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Global Exponential Stability for Impulsive Functional Differential Equations

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Abstract

This paper studies the global exponential stability of the impulsive functional differential by using Lyapunov functions and Razumikhin techniques. The obtained results extend and generalize some results existing in the literature. Moreover, the Razumikhin condition obtained is very simple.

Keywords: Impulsive functional differential equations, Global exponential stability, Razumikhin techniques, Lyapunov functions.

Subject classification codes: 34K20, 93D05, 34K38

1. INTRODUCTION

There has been a significant development in the theory of impulsive differential equations in recent years. Impulsive differential equations have attracted many researchers attention due to their wide application in many fields such as control technology, drug administration and threshold theory in biology and the like.

There are several research works which appeared in the literature on stability of impulsive functional differential equations; see [1-12]. Recently, there have been some research works in this direction, in [5-7], the authors investigated the uniform asymptotical stability and global exponential stability of impulsive functional differential equation. In [13], the authors obtained some results on exponential stabilization of impulsive functional differential equations. The method of Lyapunov functions and Razumikhin technique have been widely applied to stability analysis of various impulsive function differential equations and they have also proved to be powerful tool in the investigation of impulsive functional differential equations [5-7, 13-18].

In this paper we further study exponential stability of impulsive functional differential equations from the control point of view. Some new stability results are obtained by employing the Razumikhin technique and Lyapunov functions.

2. PRELIMINARIES

Notation. Let $\mathbb{R}$ denote the set of real numbers, $\mathbb{R}_+$ the set of nonnegative real numbers, $\mathbb{Z}_+$ the set of positive integers and $\mathbb{R}^n$ the n– dimensional real space equipped with the Euclidean norm $|\cdot|$. $K = \{a \in C(\mathbb{R}_+,\mathbb{R}_+) \mid a(0) = 0$ and $a(s) > 0$ for $s > 0$ and a is strictly increasing in s$\}$. For any $t \geq t_0 \geq 0 > \alpha \geq -\infty$, let $f(t, x(s))$ where $s \in [t+\alpha, t]$ or $f(t, x(.))$ be a Volterra type functional. In the case where α = -∞, the interval $[t + \alpha, t]$ is understood to be replaced by $(-\infty, t]$.

Consider the following impulsive functional differential equations:

$$\begin{cases} x'(t) = f(t, x(.)), & t \geq \sigma, t \neq t_{k,} \\ \Delta x_{t=t_k} = x(t_k) - x(t_k^-) = I_k(t_k, x(t_k^-)), & k \in \mathbb{Z}_+, \\ x_\sigma = \phi(s) & \alpha \leq s \leq 0, \end{cases} \tag{2.1}$$

where $\sigma \geq t_0 \geq 0$, *the impulse times* t_k *satisfy* $0 \leq t_0 < t_1 < \cdots < t_k \to \infty$ as $k \to \infty$, $sup_{k\in\mathbb{Z}_+}\{t_k - t_{k-1}\} < \infty$ and x' denote the right hand derivative of $x. \phi \in \mathbb{C}, \mathbb{C}$ being an open set in $PC([\alpha, 0], \mathbb{R}^n)$, where $PC([\alpha, 0], \mathbb{R}^n) = \{\varphi: [\alpha, 0] \to \mathbb{R}^n$ is continuous everywhere except at a finite number of points t_k, at which $\varphi(t_k^+), \varphi(t_k^-)$ exist and $\varphi(t_k^+) = \varphi(t_k)\}$. For each $t \geq t_0$, $x_t \in \mathbb{C}$ is defined by $x_t(s) = x(t+s), s \in [\alpha, 0]$. Define PCB(t)= $\{x_t \in \mathbb{C} : x_t$ is bounded$\}$. For $\varphi \in$ PCB(t), the norm of φ is defined by $||\varphi|| = sup_{\alpha \leq \theta \leq 0} |\varphi(\theta)|$.

In order to prove our main results, we need the following lemma and definitions.

Lemma 2.1: - The initial problem (2.1) exists with a unique solution which will be written in the form $x(t,\sigma,\phi)$ if the following hypotheses hold:

(H_1) $f: [t_{k-1}, t_k] \times \mathbb{C} \to \mathbb{R}^n, k \epsilon \mathbb{Z}_+,$ is continuous for all $k \epsilon \mathbb{Z}_+$ *and* for any $\varphi \epsilon \mathbb{C}$, the limit $lim_{(t,x)\to(t_k^-,\varphi)} f(t,x) = f(t_k^-,\varphi)$ exists.

(H_2) $f(t,\varphi)$ is Lipschitzian in φ in each compact set in $\mathbb{C}$.

(H_3) $I_k(t,x)$: $[t_0,\infty] \times \mathbb{R}^n \to \mathbb{R}^n$ is continuous and for any $\rho > 0$, there exists a $\rho_1 \epsilon (0,\rho)$ such that $x \epsilon S(\rho_1)$ implies that $x + I_k(t_k,x) \epsilon S(\rho)$, where $S(\rho) = \{x: |x| < \rho, x \epsilon \mathbb{R}^n\}$.

(H_4) For any any $\varphi \epsilon \mathbb{C}, f\big(t,\varphi(.)\big) \epsilon PC([t_0,\infty),\mathbb{R}^n)$.

Throughout the paper, we let $(H_1) - (H_4)$ hold. Furthermore, we assume that $f(t,0) = 0, I_k(t_k,0) = 0, k \epsilon \mathbb{Z}_+;$ then $x(t) \equiv 0$ is a solution of (2.1),which is called the trivial solution. Moreover, we will only consider the solution $x(t,\sigma,\emptyset)$ of (2.1),which can be continued to ∞ from the right of σ.

Definition 2.1. The function V :$[\alpha,\infty) \times \mathbb{C} \to \mathbb{R}_+$belongs to class v_0 if

(H_1) V is continuous on each of the sets $[t_{k-1},t_k) \times \mathbb{C}$

and $lim_{(t,\varphi_1)\to(t_k^-,\varphi_2)} V(t,\varphi_1) = V(t_k^-,\varphi_2)$ *exists*;

(H_2) $V(t,x)$is locally Lipschitzian in x and $V(t,0) \equiv 0$

Definition 2.2. Suppose $V \epsilon v_0$; for any $(t,\psi) \in [t_{k-1},t_k) \times \mathbb{C}$, the upper right-hand derivative of $V(t,x)$ along the solution of (2.1) is defined by

$$D^+V\big(t,\psi(0)\big) = \lim_{h\to 0+} \sup \frac{1}{h}\{V\big(t+h,\psi(0)+hf(t,\psi)\big) - V(t,\psi(0))\}$$

Definition 2.3. The trivial solution of (2.1) is said to be globally weakly exponentially stable if there exist functions $\alpha_1,\alpha_2 \epsilon \mathbb{K}$ and constants $\lambda > 0, \mathcal{M} \geq 1$ such that for any initial value $x_\sigma = \phi \epsilon PCB(\sigma), \alpha_1(|x(t)|) < \mathcal{M}\alpha_2(\|\phi\|)e^{-\lambda(t-\sigma)}, t \geq \sigma.$

In particular, when $\alpha_1(s) = \alpha_2(s) = s$, it is usually called globally exponentially stable.

3. MAIN RESULTS

In this section, we prove our main result with the relevant example.

Theorem 3.1 The trivial solution of (2.1) is globally exponentially stable if there exist functions $w_1, w_2 \in \mathbb{K}, V(t,x) \epsilon\, v_0$ and some contants $\lambda > 0, \beta_k \geq 0, K \epsilon \mathbb{Z}_+$ such that

(i) $w_1(\|x\|) \leq V(t,x) \leq w_2(\|x\|), (t,x) \epsilon [t_0 + \alpha, \infty) \times \mathbb{R}^n$.

(ii) For any $\sigma \geq t_0$ and $\psi \in PC([\alpha, 0], \mathbb{R}^n)$, if $V\big(t+\theta, \psi(\theta)\big)e^{-\lambda(t-\sigma)} \leq V\big(t, \psi(0)\big)$, $\theta \epsilon [\alpha, 0], t \neq t_k$, then $D^+V\big(t, \psi(0)\big) \leq -p(t)\, V\big(t, \psi(0)\big)$,

where $p(t) \in PC([t_0 - \tau, \infty), \mathbb{R}_+)$ and $inf_{t \geq \sigma + \alpha}\, p(t) \geq \lambda$.

(iii) For all, $(t_k, \psi) \epsilon \mathbb{R}_+ \times PC([\alpha, 0], \mathbb{R}^n), V\big(t_k, \psi(0) + I_k(t_k, \psi)\big) \leq (1 + \beta_k)\, V(t_k^-, \psi(0))$ with $\sum_{k=1}^{\infty} \beta_k < \infty$

Proof: Condition (i) implies that $w_1(s) < w_2(s)$, $s \epsilon \mathbb{R}^n$. So let W_1 and W_2 be continuous, strictly increasing function satisfying $W_1(s) \leq w_1(s) \leq w_2(s) \leq W_2(s)\ \forall\ s \in \mathbb{R}^n$. Thus we have for all $(t,x) \epsilon\, [\alpha, \infty) \times \mathbb{R}^n$

$$W_1(\|x\|) \leq V(t,x) \leq W_2(\|x\|)$$

From condition (ii) and (iii), we first define positive constant $\mathrm{M} = \prod_{K=1}^{\infty}(1 + \beta_k) < \infty$

For any $\sigma \geq t_0$, $let\ x(t) = x(t, \sigma, \varphi)$ be a solution of (2.1) through (σ, ϕ). For any $\phi \epsilon\, PCB(\sigma)$, we shall prove that

$$W_1\|x(t)\| < MW_2(\|\phi\|)e^{-\lambda(t-\sigma)}, t \geq \sigma \tag{3.1}$$

For convenience, we take $V(t, x(t)) = V(t)$ and next we shall show that

$$V(t) \leq W_2\|\phi\| \prod_{\sigma \leq t_k < t} (1 + \beta_k)e^{-\lambda(t-\sigma)}, t \geq \sigma$$

In order to do this we let

$$\Omega(t) = \begin{cases} V(t) - W_2\|\phi\| \prod_{\sigma \leq t_k < t} (1 + \beta_k)e^{-\lambda(t-\sigma)}, & t \geq \sigma \\ V(t) - W_2\|\phi\|e^{-\lambda(t-\sigma)}, & \sigma + \alpha \leq t < \sigma \end{cases}$$

We need to show that $\Omega(\mathrm{t}) \leq 0$ for all $t \geq \sigma$. It is clear that $\Omega(\mathrm{t}) \leq 0$ for $t \in [\sigma + \alpha, \sigma]$, since $\Omega(t) \leq V(t) - W_2\|\phi\| \leq 0$ by condition (i).

We shall show that $\Omega(\mathrm{t}) \leq 0$ for $t \epsilon [\sigma, t_1]$. In order to do this we let $q > 0$ be arbitrary and show that $\Omega(t) \leq q$ for $t \epsilon [\sigma, t_1)$. Suppose not, then there exist some

$t\,\epsilon[\sigma, t_1)$ so that $\Omega(t) > q$. Let $t^* = \inf\{t\epsilon[\sigma, t_1); \Omega(t) > q\}$, since $\Omega(t) \le 0 < q$ for $t\epsilon[\sigma+\alpha, \sigma]$, we know $t^*\epsilon(\sigma, t_1)$. Note that $\Omega(t)$ is continuous on $[\sigma, t_1)$, then $\Omega(t^*) = q$ and $\Omega(t) \le q$ for $t\,\epsilon[\sigma+\alpha, t^*]$

$V(t^*) = \Omega(t^*) + W_2\|\phi\|e^{-\lambda(t^*-\sigma)}$ and for $\theta\,\epsilon\,[\alpha, 0]$, we have

$$\begin{aligned} V(t^*+\theta) &= \Omega(t^*+\theta) + W_2\|\phi\|e^{-\lambda(t^*+\theta-\sigma)} \\ &\le q + W_2\|\phi\|e^{-\lambda(t^*+\theta-\sigma)} \\ &\le \left(q + W_2\|\phi\|e^{-\lambda(t^*-\sigma)}\right)e^{-\lambda\theta} \\ &= V(t^*)e^{-\lambda\theta} \end{aligned}$$

So, by condition (ii), we have

$$D^+V(t^*) \le -p(t^*)V(t^*),$$

so we have

$$\begin{aligned} D^+\Omega(t^*) &= D^+V(t^*) + \lambda W_2\|\phi\|e^{-\lambda(t^*-\sigma)} \\ &\le -p(t^*)V(t^*) + \lambda W_2\|\phi\|e^{-\lambda(t^*-\sigma)} \\ &\le -p(t^*)V(t^*) + W_2\|\phi\|e^{-\lambda(t^*-\sigma)}p(t^*) \\ &\le -p(t^*)(V(t^*) - W_2\|\phi\|e^{-\lambda(t^*-\sigma)}) \\ &= -p(t^*)\,q < 0 \end{aligned}$$

which contradicts the definition of t^*, so we get $\Omega(t) \le q$ for all $t\epsilon[\sigma, t_1)$.

Let $q \to 0^+$, we have $\Omega(t) \le 0$ for $t\,\epsilon\,[\sigma, t_1)$

Now we assume that

$\Omega(t) \le 0$ for $t\epsilon[\sigma, t_m)$, $m \ge 1$. We shall show that $\Omega(t) \le 0$ for $t\,\epsilon\,[\sigma, t_{m+1}]$.

By condition (iii), we have

$$\begin{aligned} \Omega(t_m) &= V(t_m) - W_2\|\phi\| \prod_{\sigma\le t_k<t_m}(1+\beta_k)e^{-\lambda(t_m-\sigma)} \\ &\le (1+\beta_m)V(t_m^-) - W_2\|\phi\| \prod_{\sigma\le t_k<t_m}(1+\beta_k)e^{-\lambda(t_m-\sigma)} \\ &= (1+\beta_m)\,\Omega(t_m^-) \le 0 \end{aligned}$$

We need to show that $\Omega(t) \le q\ for\ t\,\epsilon(t_m, t_{m+1})$.

Suppose that, let

$$t^* = \inf\{t\,\epsilon[t_m, t_{m+1})\,; \Omega(t) > q\}.$$

Since $\Omega(t_m) \leq 0 < q$, by the continuity of $\Omega(t)$, we get $t^* > t_m$, $\Omega(t^*) = q$ and $\Omega(t) \leq q$ for $t \in [\sigma, t^*]$

Since $V(t^*) = \Omega(t^*) + W_2\|\phi\| \prod_{\sigma \leq t_k < t^*}(1+\beta_k)e^{-\lambda(t^*-\sigma)}$,

then for any $\theta \in [\alpha, 0]$, we have

$$\begin{aligned} V(t^*+\theta) &\leq \Omega(t^*+\theta) + W_2\|\phi\| \textstyle\prod_{\sigma \leq t_k < t^*}(1+\beta_k)e^{-\lambda(t^*+\theta-\sigma)} \\ &\leq q + W_2\|\phi\| \textstyle\prod_{\sigma \leq t_k < t^*}(1+\beta_k)e^{-\lambda(t^*+\theta-\sigma)} \\ &\leq \left(q + W_2\|\phi\| \prod_{\sigma \leq t_k < t^*}(1+\beta_k)e^{-\lambda(t^*-\sigma)}\right)e^{-\lambda\theta} \\ &= V(t^*)e^{-\lambda\theta} < 0 \end{aligned}$$

Thus by condition (ii), we have $D^+V(t^*) \leq -p(t^*)V(t^*)$, then we have

$$\begin{aligned} D^+\Omega(t^*) &= D^+V(t^*) + \lambda W_2\|\phi\| \prod_{\sigma \leq t_k < t^*}(1+\beta_k)\, e^{-\lambda(t^*-\sigma)} \\ &\leq -p(t^*)V(t^*) + \lambda W_2\|\phi\| \prod_{\sigma \leq t_k < t^*}(1+\beta_k)\, e^{-\lambda(t^*-\sigma)} \\ &\leq -p(t^*)(V(t^*) - W_2\|\phi\| \textstyle\prod_{\sigma \leq t_k < t^*} e^{-\lambda(t^*-\sigma)}) \\ &= p(t^*)\, q < 0 \end{aligned}$$

Again this contradicts the definition of t^*, which implies Ω (t) $\leq q$ for all $t \in [t_m, t_{m+1})$.

Let $q \to 0^+$, we have $\Omega(t) \leq 0$ for all $t \in [t_m, t_{m+1})$. So $\Omega(t) \leq 0$ for all $t \in [\sigma, t_{m+1})$.

Thus by method of induction, we get

$$V(t) \leq W_2\|\phi\| \prod_{\sigma \leq t_k < t}(1+\beta_k)e^{-\lambda(t-\sigma)}, t \in [t_k, t_{k+1}),$$

By condition (i) – (iii), we have

$$\begin{aligned} W_1\|x\| < V(t) &\leq W_2\|\phi\| \prod_{\sigma \leq t_k < t}(1+\beta_k)e^{-\lambda(t-\sigma)}, \quad t \geq \sigma \\ &\leq MW_2\|\phi\|e^{-\lambda(t-\sigma)}, \quad t \geq \sigma \end{aligned}$$

which yields

$$\|x\| \leq \left(\frac{W_2 M}{W_1}\right)\|\phi\|e^{-\lambda(t-\sigma)}, \quad t \geq \sigma$$

Where $M = \prod_{i=1}^{\infty}(1+\beta_k) < \infty$, $since \; \sum_{k=1}^{\infty}\beta_k < \infty$.

Thus the proof is complete.

Corollary 3.2: The trivial solution of (2.1) is globally exponentially stable if there exist a function $V(t,x)\,\epsilon\, v_0, constants\;\; c_1 > 0, c_2 > 0, m > 0,\; \lambda > 0\; and\; \beta_k \geq 0$, $k\epsilon\,\mathbb{Z}_{+,} such\; that$

(i) $c_1|x|^m \leq V(t,x) \leq c_2|x|^m, (t,x)\epsilon[t_0+\alpha,\infty] \times \mathbb{R}^n$.

(ii) $For\; any\; \sigma \geq t_0\; and\; \psi\; \epsilon\; PC([\alpha,0],\mathbb{R}^n), if\; V\big(t+\theta,\psi(\theta)\big)e^{-\lambda(t-\sigma)}$

$\leq V\big(t,\psi(0)\big), \theta\epsilon[\alpha,0], t \neq t_k, then\; D^+V\big(t,\psi(0)\big) \leq$
$-p\, V\big(t,\psi(0)\big), p(t) \in PC([t_0-\tau,\infty),\mathbb{R}_+)$

(iii) For all $(t_k,\psi)\epsilon\mathbb{R}_+ \times PC([\alpha,0],\mathbb{R}^n), V\big(t_k,\psi(0)+I_k(t_k,\psi)\big) \leq (1+\beta_k)\;\; V\big(t_k^-,\psi(0)\big),$

with $\sum_{K=1}^{\infty}\beta_k < \infty$.

Remark 3.3 When $\alpha = -r, r$ some positive constant, system (2.1) becomes impulsive function differential equations with finite delays. We have the following result by corollary 3.4

Corollary 3.4 The trivial solution of (2.1) with $\alpha = -r$ is globally exponential stable if there exist a function $V(t,x)\epsilon\; v_0$, constants $c_1 > 0, c_2 > 0, m > 0, \lambda > 0$ and $\beta_k \geq 0, k\epsilon\mathbb{Z}_+$ such that

(i) $c_1|x|^m \leq V(t,x) \leq c_2|x|^m, (t,x)\epsilon[t_0-r,\infty] \times \mathbb{R}^n$.

(ii) $For\; any\; \sigma \geq t_0\; and\; \psi\epsilon PC([-r,0],\mathbb{R}^n), if\; V\big(t+\theta,\psi(\theta)\big)e^{-\lambda(t-\sigma)}$

$\leq V\big(t,\psi(0)\big), \theta\epsilon[-r,0], t \neq t_k, then\; D^+V\big(t,\psi(0)\big) \leq$
$-p\, V\big(t,\psi(0)\big), p(t) \in PC([t_0-\tau,\infty),\mathbb{R}_+)$

(iii) For all $(t_k,\psi)\;\epsilon\mathbb{R}_+ \times PC([-r,0],\mathbb{R}^n), V(t_k,\psi(0)+I_k(t_k,\psi))$

$$\leq (1+\beta_k)\, V\big(t_k^-,\psi(0)\big), with \sum_{k=1}^{\infty}\beta_k < \infty.$$

Remarks 3.5 It should be noted that $e^{\lambda(t-\sigma)}$in condition (ii) contains the information of infinite delay or finite delays and plays an important role in ensuring the feasibility of our results for impulsive functional differential equation for infinite and finite delays.

4. EXAMPLE

In this section, we give example to show the effectiveness and advantage of our results.

Example 4.1: Consider the impulsive delay differential equation

$$\begin{cases} x'(t) = -a(t)\,x(t) + \frac{f(t)}{1+x^2(t)}\,x(t+\alpha), & t \ge \sigma \\ x(t_k) = (1+c_k)x(t_k^-), & t_k = k, \quad k \epsilon N \\ \qquad x_{t_0} = \phi \end{cases} \tag{4.1}$$

Where $c_k > 0$ with $\sum_{k=1}^{\infty} c_k < \infty$, function $a\,\epsilon C(\mathbb{R}, \mathbb{R}_+)$ and $b\,\epsilon\, C(\mathbb{R}, \mathbb{R}), \phi\, \epsilon PC([\alpha, 0], \mathbb{R}^n)$. If $a(t) \ge |f(t)|e^{\lambda\alpha} + \lambda$, then the trivial solution of (4.1) is exponentially stable.

Proof: Set $V(x) = V(t,x) = |x|$, for all $t \ge t_0 + \alpha$,

$$D^+\,V\big(t, \varphi(0)\big) \le sgn\big(\varphi(0)\big)\left[-a(t)\varphi(0) + \frac{f(t)}{1+\varphi^2(0)}\,\varphi(\alpha)\right]$$

$$\le -a(t)|\varphi(0)| + |f(t)||\varphi(\alpha)|$$

$$\le -a(t)V\big(\varphi(0)\big) + f(t)V\big(\varphi(\alpha)\big) \tag{4.2}$$

For any solution $x(t)$ of (4.1) such that

$$V\big(t, \psi(0)\big) \ge V\big(t+\theta, \varphi(\theta)\big)e^{-\lambda(t-\sigma)} \text{for } \theta\epsilon[\alpha, 0]$$

We have

$$V\big(\varphi(\alpha)\big) \le e^{\lambda\alpha}\,V\big(\varphi(0)\big).$$

Therefore

$$D^+\,V\big(t, \varphi(0)\big) \le \big[-a(t) + f(t)e^{\lambda\alpha}\big]V\big(\varphi(0)\big)$$

Since $a(t) \ge |f(t)|e^{\lambda\alpha} + \lambda$, it follows that

$$D^+\,V\big(t, \varphi(0)\big) \le -\lambda V\big(\varphi(0)\big) \le -p(t)V\big(\varphi(0)\big).$$

Whenever $V\big(t, \varphi(0)\big) \ge V\big(t+0, \varphi(\theta)\big)e^{\lambda(t-\sigma)}$ for $\theta\epsilon[\alpha, 0]$ i.e. condition (ii) for Theorem 3.1 holds.

Moreover,

$$V\big(t_k, \varphi(0) + I_k(t_k, \varphi)\big) = (1+c_k)\,V\big(t_k^-, \varphi(0)\big)$$

Thus by Theorems 3.1, the trivial solution of system (4.1) is exponentially stable.

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