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On Complement Of Intuitionstic Product Fuzzy Graphs

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Abstract

The notion of product fuzzy graph has generalized for intuitionistic product fuzzy graph and definition of complement and ring sum of two intuitionistic product fuzzy graph have provided. Further some properties of ring sum of intuitionistic product fuzzy graph have discussed.

AMS Subject Classification: 05C72

Keywords: Fuzzy graph, product fuzzy graph, complement of fuzzy graph

INTRODUCTION

In 1975, Rosenfeld [4] discussed the concept of fuzzy graphs whose basic idea was introduced by Kauffmann in 1973. The fuzzy relations between fuzzy sets were also considered by Rosenfeld and he developed the structure of fuzzy graphs, obtaining analogs of several graph theoretical concepts. In [4] Rosendfeld and Yeh and Beng independently developed the theory of fuzzy graph. A fuzzy graph is a pair $G:(\sigma,\mu)$, where σ is a fuzzy subset of V and μ is fuzzy relation on V such that, $\mu(xy) \le \sigma(x) \wedge \sigma(y)$ for all $x,y \in V$ where $\sigma: V \to [0,1]$ and $\mu: V \times V \to [0,1]$.

Mohideen Ismail S. et al [11] have evaluated the basic definitions of intuitionistic fuzzy graph and discussed some properties of the power of an intuitionistic fuzzy graph and given the relationship between the index matrix of an intuitionistic fuzzy graph and power of an intuitionistic fuzzy graph and Nagoor Gani A. et al [2010] has introduced regular fuzzy graph.

V. Ramaswamy and Poornima B. [12] replaced ‘minimum’ in the definition of fuzzy graph by ‘product’ and call the resulting structure product fuzzy graph and proved several results which are analogous to fuzzy graphs. Further they discussed a necessary an sufficient condition for a product partial fuzzy sub graph to be the multiplication of two product partial fuzzy sub graphs.

The notion of product fuzzy graph has generalized for intuitionistic product fuzzy graph and definition of complement and ring sum of two intuitionistic product fuzzy graph have provided. Further some properties of ring sum of intuitionistic product fuzzy graph have discussed.

1. PRELIMINARY

Definition:1.1[12] Let G be a graph whose vertex set is V,σ be a fuzzy sub set of V and μ be a fuzzy sub set of V×V then the pair (σ,μ) is called product fuzzy graph if

$$\mu(x,y) \leq \sigma(x) \times \sigma(y) \quad ,\forall x,y \in V$$

Remark : If (σ,μ) is a product fuzzy graph and since $\sigma(x)$ and $\sigma(y)$ are less than or equal to 1, it follows that $\mu(x,y) \leq \sigma(x) \times \sigma(y) \leq \sigma(x) \wedge \sigma(y) \quad ,\forall\, x,y \in V$ Hence (σ,μ) is a fuzzy graph thus every product fuzzy graph is a fuzzy graph

Definition:1.2 a product fuzzy graph (σ,μ) with crisp graph (V,E) is said to be complete product fuzzy graph if $\mu(x,y) = \sigma(x) \times \sigma(y) \quad ,\forall\, x,y \in V$

Definition:1.3 A product fuzzy graph (σ,μ) with crisp graph (V,E) is said to be strong product

fuzzy graph if $\mu(x,y) = \sigma(x) \times \sigma(y) \quad ,\forall\, (x,y) \in E$

Definition:1.4[14] The complement of a product fuzzy graph (σ,μ) with crisp graph (V,E) is $(\bar{\sigma},\bar{\mu})$ where $\bar{\bar{\sigma}} = \sigma$

And
$$\bar{\mu}\,(x,y) = \sigma(x) \times \sigma(y) - \mu(x,y)$$
$$= \bar{\sigma}(x) \times \bar{\sigma}(y) - \mu(x,y)$$

It follows that $(\bar{\sigma},\bar{\mu})$ itself is a fuzzy graph. Also

$$\bar{\bar{\mu}}\,(x,y) = \sigma(x) \times \sigma(y) - \bar{\mu}(x,y)$$
$$= \sigma(x) \times \sigma(y) - [\sigma(x) \times \sigma(y) - \mu(x,y)]$$
$$= \mu(x,y)\ \forall\, x,y \in V$$

Definition:1.5 [15] Let$G_1 : (\sigma_1,\mu_1)$ and $G_2 : (\sigma_2,\mu_2)$ are product fuzzy graphs with

$G_1:(V_1,E_2)$ and $G_2:(V_2,E_2)$ and let $G = G_1 \cup G_2:(V_1 \cup V_2, E_1 \cup E_2)$ be the union of G_1 and G_2. Then the union of two fuzzy graphs G_1 and G_2 is also a product fuzzy graph and is defined as

$G = G_1 \cup G_2:(\sigma_1 \cup \sigma_2, \mu_1 \cup \mu_2)$ Where

$$(\sigma_1 \cup \sigma_2)u = \begin{cases} \sigma_{1(u)} & \text{if } u \in V_1 - V_2 \\ \sigma_2(u) & \text{if } u \in V_2 - V_1 \\ \max[\sigma_1(u), \sigma_2(u)] & \text{if } u \in V_1 \cap V_2 \end{cases}$$

And
$$(\mu_1 \cup \mu_2)uv = \begin{cases} \mu_1(uv) & \text{if } uv \in E_1 - E_2 \\ \mu_2(uv) & \text{ifuv} \in E_2 - E_1 \\ \max[\mu_1(uv), \mu_2(uv)] & \text{ifuv} \in E_1 \cap E_2 \\ 0 & \text{otherwise} \end{cases}$$

Definition:1.6 [15] Let (σ_1, μ_1) be a product partial fuzzy sub graph of $G_1 = (V_1, X_1)$ and (σ_2, μ_2) be a product fuzzy sub graph of $G_2 = (V_2, X_2)$. Let X' denot the set of all arcs joining the vertices of V_1 and V_2. Then the join of (σ_1, μ_1) and (σ_2, μ_2) is defined as $(\sigma_1 + \sigma_2, \mu_1 + \mu_2)$ where

$$(\sigma_1 + \sigma_2)u = \begin{cases} \sigma_1(u) & \text{if } u \in V_1 \\ \sigma_2(u) & \text{if } u \in V_2 \\ \sigma_1(u) \vee \sigma_2(u) & \text{if } u \in V_1 \cap V_2 \end{cases}$$

And
$$(\mu_1 + \mu_1)uv = \begin{cases} \mu_1(u,v) & \text{if } (u,v) \in X_1 \\ \mu_2(u,v) & \text{if } (u,v) \in X_2 \\ \mu_1(u,v) \vee \mu_2(u,v) & \text{if } (u,v) \in X_1 \cap X_2 \\ \sigma_1(u) \times \sigma_2(v) & \text{if } (u,v) \in X' \end{cases}$$

2. MAIN RESULTS

Definition: 2.1 An intuitionstic product fuzzy graph (IPFG), denoted by $G:\{(\sigma,\tau),(\mu,\gamma)\}$ of any crisp graph G:(V,E) is defined as follows

i) The function $\sigma: V \to [0,1]$ and $\tau: V \to [0,1]$ denote the degree of membership and nonmembership of the elements $x_i \in V$ respectively such that

$$0 \le \{\sigma(x_i) + \tau(x_i)\} \le 1 \ \ \forall x_i \in V, (i = 1,2, \dots \dots \dots n)$$

ii) The function $\mu: V \times V \to [0,1]$ and $\gamma: V \times V \to [0,1]$ are defined by

$$\mu(x_i, x_j) \le \text{product}[\sigma(x_i), \sigma(x_j)]$$

And $\gamma(x_i, x_j) \le \max[\tau(x_i), \tau(x_j)]$

Definition:2.2 An intuitionstic product fuzzy graph (IPFG) is said to be strong if and only if

$$\mu: V \times V \to [0,1] \text{ and } \gamma: V \times V \to [0,1] \text{ such that}$$

$$\mu(x_i, x_j) = \text{product}[\sigma(x_i), \sigma(x_j)]$$

$$\text{And} \quad \gamma(x_i, x_j) = \max[\tau(x_i), \tau(x_j)] \text{ for all } (x_i, x_j) \in E.$$

Definition:2.3 An intuitionstic product fuzzy graph (IPFG) is said to be complete if and only if

$$\mu: V \times V \to [0,1] \text{ and } \gamma: V \times V \to [0,1] \text{ such that}$$

$$\mu(x_i, x_j) = \text{product}[\sigma(x_i), \sigma(x_j)]$$

$$\text{And} \quad \gamma(x_i, x_j) = \max[\tau(x_i), \tau(x_j)] \text{ for all } x_i, x_j \in V.$$

Definition:2.4 Complement of An intuitionstic product fuzzy graph (IPFG) is defined by

The function $\sigma: V \to [0,1]$ and $\tau: V \to [0,1]$ denote the degree of membership and nonmembership of the elements $x_i \in V$ respectively such that

$0 \le \{\sigma(x_i) + \tau(x_i)\} \le 1 \ \forall x_i \in V, (i = 1,2, \dots \dots \dots n)$Then

$$\bar{\sigma} = \sigma \text{ and } \bar{\tau} = \tau$$

And $\mu: V \times V \to [0,1]$ and $\gamma: V \times V \to [0,1]$ such that

$$\bar{\mu}(x_i, x_j) = product[\sigma(x_i), \sigma(x_j)] - \mu(x_i, x_j)$$

$$\le product[\sigma(x_i), \sigma(x_j)]$$

$$product[\bar{\sigma}(x_i), \bar{\sigma}(x_j)]$$

i.e. $\bar{\mu}(x_i, x_j) \le product[\bar{\sigma}(x_i), \bar{\sigma}(x_j)]$

And $\bar{\gamma}(x_i, x_j) = max[\tau(x_i), \tau(x_j)] - \gamma(x_i, x_j)$

$$\le max[\tau(x_i), \tau(x_j)]$$

$$= max[\bar{\tau}(x_i), \bar{\tau}(x_j)]$$

i.e. $\bar{\gamma}(x_i, x_j) \le max[\bar{\tau}(x_i), \bar{\tau}(x_j)]$

Definition:2.5 Complement of an strong intuitionstic product fuzzy graph is defined by

$\sigma: V \to [0,1]$ and $\tau: V \to [0,1]$ such that $\bar{\sigma} = \sigma$ and $\bar{\tau} = \tau$

And $\mu: V \times V \to [0,1]$ and $\gamma: V \times V \to [0,1]$ such that

$$\bar{\mu}(x_i, x_j) = product[\sigma(x_i), \sigma(x_j)] - \mu(x_i, x_j)$$

$$= 0$$

Since $\mu(x_i, x_j) = product[\sigma(x_i), \sigma(x_j)]\ \forall (x_i, x_j) \in E$

And $\bar{\gamma}(x_i, x_j) = max[\tau(x_i), \tau(x_j)] - \gamma(x_i, x_j)$

$$= 0$$

Since $\gamma(x_i, x_j) = product[\tau(x_i), \tau(x_j)]\ \forall (x_i, x_j) \in E$

Definition:2.6 Complement of an complete intuitionstic product fuzzy graph is defined by

$\sigma: V \to [0,1]$ and $\tau: V \to [0,1]$ such that $\bar{\sigma} = \sigma$ and $\bar{\tau} = \tau$

And $\mu: V \times V \to [0,1]$ and $\gamma: V \times V \to [0,1]$ such that

$$\bar{\mu}(x_i, x_j) = product[\sigma(x_i), \sigma(x_j)] - \mu(x_i, x_j)$$

$$= 0$$

Since $\mu(x_i, x_j) = product[\sigma(x_i), \sigma(x_j)]\ \forall x_i, x_j \in V$

And $\bar{\gamma}(x_i, x_j) = max[\tau(x_i), \tau(x_j)] - \gamma(x_i, x_j)$

$$= 0$$

Since $\gamma(x_i, x_j) = product[\tau(x_i), \tau(x_j)]\ \forall x_i, x_j \in V$.

Definition:2.7 Let $G_1: \{(\sigma_1, \tau_1), \mu_1, \gamma_1)\}$ and $G_1: \{(\sigma_2, \tau_2), \mu_2, \gamma_2)\}$ be two intuitionstic product fuzzy graph with crisp graph $G_1: (V_1, E_2)$ and $G_2: (V_2, E_2)$ res. Then the ring sum of two intuitionstic product fuzzy $G_1: (\sigma_1, \mu_1)$ and $G_2: (\sigma_2, \mu_2)$ is denoted by $G = G_1 \oplus G_2: \{(\sigma_1 \oplus \sigma_2, \tau_1 \oplus \tau_2), (\mu_1 \oplus \mu_2, \gamma_1 \oplus \gamma_2)\}$ is defined as

The function $\sigma: V \to [0,1]$ and $\tau: V \to [0,1]$ denote the degree of membership and nonmembership of the elements $x_i \in V$ respectively such that

$$0 \leq \{\sigma(x_i) + \tau(x_i)\} \leq 1\ \ \forall x_i \in V, (i = 1,2, \dots \dots \dots n)$$

$$(\sigma_1 \oplus \sigma_2)x = \begin{cases} (\sigma_1)x & \text{if } x \in V_1 - V_2 \\ (\sigma_2)x & \text{if } x \in V_2 - V_1 \\ \max\{\sigma_1(x), \sigma_2(x)\} & \text{ifx} \in V_1 \cap V_2 \end{cases}$$

$$(\tau_1 \oplus \tau_2)x = \begin{cases} (\tau_1)x & if\ x \in V_1 - V_2 \\ (\tau_2)x & if\ x \in V_2 - V_1 \\ max\{\tau_1(x), \tau_2(x)\} & ifx \in V_1 \cap V_2 \end{cases}$$

And $$(\mu_1 \oplus \mu_2)xy = \begin{cases} \mu_1(xy) & if\ xy \in E_1 - E_2 \\ \mu_2(xy) & if\ xy \in E_2 - E_1 \\ 0 & otherwise \end{cases}$$

$$(\gamma_1 \oplus \gamma_2)xy = \begin{cases} \gamma_1(xy) & if\ xy \in E_1 - E_2 \\ \gamma_2(xy) & if\ xy \in E_2 - E_1 \\ 0 & otherwise \end{cases}$$

Theorem 2.8 Let $G_1: \{(\sigma_1, \tau_1), \mu_1, \gamma_1)\}$ and $G_1: \{(\sigma_2, \tau_2), \mu_2, \gamma_2)\}$ with $0 \leq \{\sigma(x_i) + \tau(x_i)\} \leq 1\ \forall x_i \in V$ be two intuitionstic product fuzzy graph of crisp graph $G_1: (V_1, E_2)$ and $G_2: (V_2, E_2)$ res. Then the ring sum of two intuitionstic product fuzzy denoted by $G = G_1 \oplus G_2: \{(\sigma_1 \oplus \sigma_2, \tau_1 \oplus \tau_2), (\mu_1 \oplus \mu_2, \gamma_1 \oplus \gamma_2)\}$ is also an intuitionstic product fuzzy graph.

Proof: Since $G_1: \{(\sigma_1, \tau_1), \mu_1, \gamma_1)\}$ and $G_1: \{(\sigma_2, \tau_2), \mu_2, \gamma_2)\}$ be two intuitionstic product fuzzy graph with $0 \leq \{\sigma(x_i) + \tau(x_i)\} \leq 1\ \forall x_i \in V$ of crisp graph $G_1: (V_1, E_2)$ and $G_2: (V_2, E_2)$ res. Then we have to show that $G = G_1 \oplus G_2: \{(\sigma_1 \oplus \sigma_2, \tau_1 \oplus \tau_2), (\mu_1 \oplus \mu_2, \gamma_1 \oplus \gamma_2)\}$ for this it is sufficient to show that

i) $(\mu_1 \oplus \mu_2)xy \leq \{(\sigma_1 \oplus \sigma_2)x \times (\sigma_1 \oplus \sigma_2)y\}$

ii) $(\gamma_1 \oplus \gamma_2)xy \leq \{(\tau_1 \oplus \tau_2)x \vee (\tau_1 \oplus \tau_2)y\}$ In each case.

Case-1 If $xy \in E_1 - E_2$ and $x, y \in V_1 - V_2$ then

$$\begin{aligned}(\mu_1 \oplus \mu_2)xy &= \mu_1(xy) \\ &\leq [\sigma_1(x) \times \sigma_1(y)] \\ &= [(\sigma_1 \cup \sigma_2)x \times (\sigma_1 \cup \sigma_2)y] \qquad \forall\, x, y \in V_1 - V_2 \\ &= [(\sigma_1 \oplus \sigma_2)x \times (\sigma_1 \oplus \sigma_2)y]\end{aligned}$$

i.e $(\mu_1 \oplus \mu_2)xy \leq [(\sigma_1 \oplus \sigma_2)x \times (\sigma_1 \oplus \sigma_2)y] \qquad \forall\, x, y \in V_1 - V_2$

And $xy \in E_1 - E_2$ and $x, y \in V_1 - V_2$ then

$$\begin{aligned}(\gamma_1 \oplus \gamma_2)xy &= \gamma_1(xy) \\ &\leq \max[\tau_1(x), \tau_1(y)] \\ &= \max[(\tau_1 \cup \tau_2)x, (\tau_1 \cup \tau_2)y] \qquad \forall\, x, y \in V_1 - V_2 \\ &= \max[(\tau_1 \oplus \tau_2)x, (\tau_1 \oplus \tau_2)y]\end{aligned}$$

i.e $(\gamma_1 \oplus \gamma_2)xy \leq \max[(\tau_1 \oplus \tau_2)x, (\tau_1 \oplus \tau_2)y] \qquad \forall\, x, y \in V_1 - V_2$

Case-2 If $xy \in E_2 - E_1$ and $x, y \in V_2 - V_1$ then

$$(\mu_1 \oplus \mu_2)xy = \mu_2(xy)$$
$$\leq [\sigma_2(x) \times \sigma_2(y)]$$
$$= [(\sigma_1 \cup \sigma_2)x \times (\sigma_1 \cup \sigma_2)y] \qquad \forall x, y \in V_2 - V_1$$
$$= [(\sigma_1 \oplus \sigma_2)x \times (\sigma_1 \oplus \sigma_2)y]$$

i.e $(\mu_1 \oplus \mu_2)xy \leq [(\sigma_1 \oplus \sigma_2)x \times (\sigma_1 \oplus \sigma_2)y] \quad \forall\, x, y \in V_2 - V_1$

And $xy \in E_2 - E_1$ and $x, y \in V_2 - V_1$ then

$$(\gamma_1 \oplus \gamma_2)xy = \gamma_2(xy)$$
$$\leq \max[\tau_2(x), \tau_2(y)]$$
$$= \max[(\tau_1 \cup \tau_2)x, (\tau_1 \cup \tau_2)y] \qquad \forall\, x, y \in V_2 - V_1$$
$$= \max[(\tau_1 \oplus \tau_2)x, (\tau_1 \oplus \tau_2)y]$$

i.e $(\gamma_1 \oplus \gamma_2)xy \leq \max[(\tau_1 \oplus \tau_2)x, (\tau_1 \oplus \tau_2)y] \quad \forall\, x, y \in V_2 - V_1$

Case-3 If $xy \in E_1 - E_2$ and $x, y \in V_1 - V_2$ then

$$(\mu_1 \oplus \mu_2)xy = \mu_1(uv)$$
$$\leq [\max(\sigma_1(x), \sigma_2(x)) \times \max(\sigma_1(y), \sigma_2(y))]$$
$$= [(\sigma_1 \cup \sigma_2)x \times (\sigma_1 \cup \sigma_2)y]$$
$$= [(\sigma_1 \oplus \sigma_2)x \times (\sigma_1 \oplus \sigma_2)y]$$

i.e $(\mu_1 \oplus \mu_2)xy \leq [(\sigma_1 \oplus \sigma_2)x \times (\sigma_1 \oplus \sigma_2)y]$ for $xy \in E_1 - E_2$ and $x, y \in V_1 - V_2$

And
$$(\gamma_1 \oplus \gamma_2)xy = \gamma_1(xy)$$
$$\leq [\max(\tau_1(x), \tau_2(x)) \vee \max(\tau_1(y), \tau_2(y))]$$
$$= \max[(\tau_1 \cup \tau_2)x, (\tau_1 \cup \tau_2)y]$$
$$= \max[(\tau_1 \oplus \tau_2)x, (\tau_1 \oplus \sigma_2)y]$$

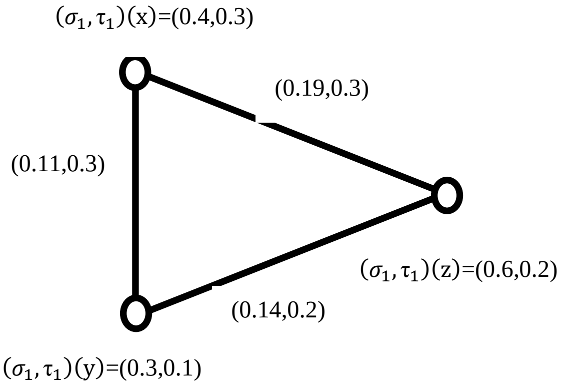
i.e $(\gamma_1 \oplus \gamma_2)xy \leq \max[(\tau_1 \oplus \tau_2)x, (\tau_1 \oplus \tau_2)y]$ for $xy \in E_1 - E_2$ and $x, y \in V_1 - V_2$

Similarly we can show that if $xy \in E_2 - E_1$ and $x, y \in V_2 - V_1$ then

$$(\mu_1 \oplus \mu_2)xy \leq [(\sigma_1 \oplus \sigma_2)x \times (\sigma_1 \oplus \sigma_2)y] \quad \text{and}$$
$$(\gamma_1 \oplus \gamma_2)xy \leq \max[(\tau_1 \oplus \tau_2)x, (\tau_1 \oplus \tau_2)y]$$

This completes the proof of the proposition.

Example 2.9



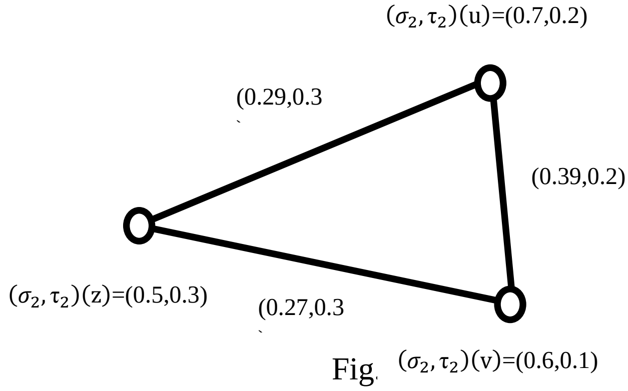


Fig.

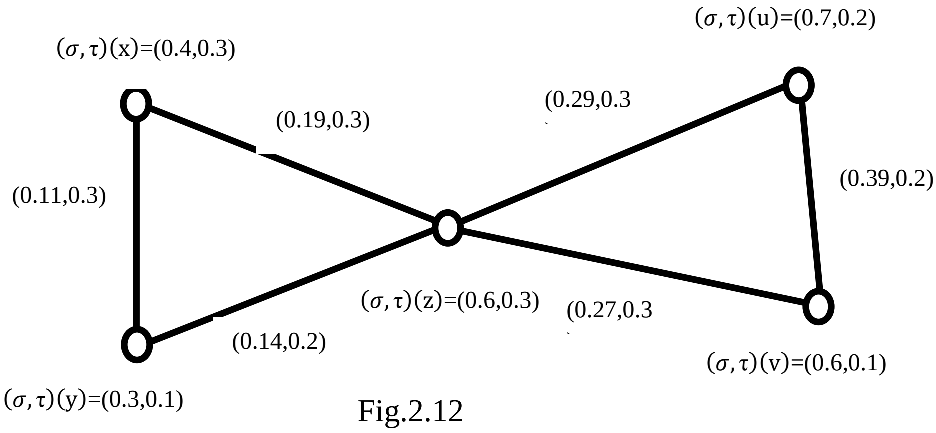


Fig.2.12

Table 2.13 Membership table for fig. 2.10,2.11 and 2.12

Membership degree for vertices			Membership degree for edges		
For Graph $G_1: \{(\sigma_1, \tau_1), \mu_1, \gamma_1)\}$	For Graph $G_1: \{(\sigma_2, \tau_2), \mu_2, \gamma_2)\}$	For Graph $G : G_1 \oplus G_2$	For Graph G_1	For Graph G_2	For Graph $G : G_1 \oplus G_2$
(σ_1, τ_1)(x)=(0.4,0.3)	(σ_2, τ_2)(u)=(0.7,0.2)	(σ_1, τ_1)(x)=(0.4,0.3)	(0.11,0.3)	(0.39,0.2)	(0.11,0.3)
(σ_1, τ_1)(y)=(0.3,0.1)	(σ_2, τ_2)(v)=(0.6,0.1)	(σ_1, τ_1)(y)=(0.3,0.1)	(0.14,0.2)	(0.27,0.3)	(0.14,0.2)
(σ_1, τ_1)(z)=(0.6,0.2)	(σ_2, τ_2)(z)=(0.5,0.3)	(σ_2, τ_2)(u)=(0.7,0.2)	(0.19,0.3)	(0.29,0.3)	(0.19,0.3)
— — — — —	— — — — —	(σ_2, τ_2)(v)=(0.6,0.1)	— — —	— — — —	(0.39,0.2)
— — — — —	— — — — —	(σ, τ)(z)=(0.6,0.3)	— — —	— — —	(0.27,0.3)
— — — — —	— — — — —	— — — — —	— — —	— — —	(0.29,0.3)

Theorem: 2.14 Let $G_1:\{(\sigma_1,\tau_1),\mu_1,\gamma_1)\}$ and $G_1:\{(\sigma_2,\tau_2),\mu_2,\gamma_2)\}$ with $0 \leq \{\sigma(x_i)+\tau(x_i)\} \leq 1\ \forall x_i \in V$ and $V_1 \cap V_2 = \emptyset$ be two (k_1,k_2) regular intuitionstic product fuzzy graph of crisp graph $G_1:(V_1,E_2)$ and $G_2:(V_2,E_2)$ res. Then $G = G_1 \oplus G_2:\{(\sigma_1\oplus\sigma_2,\tau_1\oplus\tau_2),(\mu_1\oplus\mu_2,\gamma_1\oplus\gamma_2)\}$ is also a (k_1,k_2) regular intuitionstic product fuzzy graph.

Proof: Given that $G_1:\{(\sigma_1,\tau_1),\mu_1,\gamma_1)\}$ and $G_1:\{(\sigma_2,\tau_2),\mu_2,\gamma_2)\}$ with $0 \leq \{\sigma(x_i)+\tau(x_i)\} \leq 1\ \forall x_i \in V$ and $V_1 \cap V_2 = \emptyset$ be two (k_1,k_2) regular intuitionstic product fuzzy graph.Then

$\mathbf{d_{G_1}(x)} = (k_1,k_2)\quad \forall x \in V_1$ and

$\mathbf{d_{G_2}(y)} = (k_1,k_2)\quad \forall y \in V_2$

Now given that $V_1 \cap V_2 = \emptyset$ then

$$d_{G_1\oplus G_2}(\mathbf{x}) = \mathbf{d_{G_1\oplus G_2}(x)} + \textstyle\sum_{xy\in E'}\{(\sigma_1(x)\times\sigma_2(y)),(\tau_1(x)\vee\tau_2(y)\}$$

$$= \mathbf{d_{G_1\oplus G_2}(x)} + \mathbf{0}$$

(by Definition of ring sum of intuitionstic product fuzzy graph)

$$= d_{G_1}(x)\quad \forall x \in V_1$$

$$d_{G_1}(x) = (k_1,k_2)$$

i.e. $\quad d_{G_1\oplus G_2}(x) = (k_1,k_2)\quad \forall x \in V_1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$

And $\quad d_{G_1\oplus G_2}(y) = d_{G_1\oplus G_2}(y) + \sum_{xy\in E'}\{(\sigma_1(x)\times\sigma_2(y)),(\tau_1(x)\vee\tau_2(y)\}$

$$= d_{G_1\oplus G_2}(y) + 0$$

(by Definition of ring sum of intuitionstic product fuzzy graph)

$$= d_{G_2}(y)\quad \forall y \in V_2$$

$$d_{G_2}(y) = (k_1,k_2)$$

i.e. $\quad d_{G_1\oplus G_2}(y) = (k_1,k_2)\quad \forall y \in V_2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$

Hence by equation (1) and (2) we have

$$d_{G_1\oplus G_2}(x) = d_{G_1\oplus G_2}(y)$$

So $G = G_1\oplus G_2:\{(\sigma_1\oplus\sigma_2,\tau_1\oplus\tau_2),(\mu_1\oplus\mu_2,\gamma_1\oplus\gamma_2)\}$ is a (k_1,k_2) regular intuitionstic product fuzzy graph.

Theorem:2.15 Let $G_1:\{(\sigma_1,\tau_1),\mu_1,\gamma_1)\}$ and $G_1:\{(\sigma_2,\tau_2),\mu_2,\gamma_2)\}$ with $0 \leq \{\sigma(x_i)+\tau(x_i)\} \leq 1\ \forall x_i \in V$ and $V_1 \cap V_2 = \emptyset$ and $G =$

$G_1 \oplus G_2 : \{(\sigma_1 \oplus \sigma_2, \tau_1 \oplus \tau_2), (\mu_1 \oplus \mu_2, \gamma_1 \oplus \gamma_2)\}$ is a (k_1, k_2) regular intuitionstic product fuzzy graph then $G_1 : \{(\sigma_1, \tau_1), \mu_1, \gamma_1)\}$ and $G_1 : \{(\sigma_2, \tau_2), \mu_2, \gamma_2)\}$ are also (k_1, k_2) regular intuitionstic product fuzzy graph.

Proof: Given hat $G = G_1 \oplus G_2 : \{(\sigma_1 \oplus \sigma_2, \tau_1 \oplus \tau_2), (\mu_1 \oplus \mu_2, \gamma_1 \oplus \gamma_2)\}$ is a (k_1, k_2) regular intuitionstic product fuzzy graph with $V_1 \cap V_2 = \emptyset$ then we have to show that $G_1 : \{(\sigma_1, \tau_1), \mu_1, \gamma_1)\}$ and $G_1 : \{(\sigma_2, \tau_2), \mu_2, \gamma_2)\}$ are also (k_1, k_2) regular intuitionstic product fuzzy graph.

Now given that $V_1 \cap V_2 = \emptyset$ then

$$\mathbf{d}_{\mathbf{G_1 \oplus G_2}}(\mathbf{x}) = \mathbf{d}_{\mathbf{G_1}}(\mathbf{x}) + \sum_{xy \in E'} \{(\sigma_1(x) \times \sigma_2(y)), (\tau_1(x) \vee \tau_2(y)\}$$

$$= \mathbf{d}_{\mathbf{G_1}}(\mathbf{x}) + \mathbf{0}$$

(by Definition of ring sum of intuitionstic product fuzzy graph)

$$(k_1, k_2) = \mathbf{d}_{\mathbf{G_1}}(\mathbf{x}) \quad \forall x \in V_1$$

Since $G_1 \oplus G_2$ is (k_1, k_2) regular intuitionstic product fuzzy graph

i.e. $$d_{G_1}(x) = (k_1, k_2) \quad \forall x \in V_1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

And $$d_{G_1 \oplus G_2}(y) = d_{G_2}(y) + \sum_{xy \in E'} \{(\sigma_1(x) \times \sigma_2(y)), (\tau_1(x) \vee \tau_2(y)\}$$

(by Definition of ring sum of intuitionstic product fuzzy graph)

$$d_{G_1 \oplus G_2}(y) = d_{G_2}(y) + 0$$

$$d_{G_2}(y) = (k_1, k_2) \quad \forall y \in V_2$$

Since $G_1 \oplus G_2$ is (k_1, k_2) regular intuitionistic product fuzzy graph

i.e.

$$d_{G_2}(y) = (k_1, k_2) \quad \forall y \in V_2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Hence; by equation (1) and (2) $G_1 : \{(\sigma_1, \tau_1), \mu_1, \gamma_1)\}$ and $G_2 : \{(\sigma_2, \tau_2), \mu_2, \gamma_2)\}$ are also (k_1, k_2) regular intuitionstic product fuzzy graph.

CONCLUSION

The notion of product fuzzy graph has generalized for intuitionistic product fuzzy graph and definition of complement and ring sum of two intuitionistic product fuzzy graph have provided with example. Further we have proved that, If two product fuzzy graph are (k_1, k_2) regular intuitionistic product fuzzy graph then ring sum of regular intuitionistic product fuzzy graph is also a (k_1, k_2) regular intuitionstic product fuzzy graph and conversely.

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