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Analysis Of System Reliability Using Markov Technique

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Abstract

The Reliability rate of Human Error and other equipment error have been studied over the past years. There are many models that have been developed on the Reliability Analysis using Markov Technique. Markov models are very much useful in finding the System Reliability in various life situations. In this Paper, we present the Markov Model for a system comprising 4-Elements to determine the Reliability R(t), Failure Probability F(t) and Mean Time to Failure MTTF of the system using human error rate. Numerical illustration is given and Conclusion is derived.

Key Words: Reliability, Markov Model, States, Mean Time to Failure, Failure Rate.

1. INTRODUCTION

Reliability is the probability of a device performing its purpose adequately for the period intended under the given operating conditions. In Reliability models, the states usually represent the various working and failed conditions of the system.[9]

Markov Processes are widely used in Engineering, Science and Business Modeling. Markov Modeling is a widely used technique in the study of Reliability analysis of system. They are used to model systems that have a limited memory of their past. In a Markov Process, if the present state of the process is given, the future state is independent of the past. This property is usually referred to as the Markov Process.[1]

Andrei.A.Markov, a great Mathematician, after whom the term 'Markov Model have been named.[5]

Markov modeling is a modeling technique that is widely useful for the Reliability analysis of complex systems. It is very flexible in the type of systems and system behavior it can model. This modeling technique is very helpful in most of the situations .The model is quite useful to modeling operation system with dependent failure and repair models. In fact it is widely used to perform Reliability and Availability analysis of responsible system with constant failure and repair rates. From time to time the Markov method is also used to perform human Reliability Analysis.[7]

The Paper is organized as follows. In Section 1, we give the Introduction, In Section 2, we give the definitions and basic Concepts, In section 3, we present the Markov Model related to our study, In Section 4, we discuss with a Numerical Example in Medical Field, In Section 5, we draw the Conclusion.

2. BASIC DEFINITIONS AND RELATED CONCEPTS

Reliability

Reliability is the Probability that an item will carry out its function satisfactorily for the stated period when used according to the specified conditions.

Reliability is defined as

$$R(t) = \int_t^\infty f(x)dx$$

Failure Rate

Failure Rate is the frequency with which an engineered system or component fails, expressed in failures per unit of time. It is often denoted by the Greek Letter λ and is highly used in Reliability Engineering.

Human Error

This is the failure to carry out a required task that could result in disruption of scheduled operations or damage to equipment or property.

3. MARKOV MODEL FOR A SYSTEM WITH 4 ELEMENTS

We discuss the Markov Model with 4-Elements. The Reliability of Markov Model for a System with 3-Elements have been already derived.[8]

In the 4-Elements Markov Model, each element has two states - good and failed state. The states of the Model are generated based on the elements being in one of these two states. An element with constant failure rate has a transition Probability that is approximated by $\lambda\Delta t$. The Probability for more than one element failure in time t is considered to be negligible.

The transition diagram for four element Markov Model Process is given below.

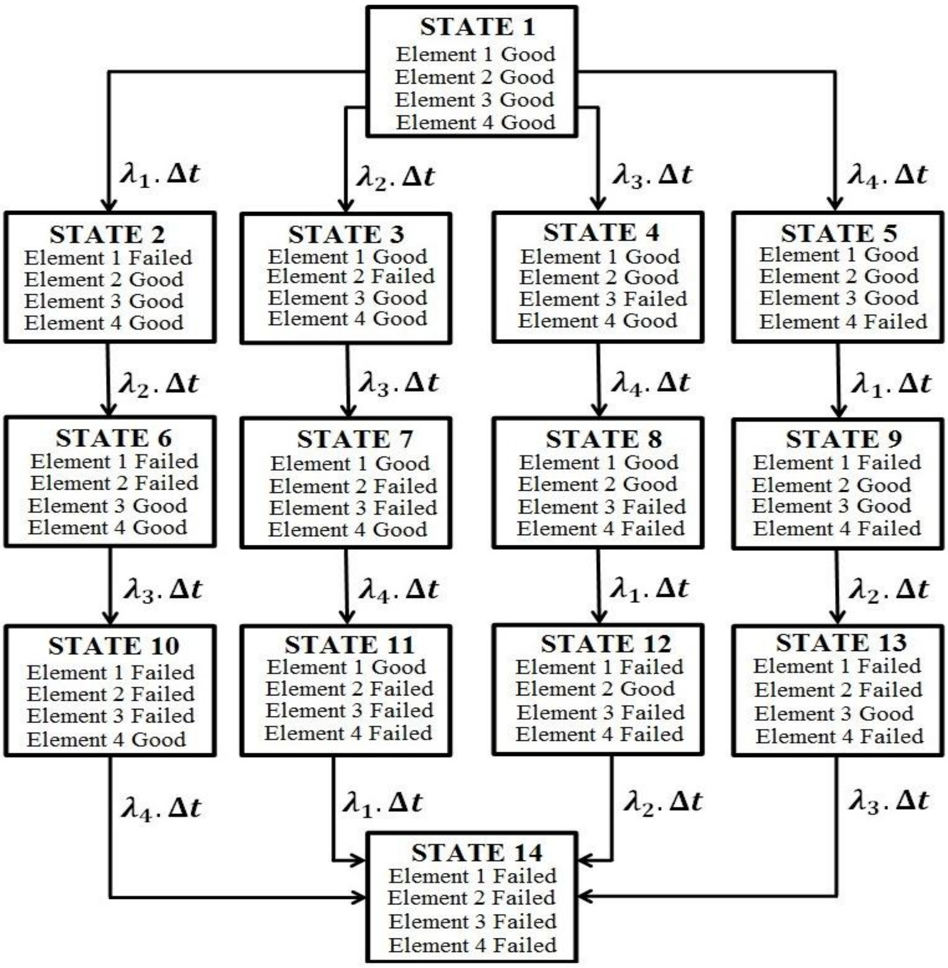

Fig 1. Transition diagram for 4 element Markov Model

The Markov differential equation is developed by giving the probability of each state at time $t + \Delta t$ as a function of that state at time t. The Probability of being in state 1 at some time $t + \Delta t$ is equal to the probability of being in state 1 at time t and not transitioning out during time Δt. Thus, we get the equation,

$P_1(t+\Delta t) = P_1(t)[1-(\lambda_1+\lambda_2+\lambda_3+\lambda_4)\Delta t]$

The probability of being in state 2 at time $t+\Delta t$ is equal to the probability of being in state 1 at time t and transitioning to state 2 at time Δt with the probability of being in state 2 at time t and not transitioning out during time Δt. Thus, we get the equation,

$P_2(t+\Delta t) = P_1(t)\lambda_1\Delta t + [1-(\lambda_2+\lambda_3+\lambda_4)\Delta t]P_2(t)$

The other state equations are as follows,

$P_3(t+\Delta t) = P_1(t)\lambda_2\Delta t + [1-(\lambda_1+\lambda_3+\lambda_4)\Delta t]P_3(t)$

$P_4(t+\Delta t) = P_1(t)\lambda_3\Delta t + [1-(\lambda_1+\lambda_2+\lambda_4)\Delta t]P_4(t)$

$P_5(t+\Delta t) = P_1(t)\lambda_4\Delta t + [1-(\lambda_1+\lambda_2+\lambda_3)\Delta t]P_5(t)$

$P_6(t+\Delta t) = P_2(t)\lambda_2\Delta t + [1-(\lambda_3+\lambda_4)\Delta t]P_6(t)$

$P_7(t+\Delta t) = P_3(t)\lambda_3\Delta t + [1-(\lambda_1+\lambda_4)\Delta t]P_7(t)$

$P_8(t+\Delta t) = P_4(t)\lambda_4\Delta t + [1-(\lambda_1+\lambda_2)\Delta t]P_8(t)$

$P_9(t+\Delta t) = P_5(t)\lambda_1\Delta t + [1-(\lambda_2+\lambda_3)\Delta t]P_9(t)$

$P_{10}(t+\Delta t) = P_6(t)\lambda_3\Delta t + [1-\lambda_4\Delta t]P_{10}(t)$

$P_{11}(t+\Delta t) = P_7(t)\lambda_4\Delta t + [1-\lambda_1\Delta t]P_{11}(t)$

$P_{12}(t+\Delta t) = P_8(t)\lambda_1\Delta t + [1-\lambda_2\Delta t]P_{12}(t)$

$P_{13}(t+\Delta t) = P_9(t)\lambda_2\Delta t + [1-\lambda_3\Delta t]P_{13}(t)$

$P_{14}(t+\Delta t) = P_{10}(t)\lambda_4\Delta t + P_{11}(t)\lambda_1\Delta t + P_{12}(t)\lambda_2\Delta t + P_{13}(t)\lambda_3\Delta t + 1.P_{14}(t)$

Solving the above equations with the initial conditions,

$P_1(0)=1$, $P_2(0)=0,\ldots P_{14}(0)=0$

We get the following set of equations

$$P_1(t) = e^{-(\lambda_1+\lambda_2+\lambda_3+\lambda_4)t}$$

$$P_2(t) = -e^{-(\lambda_1+\lambda_2+\lambda_3+\lambda_4)t} + e^{-(\lambda_2+\lambda_3+\lambda_4)t}$$

$$P_3(t) = -e^{-(\lambda_1+\lambda_2+\lambda_3+\lambda_4)t} + e^{-(\lambda_1+\lambda_3+\lambda_4)t}$$

$$P_4(t) = -e^{-(\lambda_1+\lambda_2+\lambda_3+\lambda_4)t} + e^{-(\lambda_1+\lambda_2+\lambda_4)t}$$

$$P_5(t) = -e^{-(\lambda_1+\lambda_2+\lambda_3+\lambda_4)t} + e^{-(\lambda_1+\lambda_2+\lambda_3)t}$$

$$P_6(t) = \frac{\lambda_2}{\lambda_1+\lambda_2}e^{-(\lambda_1+\lambda_2+\lambda_3+\lambda_4)t} - e^{-(\lambda_2+\lambda_3+\lambda_4)t} + \frac{\lambda_1}{\lambda_1+\lambda_2}e^{-(\lambda_3+\lambda_4)t}$$

$$P_7(t) = \frac{\lambda_3}{\lambda_2+\lambda_3} e^{-(\lambda_1+\lambda_2+\lambda_3+\lambda_4)t} - e^{-(\lambda_1+\lambda_3+\lambda_4)t} + \frac{\lambda_2}{\lambda_2+\lambda_3} e^{-(\lambda_1+\lambda_4)t}$$

$$P_8(t) = \frac{\lambda_4}{\lambda_3+\lambda_4} e^{-(\lambda_1+\lambda_2+\lambda_3+\lambda_4)t} - e^{-(\lambda_1+\lambda_2+\lambda_4)t} + \frac{\lambda_3}{\lambda_3+\lambda_4} e^{-(\lambda_1+\lambda_2)t}$$

$$P_9(t) = \frac{\lambda_1}{\lambda_1+\lambda_4} e^{-(\lambda_1+\lambda_2+\lambda_3+\lambda_4)t} - e^{-(\lambda_1+\lambda_2+\lambda_3)t} + \frac{\lambda_4}{\lambda_1+\lambda_4} e^{-(\lambda_2+\lambda_3)t}$$

$$\begin{aligned} P_{10}(t) = &-\frac{\lambda_2\lambda_3}{(\lambda_1+\lambda_2)(\lambda_1+\lambda_2+\lambda_3)} e^{-(\lambda_1+\lambda_2+\lambda_3+\lambda_4)t} + \frac{\lambda_3}{\lambda_2+\lambda_3} e^{-(\lambda_2+\lambda_3+\lambda_4)t} \\ &- \frac{\lambda_1\lambda_4}{\lambda_4(\lambda_2+\lambda_3)} e^{-(\lambda_1+\lambda_4)t} \\ &+ \left[\frac{\lambda_2\lambda_3}{(\lambda_1+\lambda_2)(\lambda_1+\lambda_2+\lambda_3)} - \frac{\lambda_3}{\lambda_2+\lambda_3} + \frac{\lambda_1\lambda_4}{\lambda_4(\lambda_2+\lambda_3)}\right] e^{-\lambda_4 t} \end{aligned}$$

$$\begin{aligned} P_{11}(t) = &-\frac{\lambda_3\lambda_4}{(\lambda_2+\lambda_3)(\lambda_2+\lambda_3+\lambda_4)} e^{-(\lambda_1+\lambda_2+\lambda_3+\lambda_4)t} + \frac{\lambda_4}{\lambda_3+\lambda_4} e^{-(\lambda_1+\lambda_3+\lambda_4)t} \\ &- \frac{\lambda_2\lambda_4}{\lambda_4(\lambda_2+\lambda_3)} e^{-(\lambda_1+\lambda_4)t} \\ &+ \left[\frac{\lambda_3\lambda_4}{(\lambda_2+\lambda_3)(\lambda_2+\lambda_3+\lambda_4)} - \frac{\lambda_4}{\lambda_3+\lambda_4} + \frac{\lambda_2\lambda_4}{\lambda_4(\lambda_2+\lambda_3)}\right] e^{-\lambda_1 t} \end{aligned}$$

$$\begin{aligned} P_{12}(t) = &-\frac{\lambda_1\lambda_4}{(\lambda_3+\lambda_4)(\lambda_1+\lambda_3+\lambda_4)} e^{-(\lambda_1+\lambda_2+\lambda_3+\lambda_4)t} + \frac{\lambda_1}{\lambda_1+\lambda_4} e^{-(\lambda_1+\lambda_2+\lambda_4)t} \\ &- \frac{\lambda_1\lambda_3}{\lambda_1(\lambda_3+\lambda_4)} e^{-(\lambda_1+\lambda_2)t} + [\frac{\lambda_1\lambda_4}{(\lambda_3+\lambda_4)(\lambda_1+\lambda_3+\lambda_4)} - \frac{\lambda_1}{\lambda_1+\lambda_4} \\ &+ \frac{\lambda_1\lambda_3}{\lambda_1(\lambda_3+\lambda_4)}] e^{-\lambda_2 t} \end{aligned}$$

$$\begin{aligned} P_{13}(t) = &-\frac{\lambda_1\lambda_2}{(\lambda_1+\lambda_4)(\lambda_1+\lambda_2+\lambda_4)} e^{-(\lambda_1+\lambda_2+\lambda_3+\lambda_4)t} + \frac{\lambda_2}{\lambda_1+\lambda_2} e^{-(\lambda_1+\lambda_2+\lambda_3)t} \\ &- \frac{\lambda_2\lambda_4}{\lambda_2(\lambda_1+\lambda_4)} e^{-(\lambda_2+\lambda_3)t} + [\frac{\lambda_1\lambda_2}{(\lambda_1+\lambda_4)(\lambda_1+\lambda_2+\lambda_4)} - \frac{\lambda_2}{\lambda_1+\lambda_2} \\ &+ \frac{\lambda_2\lambda_4}{\lambda_2(\lambda_1+\lambda_4)}] e^{-\lambda_3 t} \end{aligned}$$

$$P_{14}(t) = 1 - P_1(t) - P_2(t) - \cdots - P_{13}(t)$$

The Reliability of the System at time t is the sum of all the probabilities,

$$R(t) = \sum_{i=1}^{n-1} P_i(t), n = 14$$

$$\begin{aligned} R(t) = {} & -3e^{-(\lambda_1+\lambda_2+\lambda_3+\lambda_4)t} \\ & + e^{-(\lambda_1+\lambda_2+\lambda_3+\lambda_4)t}\left[\frac{\lambda_2}{\lambda_1+\lambda_2+\lambda_3} + \frac{\lambda_3}{\lambda_2+\lambda_3+\lambda_4} + \frac{\lambda_4}{\lambda_1+\lambda_3+\lambda_4} \right. \\ & \left. + \frac{\lambda_1}{\lambda_1+\lambda_2+\lambda_4}\right] + e^{-(\lambda_2+\lambda_3+\lambda_4)t}\left(\frac{\lambda_3}{\lambda_2+\lambda_3}\right) + e^{-(\lambda_1+\lambda_2+\lambda_4)t}\left(\frac{\lambda_1}{\lambda_1+\lambda_4}\right) \\ & + e^{-(\lambda_1+\lambda_2+\lambda_3)t}\left(\frac{\lambda_2}{\lambda_1+\lambda_2}\right) + e^{-(\lambda_1+\lambda_3+\lambda_4)t}\left(\frac{\lambda_4}{\lambda_3+\lambda_4}\right) \\ & + \left[\frac{\lambda_1\lambda_2}{(\lambda_2+\lambda_3)(\lambda_1+\lambda_2+\lambda_3)}\right]e^{-\lambda_4 t} + \left[\frac{\lambda_2\lambda_3}{(\lambda_3+\lambda_4)(\lambda_2+\lambda_3+\lambda_4)}\right]e^{-\lambda_1 t} \\ & + \left[\frac{\lambda_3\lambda_4}{(\lambda_1+\lambda_4)(\lambda_1+\lambda_3+\lambda_4)}\right]e^{-\lambda_2 t} + \left[\frac{\lambda_1\lambda_4}{(\lambda_1+\lambda_2)(\lambda_1+\lambda_2+\lambda_4)}\right]e^{-\lambda_3 t} \end{aligned}$$

The Failure Rate is given by

$$F(t) = 1 - R(t)$$

The Mean Time to Failure is given by

$$MTTF = \int_0^\infty R(t)dt$$

$$\begin{aligned} MTTF = {} & -\frac{3}{\lambda_1+\lambda_2+\lambda_3+\lambda_4} \\ & + \left[\frac{1}{\lambda_1+\lambda_2+\lambda_3+\lambda_4}\right]\left[\frac{\lambda_2}{\lambda_1+\lambda_2+\lambda_3} + \frac{\lambda_3}{\lambda_2+\lambda_3+\lambda_4} + \frac{\lambda_4}{\lambda_1+\lambda_3+\lambda_4} \right. \\ & \left. + \frac{\lambda_1}{\lambda_1+\lambda_2+\lambda_4}\right] + \frac{\lambda_3}{(\lambda_2+\lambda_3)(\lambda_2+\lambda_3+\lambda_4)} \\ & + \frac{\lambda_1}{(\lambda_1+\lambda_4)(\lambda_1+\lambda_2+\lambda_4)} + \frac{\lambda_2}{(\lambda_1+\lambda_2)(\lambda_1+\lambda_2+\lambda_3)} \\ & + \frac{\lambda_4}{(\lambda_3+\lambda_4)(\lambda_1+\lambda_3+\lambda_4)} + \frac{\lambda_1\lambda_2}{\lambda_4(\lambda_2+\lambda_3)(\lambda_1+\lambda_2+\lambda_3)} \\ & + \frac{\lambda_2\lambda_3}{\lambda_1(\lambda_3+\lambda_4)(\lambda_2+\lambda_3+\lambda_4)} + \frac{\lambda_3\lambda_4}{\lambda_2(\lambda_1+\lambda_4)(\lambda_1+\lambda_3+\lambda_4)} \\ & + \frac{\lambda_1\lambda_4}{\lambda_3(\lambda_1+\lambda_2)(\lambda_1+\lambda_2+\lambda_4)} \end{aligned}$$

4. NUMERICAL EXAMPLE

A Medical Laboratory consists of 4 equipments maintained by four Lab Technicians. We assume that each of the four Technicians have some error rate in doing their jobs successfully and their error rates are found based on their Past Records. Using their error rates we calculate the Reliability of the System and Failure Probability of the Medical Laboratory at any particular time period t. At time t=2,3,4,5 years the Probabilities are tabulated and given below.

Assume that $\lambda_1 = 0.10$, $\lambda_2 = 0.15$, $\lambda_3 = 0.20$, $\lambda_4 = 0.25$

Table 1. Probabilities of different states at time t

Probabilities	t=2	3	4	5
$P_1(t)$	0.2465	0.1224	0.0608	0.0301
$P_2(t)$	0.0546	0.0428	0.0299	0.0196
$P_3(t)$	0.0863	0.0696	0.0500	0.0338
$P_4(t)$	0.1213	0.1007	0.0745	0.0519
$P_5(t)$	0.1600	0.1368	0.1044	0.0752
$P_6(t)$	0.0094	0.0118	0.0117	0.0104
$P_7(t)$	0.0207	0.0278	0.0295	0.0276
$P_8(t)$	0.0345	0.052	0.0599	0.0605
$P_9(t)$	0.0185	0.0255	0.0821	0.0272
$P_{10}(t)$	0.0014	0.003	0.0046	0.0054
$P_{11}(t)$	0.0043	0.0098	0.0159	0.0313
$P_{12}(t)$	0.003	0.0071	0.0116	0.0157
$P_{13}(t)$	0.0021	0.005	0.0077	0.0101

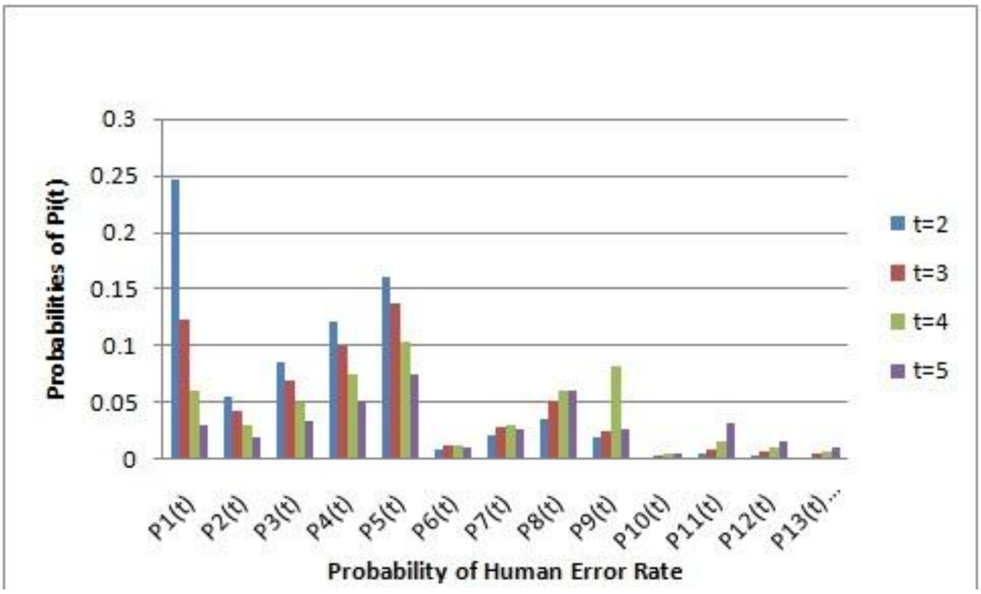

Fig 2. Reliability of System with Human Error

Table 2. Reliability and Failure Probability at time t

Probability	t=2	3	4	5
R(t)	0.7626	0.6143	0.5426	0.4588
F(t)	0.2374	0.3857	0.4574	0.5412

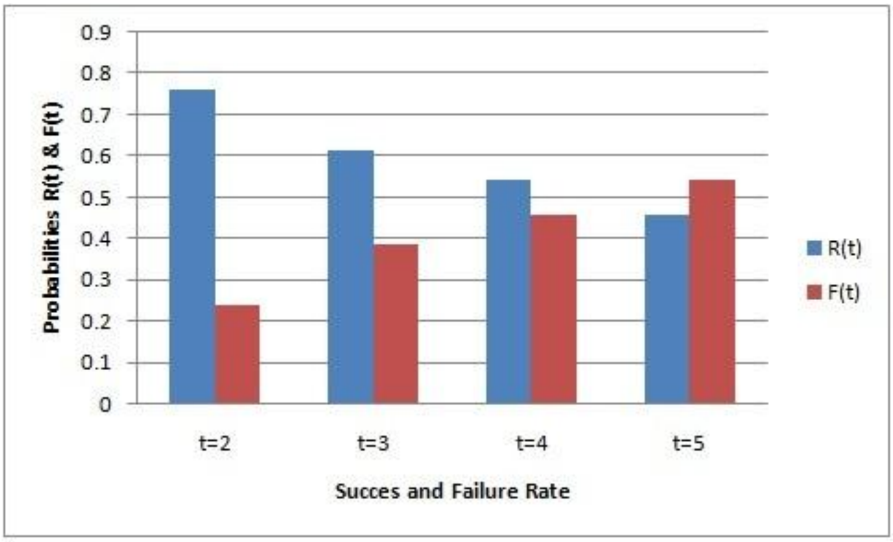

Fig 3. Success and Failure Probabilities at time t

5. CONCLUSION

Reliability of the system can be calculated by many ways. Markov Modeling technique is the one which is more effective in calculating the System Reliability. In this Paper, we analyze the Reliability of the System based on the human error rate. We determine the Reliability, Failure Probability and Mean Time to Failure of the System, which would enable us to improve the effectiveness of the Health Care System and to take care of Patients Safety. We observe that when the value of t increases, the Reliability Rate R(t) decreases and when t increases, Failure Rate F(t) also increases.

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