + a_{22}a_{33}$$

$$\tilde{\tau}_{ZZ} = \sum_{i=1}^{3} L_i \cosh m_i Z \left[a_{42}\alpha + a_{41}V_i - a_{43}S_i \right] \qquad (37)$$

$$\tilde{\tau}_{RZ} = a_{44} \left(\sum_{i=1}^{3} L_i \sinh m_i Z (1 - \alpha V_i \right) \qquad (38)$$

where $a_{41} = p\delta_0^*$, $a_{42} = (1 - 2\delta^2) p\alpha_0^*$, $a_{43} = p\beta_0^*$, $a_{44} = p\delta^2\alpha_1^*$

Taking Inverse Hankel Transform of equations (34) to (38) we get

$$\overline{U} = \int_0^\infty \left[\sum_{i=1}^{3} L_i \cosh m_i Z \right] J_1(\alpha R) d\alpha \qquad (39)$$

$$\overline{W} = \int_0^\infty \left[\sum_{i=1}^{3} L_i \, V_i \sinh \, m_i Z \right] \alpha J_0(\alpha R) d\alpha \tag{40}$$

$$\overline{\theta} = \int_0^\infty \left[\sum_{i=1}^{3} L_i \, S_i \cosh \, m_i Z \right] \alpha J_0(\alpha R) d\alpha \tag{41}$$

$$\overline{\tau}_{ZZ} = \int_0^\infty \sum_{i=1}^{3} L_i \cosh \, m_i Z \left[a_{42}\alpha + a_{41}V_i - a_{43}S_i \right] \alpha \, J_0(\alpha R) \, d\alpha \tag{42}$$

$$\overline{\tau}_{RZ} = \int_0^\infty a_{44} \left(\sum_{i=1}^{3} L_i \sinh \, m_i Z (1 - \alpha V_i) \right) J_1(\alpha R) \, d\alpha \tag{43}$$

After applying the Hankel transform, the boundary conditions become,

$$\tilde{\tau}_{ZZ} = \tilde{f}(\alpha\,,\,p) \quad , \quad \tilde{\tau}_{RZ} = 0 \quad , \quad \tilde{\theta} = 0 \quad \text{at} \quad Z = \pm h \tag{44}$$

Thermal boundary conditions

$$\tilde{\tau}_{ZZ} = 0 \quad , \quad \tilde{\tau}_{RZ} = 0 \quad , \quad \tilde{\theta},_{Z} = \tilde{f}(\alpha\,,\,p) \quad \text{at} \quad Z = \pm h \tag{45}$$

Case I: Mechanical load on the surface:

On applying boundary conditions in equations (44) to determine unknown parameters, we get

$$\sum_{i=1}^{3} L_i \cosh \, m_i h \left[a_{42}\alpha + a_{41}V_i - a_{43}S_i \right] \alpha = \bar{f}(\alpha\,,\,p) \tag{46}$$

$$a_{44} \left(\sum_{i=1}^{3} L_i \sinh \, m_i Z (1 - \alpha V_i \right) = 0 \tag{47}$$

$$\alpha \left[\sum_{i=1}^{3} L_i \, S_i \cosh \, m_i Z \right] = 0 \tag{48}$$

Case II: Thermal load on the surface

On applying boundary conditions in equations (45) to determine unknown parameters, we get

$$\sum_{i=1}^{3} L_i \cosh \, m_i h \, (a_{42}\alpha + a_{41}V_i - a_{43}S_i) \alpha = 0 \tag{49}$$

$$a_{44}\left(\sum_{i=1}^{3} L_i \sinh m_i Z(1-\alpha V_i\right)=0 \tag{50}$$

$$\alpha\left(\sum_{i=1}^{3} L_i\, S_i\, m_i \sinh\, m_i Z\right)=\bar{f}(\alpha\ ,\ p) \tag{51}$$

On solving the equations (39) – (43) and (46) – (51) analytically and numerically we obtain the solution of the given problem in the transformed domain.

8. INVERSE OF DOUBLE TRANSFORMS

In order to determine the solution of the problem in physical domain, we take inverse transformations of equations (46) to (51). To remove the complexity of the solution in the Laplace transform in physical domain we find the inverse of Laplace transform which is obtained by using the Gaver – Stehfast algorithm [25, 26]. The work completed which have been done by Widder [27] who developed an inversion operator for Laplace transform. Gaver – Stehfast modified this operator and derived the formula

$$f(t)=\frac{\ln 2}{\tau}\sum_{j=1}^{k} D(j\ ,\ k)F\left(j\frac{\ln 2}{\tau}\right) \tag{52}$$

with

$$D(j\ ,\ k)=(-1)^{j+M}\sum_{n=m}^{\min(j\ ,\ M)}\frac{n^M(2n)!}{(M-n)!n!(n-1)!(j-n)!(2n-j)!} \tag{53}$$

where k is an even integer, whose value depends on the word length the computer used. $M=\frac{k}{2}$ and m is the integer part of the $\frac{(j+1)}{2}$. The optimal value of k was chosen as described in Stehfast algorithm, for the fast convergence of results with the desired accuracy. The numerical integration technique Press et al. [28] have been done with variable step size was used to evaluate the integrals involved in related equations. Computer analyzed and simulated results have been obtained by the use of MATLAB software tools.

9. NUMERICAL RESULTS AND DISCUSSION

For numerical computation we are supposed to take

$$f(R, \tau) = \begin{cases} \sigma_0 F(a-R) F(\tau) & \text{for} \quad \text{mechanical loads} \\ \theta_0 H(a-R) H(\tau) & \text{for} \quad \text{thermal loads} \end{cases} \tag{54}$$

Here σ_0 and θ_0 are constants. On taking Laplace and Hankel transformations in equation (53) we get:

$$\tilde{f}(\alpha, p) = \begin{cases} (\sigma_0\, a\, J_1(\alpha\, a)) / \alpha p \\ (\theta_0\, a\, J_1(\alpha\, a)) / \alpha p \end{cases} \tag{55}$$

In order to demonstrate the problem, the copper material has been taken for the computation purpose whose physical data is given in Table 1 below:

Table 1: Physical data of copper material

S. No.	Coefficient	Units	Value	References
1.	ρ	$kg\, m^{-3}$	8954	[29,30]
2.	T_0	K	293	[29,30]
3.	λ	GPa	77.6	[29,30]
4.	μ	GPa	38.6	[29,30]
5.	C_e	$J\, kg^{-1} K^{-1}$	383.1	[29,30]
6.	K	$W\, m^{-1} K^{-1}$	386	[29,30]
7.	α_T	K^{-1}	1.78×10^{-5}	[29,30]
8.	$\alpha_0 = \alpha_1$	s	6.8831×10^{-13}	[16]

Here Figs. 1 – 8 have been presented for the cases of viscothermoelastic (VTE), thermoelastic (TE), viscoelastic (VE) and elastic (E). Here Fig. 1 and Fig. 2 have been presented for radial and axial displacements. Figs. 3 – 5 have plotted for mechanical sources and Figs. 6 – 8 have been drawn for heat sources. Fig. 1 has been plotted for radial displacement versus radius (R) . It can be inferred from Fig. 1, that initially the variation is very high in all the cases and with increase in radius (R) the variation of vibrations go on decreasing and die out. The variation is highest in thermoelatic (TE) case rather than others due to thermal effects.

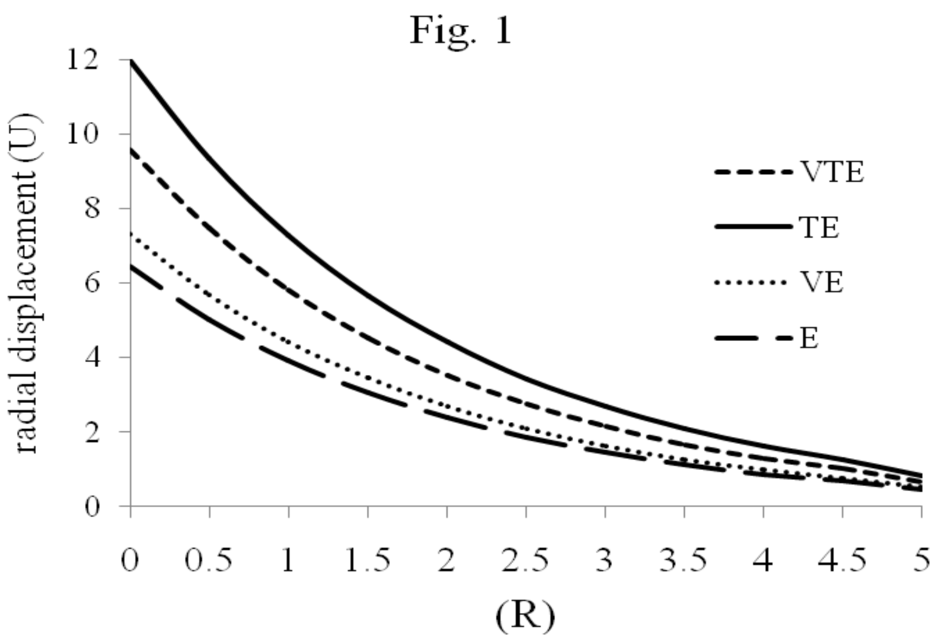

Fig. 1: Variation of radial displacement (U) versus radius (R) .

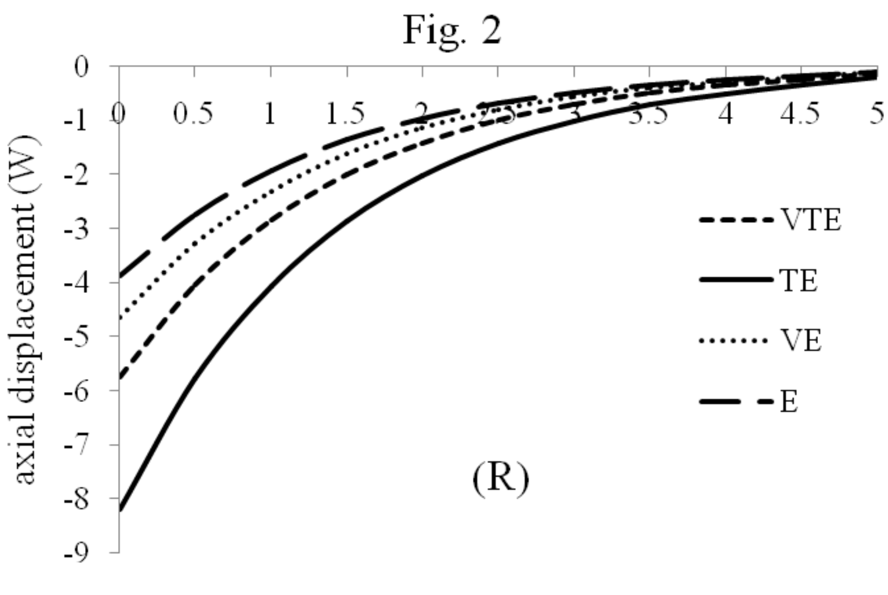

Fig. 2: Variation of axial displacement (U) versus radius (R) .

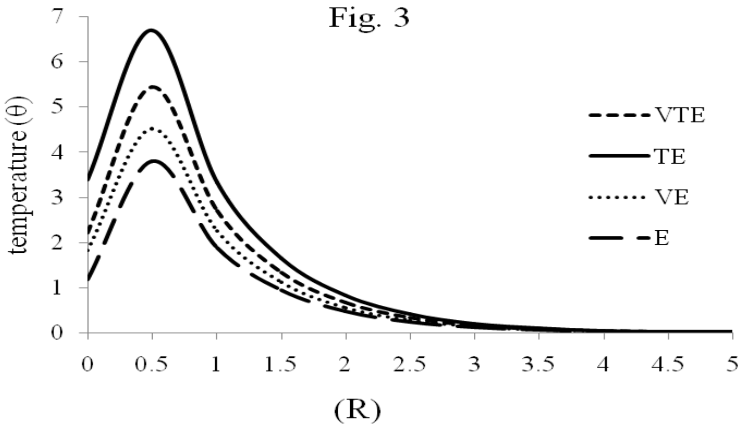

Fig. 3: Variation of temperature (θ) versus radius (R) .

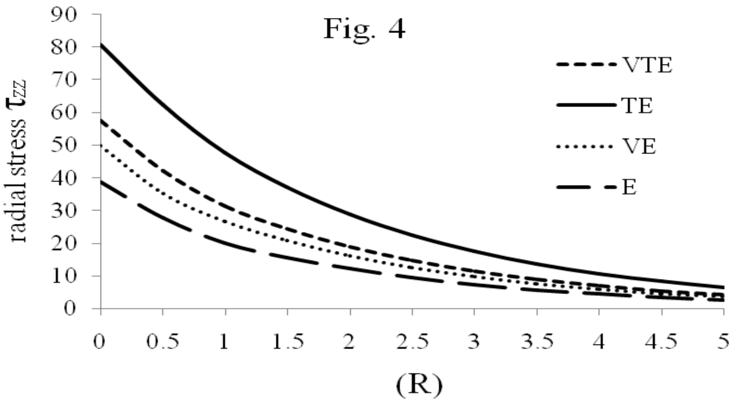

Fig. 4: Variation of radial stress (τ_{RZ}) versus radius (R) for mechanical loads.

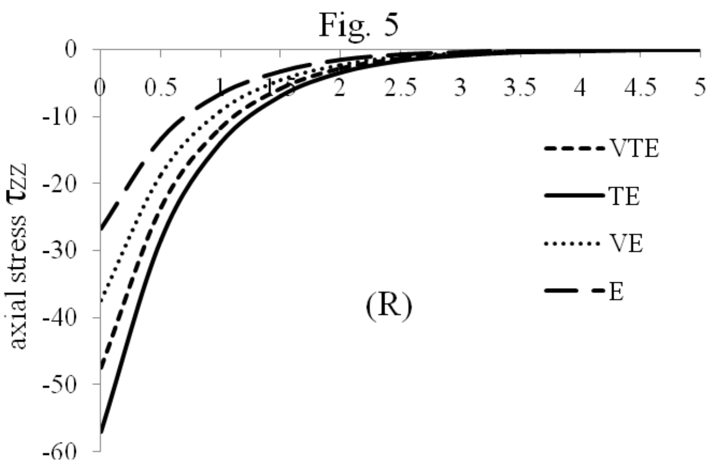

Fig. 5: Variation of axial stress (τ_{ZZ}) versus radius (R) for mechanical loads..

Fig. 2 has been drawn for axial displacement versus radius (R). The variations are apposite to that of radial displacement. Here the variations are initially low and with increase in the value of (R) the variation of vibrations die out. Fig. 3 repersents the variation of temperature versus radius (R) in case of mechanical loads. At $(R=0)$ the variation meager, maximum at $(R=0.5)$ and with increase in value of (R) the variations die out. Fig. 4 and Fig. 5 have been presented for radial stresses and axial stresses for versus (R) for viscothermoelastic (VTE), thermoelastic (TE), viscoelastic (VE) and elastic (E) in case of mechanical loading. It is revealed from these Figs. that at $(R=0)$ the variations are highest and with increase in value of (R) the variation of vibrations go on decreasing and die out.

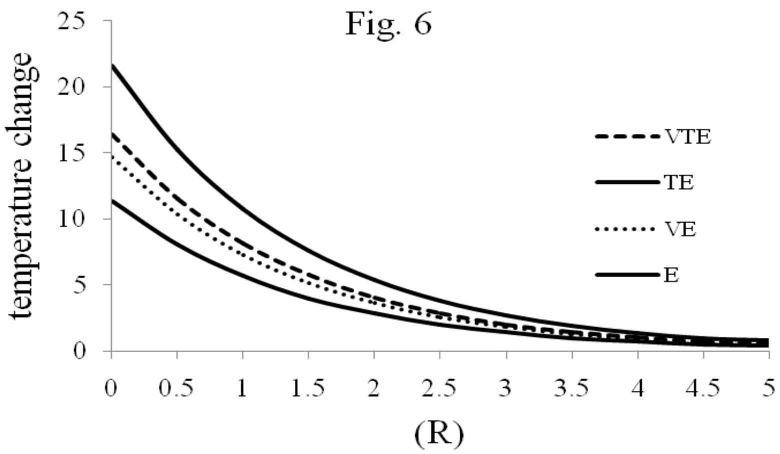

Fig. 6: Variation of temperature change (θ) versus radius (R).

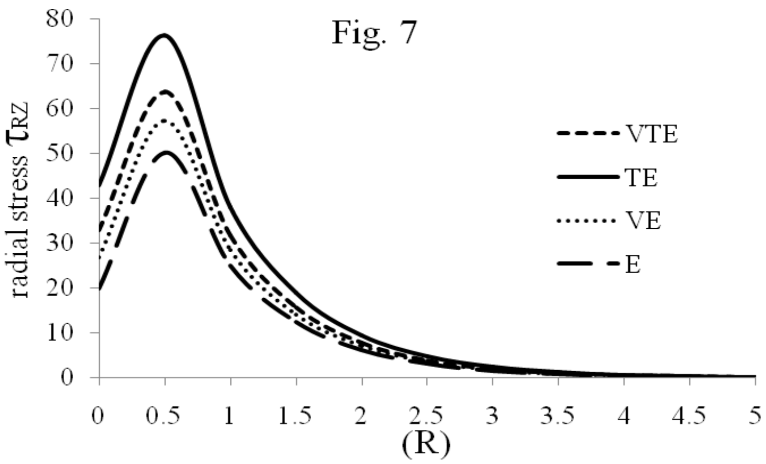

Fig. 7: Variation of radialaxial stress (τ_{RZ}) versus radius (R) for mechanical loads..

Figs 6 to 8 have been presented for variation of temperature change, radial stresses and axial stresses for versus (R) for viscothermoelastic (VTE), thermoelastic (TE), viscoelastic (VE) and elastic (E) in case of thermal loading. It is inferred from Fig. 6 that initially the variation at $(R=0)$ is high and with increase in value of radius that the variation of temperature die out. It is noticed from Fig. 7 and Fig. 8 that the variations is meager initially and highest at $(R=0.5)$ in radial as well as axial stresses and with increase in the value of (R) the variation of vibrations go on decreasing and die out. It is concluded that the behavior in Fig. 4 and Fig. 5 have reverse variation and same behaviour have been observed in Fig. 7 and Fig. 8.

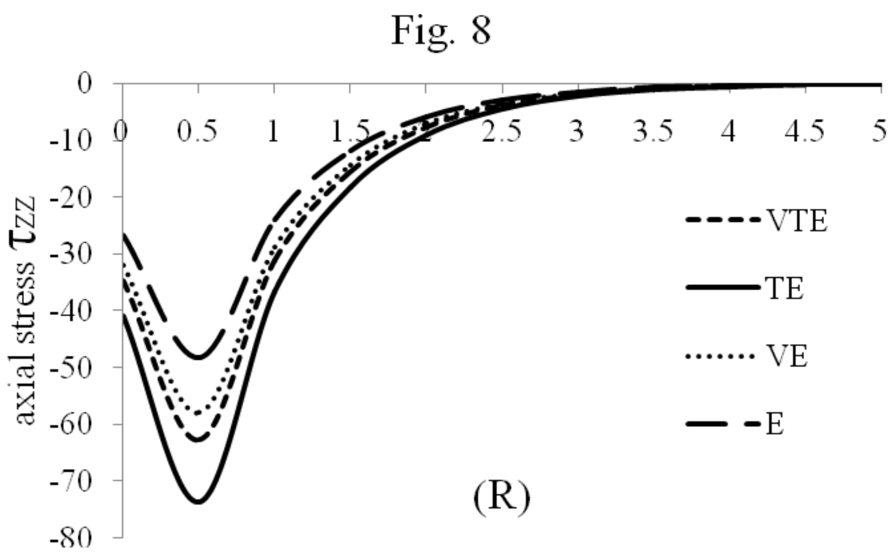

Fig. 8: Variation of axial stress (τ_{ZZ}) versus radius (R) for thermal loads..

CONCLUSION:

The problem have been solved the semi infinite cylinder with mechanical forces and thermal heat sources and presented graphically for copper material. We have directly calculated the solutions of field functions without finding potential functions. The numerical inversion techniques have been used are fast and accurate than other methods. From the behaviour of the graphs, it has been concluded from al Figs. that the variation of the vibrations are very high at origin and as we move away from origin the value of (R) the variation of vibrations are decreasing and die out. It is also observed from all the Figs. that the variation is very high in thermoelastic (TE) case as compared to viscothermoelastic (VTE), viscoelastic (VE) and elastic (E) cases due to thermal effects. The study may find useful and wide range of applications in the design and construction of sensors and other acoustic waves to possible bio industries.

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