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Markovian Ration Model – Two commodities perishable inventory system

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Abstract

In this article, a continuous review of two commodities ration model inventory system has been taken up for consideration. We assume S_1 and S_2 are denoted the maximum level for the first commodity and second commodity respectively. These items are issued by the server. The arrivals of demands for the first commodity are assumed to follow Poisson process. Also we assume that the nature of the first commodity items are perishable. The life time of the items are assumed to follow the negative exponential distribution. Whenever the on hand inventory level for the first commodity drops to the prefixed level s, the server places an order for $Q(= S_1 - s)$ units. These items are replenished after some random interval of time. The replenishment times are assumed to be exponential. When the inventory level of the first commodity drops to zero, the server is available to serve the second commodity. The inter arrival times of demands for the second commodity is assumed to be independent exponential distribution. For the second commodity, we assume the instantaneous supply of orders. The server finds the positive inventory level for the first commodity and immediately he terminates his service of providing the second commodity and be available to serve for the first commodity. The matrix analytic method is used for finding the steady state distribution of the model. Various performance measures and the cost analysis are shown with numerical results.

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Keywords: Continuous review Inventory Systems, Perishable items, Instantaneous supply, Two commodity.

1. Introduction

In inventory theory, the inventory is supplied to the demands immediately if it is available. In some occasions the items are delivered after sometime due to the service on the items, like installation time for computer products, packing time in super markets etc. In most of the models the author assumes the nature of the item is perishable. The items are replenished instantaneously or after sometime due to the operating policy for the model. The items are replenished after sometime means that we have a chance to meet the stock-out period. The demands that occur during the stock-out period either may lost or retry after some time. There are many different models available in the literature which handle the stock-out period properly in the sense of maximizing the profit.

There are many researcher have analysed the inventory model under Markovian environment. There by they relate the input and the output processes to the suitable random variables and that the random variables are having the special property, that is, Markovian property. These models are constructed and developed that are applicable in the real life.

In this article we consider the two commodities inventory model. The operating policy for the first commodity is (s, S_1) ordering policy. According to the policy the server order for $Q(= S_1 - s)$ items when the inventory level depletes to the prefixed level s. Here we assume the items in the first commodity are perishable in nature. So the inventory level for the first commodity is reduced by the arrival of demand for the first commodity or perish. Further the arrival of demands for the second commodity occurs only when the inventory level for the first commodity is zero. For commodity two, we applied the operating policy as instantaneous supply of orders. According to the instantaneous supply of orders, the inventory replenished immediately after the last item sold and bring the inventory level of the second commodity to maximum.

The rest of the paper is organized as follows, the first section narrates the problem formulation in the next section pertaining to the model description, section 3 deals the problem mathematically. In section 4, we calculated the limiting probabilities in steady state case and in the next section we derived the important system performance measures. The paper concludes with numerical illustration.

Notation:

- $[A]_{ij}$: (i, j) th position of A
- $\mathbf{0}$: Zero vector
- I_k : Identity matrix of order k
- $\mathbf{e}$: a column vector of 1's with appropriate dimension
- $\delta_{i,j}$: Kronecker delta function

2. Problem formulation

A continuous review of two commodities ration model inventory system has been taken up for consideration. We assume S_1 and S_2 are the maximum level for the commodity one and commodity two respectively. These items are issued by the server one by one. The arrivals of demands for the commodity one are assumed to follow Poisson process with parameter λ. Also we assume that the nature of the first commodity is perishable. The life time of the items are assumed to follow exponential distribution with rate γ. Whenever the on hand inventory level for the commodity one drops to the prefixed level s, he places an order for $Q(= S_1 - s)$ units. These items are replenished after some random interval of time. The replenishment times are assumed to be exponential with mean $\frac{1}{\mu}$. When the inventory level of commodity one drops to zero, he is available to serve commodity two. The inter arrival times of demands for the commodity two is assumed to be independent exponential distribution with parameter ν. For commodity two, we assume the instantaneous supply of orders. The server finds the positive inventory level of commodity one he immediately terminates his service of providing commodity two and be available to serve the commodity one.

3. Model Description

Let $A(t)$ and $B(t)$ denote the inventory level of the first and second commodity respectively at time t. From the assumptions made on the input and output processes, it can be shown that the stochastic process $(A, B) = \{(A(t), B(t)), t \geq 0\}$ is a continuous time Markov process with state space E and it is given by

$$E = \{(i, j) : i = 0, 1, \ldots, S_1, \ j = 1, 2, \ldots, S_2\}.$$

$$P((i, j), (k, l)) = Pr[A(t + \Delta t) = k, B(t + \Delta t) = l | A(t) = i, B(t) = j].$$

Group the states as $\{(0), (1), \ldots, (S_1)\}$ where each entry $(i), \ i = 0, 1, \ldots, S_1$ represents the vector $\{(i, 1), (i, 2), \ldots, (i, S_2)\}$. Then the infinitesimal generator P can be conveniently expressed in block partitioned matrix,

$$\mathbb{P} = \begin{array}{c} \\ (0) \\ (1) \\ \vdots \\ (s) \\ \vdots \\ (Q) \\ (Q+1) \\ \vdots \\ (S) \end{array} \begin{array}{c} \begin{array}{cccccccccc} (0) & (1) & (2) & \cdots & (s) & \cdots & (Q) & (Q+1) & \cdots & (S) \end{array} \\ \left(\begin{array}{cccccccccc} A_0 & & & & & & C_0 & & & \\ B_1 & A_1 & & & & & & C_1 & & \\ & & \ddots & \ddots & & & & & \ddots & \\ & & & B_s & A_s & & & & & C_1 \\ & & & & \ddots & \ddots & & & & \\ & & & & & B_Q & A_Q & & & \\ & & & & & & B_{Q+1} & A_{Q+1} & & \\ & & & & & & & \ddots & \ddots & \\ & & & & & & & & B_S & A_S \end{array} \right) \end{array}$$

where,

$$A_0 = \begin{array}{c} (0,1) \\ (0,2) \\ (0,3) \\ \vdots \\ (0,S_2-1) \\ (0,S_2) \end{array} \overset{\begin{array}{cccccc} (0,1) & (0,2) & (0,3) & \cdots & (0,S_2-1) & (0,S_2) \end{array}}{\begin{pmatrix} -\nu-\mu & & & & & \nu \\ \nu & -\nu-\mu & & & & \\ & \nu & -\nu-\mu & & & \\ & & \ddots & \ddots & & \\ & & & \nu & -\nu-\mu & \\ & & & & \nu & -\nu-\mu \end{pmatrix}}$$

Matrix	Description	Dimension
$A_i,\ i = 1, 2, \ldots, s$	$-(\lambda + \mu + i\gamma)I_{S_2}$	S_2
$A_i,\ i = s+1, s+2, \ldots, S$	$-(\lambda + i\gamma)I_{S_2}$	S_2
$B_i,\ i = 1, 2, \ldots, S$	$(\lambda + i\gamma)I_{S_2}$	S_2
C	μI_{S_2}	S_2

4. Steady State Analysis

It can be seen from the structure of $\mathbb{P}$ that the homogeneous Markov process $\{A(t), B(t) : t \geq 0\}$ on the finite space E is irreducible. Hence the limiting distribution

$$\begin{aligned} \pi^{(i,k)} &= \lim_{t\to\infty} Pr[A(t) = i, B(t) = k | A(0), B(0)], \text{ exists.} \\ \text{Let } \pi^{(i)} &= (\pi^{(i,1)}, \pi^{(i,2)}, \ldots, \pi^{(i,S_2)}),\ i = 0, 1, \ldots, S_1, \\ \text{and } \Pi &= (\pi^{(0)}, \pi^{(1)}, \ldots, \pi^{(S_1)}). \end{aligned}$$

Then the vector of limiting probabilities Π satisfies

$$\Pi\mathbb{P} = 0 \text{ and } \Pi = 1. \tag{4.1}$$

The first equation of the above yields the following set of equations:

$$\begin{aligned} \pi^{(i+1)}B_{i+1} + \pi^{(i)}A_i &= \mathbf{0}, \quad & i &= 0, 1, \ldots, Q-1, \\ \pi^{(i+1)}B_{i+1} + \pi^{(i)}A_i + \pi^{(i-Q)}C &= \mathbf{0}, \quad & i &= Q, Q+1, \ldots, S_1-1, \\ \pi^{(i)}A_i + \pi^{(i-Q)}C &= \mathbf{0}, \quad & i &= S_1. \end{aligned}$$

The equations can be recursively solved to get

$$\pi^{(i)} = \pi^{(Q)}\Omega_i, \quad i = 0, 1, \ldots, S_1,$$

where

$$\Omega_i = \begin{cases} (-1)^{Q-i} B_Q A_{Q-1}^{-1} B_{Q-1} \cdots B_{i+1} A_i^{-1}, & i = 0, 1, \ldots, Q-1, \\ I, & i = Q, \\ (-1)^{2Q-i+1} \sum_{j=0}^{S_1-i} \Big[\Big(B_Q A_{Q-1}^{-1} B_{Q-1} \cdots B_{s+1-j} A_{s-j}^{-1} \Big) C A_{S_1-j}^{-1} & \\ \quad \times \Big(B_{S_1-j} A_{S_1-j-1}^{-1} B_{S_1-j-1} \cdots B_{i+1} A_i^{-1} \Big) \Big], & i = Q+1, \ldots, S_1, \end{cases}$$

and $\pi^{(Q)}$ can be obtained by solving

$$\pi^{(Q)}\Bigg[(-1)^Q\sum_{j=0}^{s-1}\Big[\Big(B_Q A_{Q-1}^{-1}B_{Q-1}\cdots B_{s+1-j}A_{s-j}^{-1}\Big)CA_{S_1-j}^{-1} \times\Big(B_{S_1-j}A_{S_1-j-1}^{-1}B_{S_1-j-1}\cdots B_{Q+2}A_{Q+1}^{-1}\Big)\Big]B_{Q+1}+A_Q +(-1)^Q B_Q A_{Q-1}^{-1}B_{Q-1}\cdots B_1A_0^{-1}C\Bigg]=\mathbf{0},$$

and

$$\pi^{(Q)}\Bigg[\sum_{i=0}^{Q-1}\Big((-1)^{Q-i}B_QA_{Q-1}^{-1}B_{Q-1}\cdots B_{i+1}A_i^{-1}\Big)+I + \sum_{i=Q+1}^{S_1}\Bigg((-1)^{2Q-i+1}\sum_{j=0}^{S_1-i}\Big[\Big(B_QA_{Q-1}^{-1}B_{Q-1}\cdots B_{s+1-j}A_{s-j}^{-1}\Big)CA_{S_1-j}^{-1} \times\Big(B_{S_1-j}A_{S_1-j-1}^{-1}B_{S_1-j-1}\cdots B_{i+1}A_i^{-1}\Big)\Big]\Bigg)\Bigg]\mathbf{e}=1.$$

5. System performance measures

In this section, we derive some system performance measures in the steady-state case.

5.1. Expected inventory level

Let $E[Inv_1]$ denote the expected inventory level for the first commodity in the steady-state. Then $E[Inv_1]$ is given by

$$E[Inv_1]=\sum_{i=1}^{S_1}\sum_{j=1}^{s_2}i\pi^{(i,j)}$$

Let $E[Inv_2]$ denote the expected inventory level for the first commodity in the steady-state. Then $E[Inv_2]$ is given by

$$E[Inv_2]=\sum_{i=0}^{S_1}\sum_{j=1}^{s_2}j\pi^{(i,j)}$$

5.2. Expected reorder rate

Let $E[R]$ denote the expected reorder level in the steady-state. Then $E[R]$ is given by

$$E[R]=\sum_{j=1}^{S_2}(\lambda+\gamma)\pi^{(s+1,j)}$$

5.3. Expected Perishable rate

Let $E[P_r]$ denote the expected reorder level in the steady-state. Then $E[R]$ is given by

$$E[P_r] = \sum_{i=1}^{S_1} \sum_{j=1}^{s_2} i\gamma\pi^{(i,j)}$$

5.4. Cost function

The long-run total expected cost rate for this model is defined to be

$$E[C] = H_{c_1}E[Inv_1] + H_{c_2}E[Inv_2] + S_cE[R] + F_cE[P_r].$$

where,

H_{c_1}: The inventory carrying cost for the commodity 1 per unit item per unit time
H_{c_2}: The inventory carrying cost for the commodity 2 per unit item per unit time
S_c : Set-up cost per order per unit time
F_c : Cost per unit failure (Commodity 1)

6. Numerical Illustration

Figure 1 gives the effect of the arrival rate for the first commodity and the replenishment rate for the first commodity on the long run total expected cost rate $E[C]$ and Figure 2 gives the effect of the arrival rate for the first commodity and the failure rate for the first commodity on the long run total expected cost rate $E[C]$.

Table 1 gives the variation on the long run total expected cost rate $E[C]$ by varying the holding cost for the first commodity (H_{C_1}), the holding cost for the second commodity

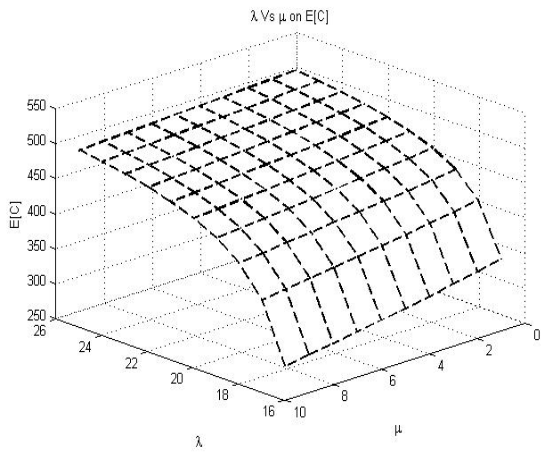

Figure 1: λ vs μ variation on $E[C]$

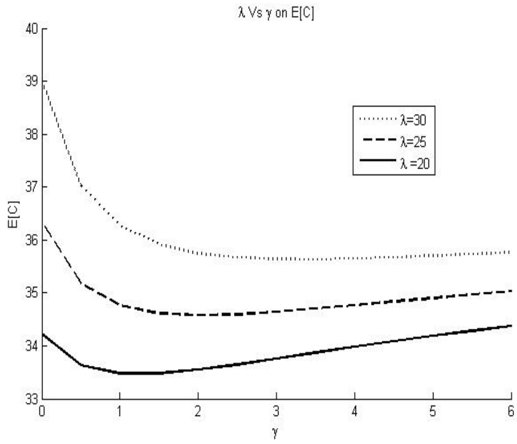

Figure 2: γ vs λ variation on $E[C]$

Table 1: Effect of cost parameters on long run total expected cost rate $E[C]$

H_{C_1}	H_{C_2}	S_c	F_c			
			0.1	0.2	0.3	0.4
2	1	15	48.765845	48.946897	49.127950	49.309002
		20	50.425699	50.606751	50.787803	50.968856
		25	52.085552	52.266604	52.447657	52.628709
	2	15	74.265845	74.446897	74.627950	74.809002
		20	75.925699	76.106751	76.287803	76.468856
		25	77.585552	77.766604	77.947657	78.128709
	3	15	99.765845	99.946897	100.127950	100.309002
		20	101.425699	101.606751	101.787803	101.968856
		25	103.085552	103.266604	103.447657	103.628709
4	1	15	66.871077	67.052130	67.233182	67.414234
		20	68.530931	68.711983	68.893035	69.074088
		25	70.190784	70.371837	70.552889	70.733941
	2	15	92.371077	92.552130	92.733182	92.914234
		20	94.030931	94.211983	94.393035	94.574088
		25	95.690784	95.871837	96.052889	96.233941
	3	15	117.871077	118.052130	118.233182	118.414234
		20	119.530931	119.711983	119.893035	120.074088
		25	121.190784	121.371837	121.552889	121.733941
6	1	15	84.976310	85.157362	85.338414	85.519467
		20	86.636163	86.817215	86.998268	87.179320
		25	88.296017	88.477069	88.658121	88.839174
	2	15	110.476310	110.657362	110.838414	111.019467
		20	112.136163	112.317215	112.498268	112.679320
		25	113.796017	113.977069	114.158121	114.339174
	3	15	135.976310	136.157362	136.338414	136.519467
		20	137.636163	137.817215	137.998268	138.179320
		25	139.296017	139.477069	139.658121	139.839174

(H_{C_2}), set-up cost for the first commodity (S_c) and the failure cost(F_C) (per unit item for the first commodity).

From Figure 1 and 2 we observe that the long run total expected cost rate $E[C]$ increases when the arrival rate (λ) for the first commodity increases, the long run total expected cost rate $E[C]$ decreases when the replenishment rate increases and the long run total expected cost rate $E[C]$ increases when the perishable rate for the first commodity (γ) increases. From Table 1, We conclude that the long run total expected cost rate $E[C]$ increases, when H_{C_1}, H_{C_2}, S_C and F_C increase.

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