

Now we shall discuss some of the binary soft hereditary properties of τ_{Δ_i} ($i= 0,1$) spaces.

Theorem 3.21. Let $(U_1, U_2, \tau_{\Delta}, E)$ be a binary soft topological space on \tilde{X} over $(U_1 \times U_2)$ and $\tilde{Y} \subseteq \tilde{X}$. Then, if $(U_1, U_2, \tau_{\Delta}, E)$ is a binary soft τ_{Δ_0} -space then $(U_1, U_2, \tau_{\Delta_y}, E)$ is a binary soft τ_{Δ_0} -space.

Proof: $F_e, G_e \in \tilde{Y}$ such that $F_e \neq G_e$. Then $F_e, G_e \in \tilde{X}$. Since $(U_1, U_2, \tau_{\Delta}, E)$ is a binary soft τ_{Δ_0} -space, thus there exists binary soft open sets (F, E) and (G, E) in $(U_1, U_2, \tau_{\Delta}, E)$ such that $F_e \in (F, E)$ and $G_e \notin (F, E)$ or $G_e \in (G, E)$ and $F_e \notin (G, E)$. Therefore $F_e \in \tilde{Y} \cap (F, E) = {}^Y(F, E)$. Similarly it can be shown that if $G_e \in (G, E)$ and $F_e \notin (G, E)$, then $G_e \in {}^Y(G, E)$ and $F_e \notin {}^Y(G, E)$. Thus $(U_1, U_2, \tau_{\Delta_y}, E)$ is binary soft τ_{Δ_0} -space.

Theorem 3.22. Let $(U_1, U_2, \tau_{\Delta}, E)$ be a binary soft topological space on \tilde{X} over $(U_1 \times U_2)$ and $\tilde{Y} \subseteq \tilde{X}$. If $(U_1, U_2, \tau_{\Delta}, E)$ is a binary soft τ_{Δ_1} -space then $(U_1, U_2, \tau_{\Delta_y}, E)$ is a binary soft τ_{Δ_1} -space.

Proof: The proof is similar to the proof of Theorem 3.21.

Theorem 3.23. Let $(U_1, U_2, \tau_{\Delta}, E)$ be a binary soft topological space on \tilde{X} over $(U_1 \times U_2)$. If $(U_1, U_2, \tau_{\Delta}, E)$ is a binary soft τ_{Δ_2} -space on \tilde{X} over $(U_1 \times U_2)$, then $(U_1, U_2, \tau_{\Delta_e}, E)$ is a binary soft τ_{Δ_2} -space for each $e \in E$.

Proof: Let $(U_1, U_2, \tau_{\Delta}, E)$ be a binary soft topological space on \tilde{X} over $(U_1 \times U_2)$. For any $e \in E$, $\tau_{\Delta_e} = \{F(e) : (F, E) \in \tau_{\Delta}\}$ is a binary soft topology on \tilde{X} over $(U_1 \times U_2)$. Let $x, y \in \tilde{X}$ such that $x \neq y$, since $(U_1, U_2, \tau_{\Delta}, E)$ is a binary soft τ_{Δ_2} -space, therefore binary soft points $F_e, G_e \in \tilde{X}$ such that $F_e \neq G_e$ and $x \in F(e)$, $y \in G(e)$, there exists binary soft open sets (F_1, E) and (F_2, E) such that

$F_e \tilde{\in} (F_1, E)$, $G_e \tilde{\in} (F_2, E)$ and $(F_1, E) \tilde{\cap} (F_2, E) = \tilde{\phi}$. Which implies that $x \tilde{\in} F(e) \tilde{\subseteq} F_1(e)$, $y \tilde{\in} G(e) \tilde{\subseteq} F_2(e)$ and $F_1(e) \tilde{\cap} F_2(e) = \tilde{\phi}$. This proves that $(U_1, U_2, \tau_{\Delta_e}, E)$ is a binary soft τ_{Δ_2} -space.

Theorem 3.24. Let $(U_1, U_2, \tau_{\Delta}, E)$ be a binary soft topological space on \tilde{X} over $(U_1 \times U_2)$ and $\tilde{Y} \tilde{\subseteq} \tilde{X}$. If $(U_1, U_2, \tau_{\Delta}, E)$ is a binary soft τ_{Δ_2} -space then $(U_1, U_2, \tau_{\Delta_y}, E)$ is a binary soft τ_{Δ_2} -space and $(U_1, U_2, \tau_{\Delta_e}, E)$ is a binary soft τ_{Δ_2} -space for each $e \tilde{\in} E$.

Proof: Let $F_e, G_e \tilde{\in} \tilde{Y}$ such that $F_e \neq G_e$. Then $F_e, G_e \tilde{\in} \tilde{X}$, since $(U_1, U_2, \tau_{\Delta}, E)$ is a binary soft τ_{Δ_2} therefore there exists binary soft open sets (F_1, E) and (F_2, E) such that $F_e \tilde{\in} (F_1, E)$, $G_e \tilde{\in} (F_2, E)$ and $(F_1, E) \tilde{\cap} (F_2, E) = \tilde{\phi}$. Thus $F_e \tilde{\in} \tilde{Y} \tilde{\cap} (F_1, E) = {}^Y(F_1, E)$,

$G_e \tilde{\in} \tilde{Y} \tilde{\cap} (F_2, E) = {}^Y(F_2, E)$ and ${}^Y(F_2, E) \tilde{\cap} {}^Y(F_2, E) = \tilde{\phi}$. Thus it proves that $(U_1, U_2, \tau_{\Delta_y}, E)$ is a binary soft τ_{Δ_2} -space.

Theorem 3.25. Let $(U_1, U_2, \tau_{\Delta}, E)$ be a binary soft topological space on \tilde{X} over $(U_1 \times U_2)$. If $(U_1, U_2, \tau_{\Delta}, E)$ is a binary soft τ_{Δ_2} -space and for any two binary soft points $F_e, G_e \tilde{\in} \tilde{X}$ such that $F_e \neq G_e$, then there exists binary soft closed sets (F_1, E) and (F_2, E) such that $F_e \tilde{\in} (F_1, E)$, $G_e \tilde{\notin} (F_1, E)$ and $F_e \tilde{\notin} (F_2, E)$, $G_e \tilde{\in} (F_2, E)$ and $(F_1, E) \tilde{\cup} (F_2, E) = \tilde{X}$.

Proof: Let $(U_1, U_2, \tau_{\Delta}, E)$ be a binary soft topological space on \tilde{X} over $(U_1 \times U_2)$. Since $(U_1, U_2, \tau_{\Delta}, E)$ is binary soft τ_{Δ_2} -space and $F_e, G_e \tilde{\in} \tilde{X}$ such that $F_e \neq G_e$, there exists binary soft open sets (H, E) and (L, E) such that $F_e \tilde{\in} (H, E)$ and $G_e \tilde{\in} (L, E)$ and $(H, E) \tilde{\cap} (L, E) = \tilde{\phi}$. Clearly $(H, E) \tilde{\subseteq} (L, E)^c$ and $(L, E) \tilde{\subseteq} (H, E)^c$. Hence $F_e \tilde{\in} (L, E)^c$, put $(L, E)^c = (F_1, E)$ which gives $F_e \tilde{\in} (F_1, E)$ and $G_e \tilde{\notin} (F_1, E)$. Also $G_e \tilde{\in} (F_1, E)^c$, then put $(H, E)^c = (F_2, E)$. Therefore $F_e \tilde{\in} (F_1, E)$ and $G_e \tilde{\in} (F_2, E)$.

Moreover $(F_1, E) \tilde{\subset} (F_2, E) = (L, E)^c \tilde{\subset} (H, E)^c = \tilde{X}$. Which completes the proof.

4. Binary Soft Regular, Binary Soft Normal and Binary Soft τ_{Δ_i} (i=4,3) Spaces

In this section, we define binary soft regular and binary soft τ_{Δ_3} - spaces using binary soft points. We also characterize binary soft regular and binary soft normal spaces. Moreover we prove that binary soft regular and binary soft τ_{Δ_3} properties are binary soft hereditary, where as binary soft normal and binary soft τ_{Δ_4} are binary soft closed hereditary properties.

Now we define binary soft regular space as follows:

Definition 4.1. Let $(U_1, U_2, \tau_{\Delta}, E)$ be a binary soft topological space on \tilde{X} over $(U_1 \times U_2)$. Let (F, E) be a binary soft closed set in $(U_1, U_2, \tau_{\Delta}, E)$ and $F_e \tilde{\in} \tilde{X}$, such that $F_e \tilde{\not\in} (F, E)$. If there exist binary soft open sets (G, E) and (H, E) , such that $F_e \tilde{\in} (G, E)$, $(F, E) \tilde{\subseteq} (H, E)$ and $(G, E) \tilde{\cap} (H, E) = \tilde{\phi}$, then $(U_1, U_2, \tau_{\Delta}, E)$ is called a binary soft regular space.

Theorem 4.2. Let $(U_1, U_2, \tau_{\Delta}, E)$ be a binary soft topological space on \tilde{X} over $(U_1 \times U_2)$. Then the following statements are equivalent:

- (i) $(U_1, U_2, \tau_{\Delta}, E)$ is binary soft regular.
- (ii) For any binary soft open set (F, E) in $(U_1, U_2, \tau_{\Delta}, E)$ and $G_e \tilde{\in} (F, E)$, there is a binary soft open set (G, E) containing G_e such that $G_e \tilde{\in} \overline{(G, E)} \tilde{\subseteq} (F, E)$.
- (iii) Each binary soft point in $(U_1, U_2, \tau_{\Delta}, E)$ has a binary soft neighborhood base consisting of binary soft closed sets.

Proof: (i) \Rightarrow (ii)

Let (F, E) be a binary soft open set in $(U_1, U_2, \tau_{\Delta}, E)$ and $G_e \tilde{\in} (F, E)$. Then $(F, E)^c$ is binary soft closed set such that $G_e \tilde{\not\in} (F, E)^c$. By the binary soft regularity of $(U_1, U_2, \tau_{\Delta}, E)$, there are binary soft open sets (F_1, E) , (F_2, E) such that $G_e \tilde{\in} (F_1, E)$, $(F, E)^c \tilde{\subseteq} (F_2, E)$ and $(F_1, E) \tilde{\cap} (F_2, E) = \tilde{\phi}$. Clearly $(F_2, E)^c$ is a binary

soft closed set contained in (F, E) . Thus $(F_1, E) \tilde{\subseteq} (F_2, E)^c \tilde{\subseteq} (F, E)$. This gives $\overline{\overline{(F_1, E)}} \tilde{\subseteq} (F_2, E)^c \tilde{\subseteq} (F, E)$, put $(F_1, E) = (G, E)$. Consequently, $G_e \tilde{\in} (G, E)$ and $\overline{\overline{(G, E)}} \tilde{\subseteq} (F, E)$. This proves (ii).

(ii) \Rightarrow (iii)

Let $G_e \tilde{\in} \tilde{X}$, for binary soft open set (F, E) in $(U_1, U_2, \tau_\Delta, E)$, there is a binary soft open set (G, E) containing G_e such that $G_e \tilde{\in} (G, E)$, $\overline{\overline{(G, E)}} \tilde{\subseteq} (F, E)$. Thus for each $G_e \tilde{\in} \tilde{X}$, the sets $\overline{\overline{(G, E)}}$ form a binary soft neighborhood base consisting of binary soft closed sets of $(U_1, U_2, \tau_\Delta, E)$ which proves (3).

(iii) \Rightarrow (i)

Let (F, E) be a binary soft closed set such that $G_e \tilde{\notin} (F, E)$. Then $(F, E)^c$ is a binary soft open neighborhood of G_e . By (iii), there is binary soft closed set (F_1, E) which contains G_e and is a binary soft neighborhood of G_e with $(F_1, E) \tilde{\subseteq} (F_1, E)^c$. Then $G_e \tilde{\notin} (F, E)^c$, $(F, E) \tilde{\subseteq} (F_1, E)^c = (F_2, E)$ and $(F_1, E) \tilde{\cap} (F_2, E) = \tilde{\phi}$. Therefore $(U_1, U_2, \tau_\Delta, E)$ is binary soft regular.

Theorem 4.3. Let $(U_1, U_2, \tau_\Delta, E)$ be a binary soft regular space on \tilde{X} over $(U_1 \times U_2)$. Then every binary soft subspace of $(U_1, U_2, \tau_\Delta, E)$ is binary soft regular.

Proof: Let $(U_1, U_2, \tau_{\Delta_y}, E)$ be a binary soft subspace of a binary soft regular space $(U_1, U_2, \tau_\Delta, E)$. Suppose, (F, E) is a binary soft closed set in $(U_1, U_2, \tau_{\Delta_y}, E)$ and $F_e \tilde{\in} \tilde{Y}$ such that $F_e \tilde{\notin} (F, E)$. Then $(F, E) = (G, E) \tilde{\cap} \tilde{Y}$; where (G, E) is binary soft closed set in $(U_1, U_2, \tau_\Delta, E)$. Then $F_e \tilde{\notin} (G, E)$, since $(U_1, U_2, \tau_\Delta, E)$ is binary soft regular, there exists binary soft disjoint binary open sets $(F_1, E), (F_2, E)$ in $(U_1, U_2, \tau_\Delta, E)$ such that $F_e \tilde{\in} (F_1, E)$, $(G, E) \tilde{\subseteq} (F_2, E)$. Clearly, $F_e \tilde{\in} (F_1, E) \tilde{\cap} \tilde{Y} = {}^Y(F_2, E)$ and $(F, E) \tilde{\subseteq} (F_2, E) \tilde{\cap} \tilde{Y} = {}^Y(F_2, E)$ such that ${}^Y(F_1, E) \tilde{\cap} {}^Y(F_2, E) = \tilde{\phi}$. Therefore it proves that $(U_1, U_2, \tau_{\Delta_y}, E)$ is a binary soft regular subspace of $(U_1, U_2, \tau_\Delta, E)$.

Theorem 4.4. Let $(U_1, U_2, \tau_\Delta, E)$ be a binary soft regular space on \tilde{X} over $(U_1 \times U_2)$. A binary space $(U_1, U_2, \tau_\Delta, E)$ is binary soft regular if and only if for each

$F_e \tilde{\in} \tilde{X}$ and a binary soft closed set (F, E) in $(U_1, U_2, \tau_\Delta, E)$ such that $F_e \tilde{\notin} (F, E)$, there exist binary soft open sets $(F_1, E), (F_2, E)$ in $(U_1, U_2, \tau_\Delta, E)$ such that $F_e \tilde{\in} (F_1, E)$, $(F_1, E) \tilde{\subseteq} (F_2, E)$ and $\overline{\overline{(F_1, E)}} \tilde{\cap} \overline{\overline{(F_2, E)}} = \tilde{\phi}$.

Proof: For each $F_e \tilde{\in} \tilde{X}$ and a binary soft closed set (G, E) such that $F_e \tilde{\notin} (G, E)$, by Theorem 4.2 there is a binary soft open (G, E) such that $F_e \tilde{\in} (G, E)$, $\overline{\overline{(G, E)}} \tilde{\subseteq} (F_1, E)^c$. Again by Theorem 4.2 there is a binary soft open (F_1, E) containing F_e such that $\overline{\overline{(F_1, E)}} \tilde{\subseteq} (G, E)$. Let $(F_2, E) = \overline{\overline{(G, E)}}^c$, then $\overline{\overline{(F_1, E)}} \tilde{\subseteq} (G, E) \tilde{\subseteq} \overline{\overline{(G, E)}} \tilde{\subseteq} (F, E)^c$ implies $(F, E) \tilde{\subseteq} \overline{\overline{(G, E)}}^c = (F_2, E)$ or $(F, E) \tilde{\subseteq} (F_2, E)$.

Also

$\overline{\overline{(F_1, E)}} \tilde{\cap} \overline{\overline{(F_2, E)}} = \overline{\overline{(F_1, E)}} \tilde{\cap} \overline{\overline{(G, E)}}^c \tilde{\subseteq} (G, E) \tilde{\cap} \overline{\overline{(G, E)}}^c \tilde{\subseteq} \overline{\overline{(G, E)}} \tilde{\cap} \overline{\overline{(G, E)}}^c = \tilde{\phi} = \phi$. Thus (F_1, E) , (F_2, E) are the required binary soft open sets in $(U_1, U_2, \tau_\Delta, E)$. This proves the necessity. The sufficiency is immediate.

Definition 4.5. Let $(U_1, U_2, \tau_\Delta, E)$ be a binary soft topological space on \tilde{X} over $(U_1 \times U_2)$, (F, E) and (G, E) are binary soft closed sets over $(U_1 \times U_2)$ such that $(F, E) \tilde{\cap} (G, E) = \tilde{\phi}$. If there exist binary soft open sets (F_1, E) and (F_2, E) such that $(F, E) \tilde{\subseteq} (F_1, E)$, $(G, E) \tilde{\subseteq} (F_2, E)$ and $(F_1, E) \tilde{\cap} (F_2, E) = \phi$, then $(U_1, U_2, \tau_\Delta, E)$ is called a binary soft normal space.

Definition 4.6. Let $(U_1, U_2, \tau_\Delta, E)$ be a binary soft topological space on \tilde{X} over $(U_1 \times U_2)$. Then $(U_1, U_2, \tau_\Delta, E)$ is said to be binary soft τ_{Δ_3} -space if it is binary soft regular and a binary soft τ_{Δ_1} -space.

Remark 4.7. (i) A binary soft τ_{Δ_3} -space may not be a binary soft τ_{Δ_2} -space.

(ii) If $(U_1, U_2, \tau_\Delta, E)$ is a binary soft τ_{Δ_3} -space, then $(U_1, U_2, \tau_{\Delta_e}, E)$ may not be a binary soft τ_{Δ_3} -space for each parameter $e \tilde{\in} E$

Proposition 4.8. Let $(U_1, U_2, \tau_\Delta, E)$ be a binary soft topological space on \tilde{X} over $(U_1 \times U_2)$ and $Y \tilde{\subseteq} \tilde{X}$. If $(U_1, U_2, \tau_\Delta, E)$ is a binary soft τ_{Δ_3} -space then

$(U_1, U_2, \tau_{\Delta_y}, E)$ is a binary soft τ_{Δ_3} -space.

Definition 4.9. A binary soft topological space $(U_1, U_2, \tau_{\Delta}, E)$ on \tilde{X} over $(U_1 \times U_2)$ is said to be a binary soft τ_{Δ_4} -space, if it is binary soft normal and binary soft τ_{Δ_1} -space.

Theorem 4.10. A binary soft topological space $(U_1, U_2, \tau_{\Delta}, E)$ is binary soft normal if and only if for any binary soft closed set (F, E) and binary soft open set (G, E) such that $(F, E) \tilde{\subseteq} (G, E)$, there exist at least one binary soft open set (H, E) containing (F, E) such that $(F, E) \tilde{\subseteq} (H, E) \tilde{\subseteq} \overline{(H, E)} \tilde{\subseteq} (G, E)$.

Proof: Let us suppose that $(U_1, U_2, \tau_{\Delta}, E)$ is a binary soft normal space and (F, E) is any binary soft closed subset of $(U_1, U_2, \tau_{\Delta}, E)$ and (G, E) is a binary soft open set such that $(F, E) \tilde{\subseteq} (G, E)$. Then $(G, E)^c$ is binary soft closed and $(F, E) \tilde{\cap} (G, E)^c = \phi$. So by supposition, there are binary soft open sets (H, E) and (K, E) such that $(F, E) \tilde{\subseteq} (H, E)$, $(G, E)^c \tilde{\subseteq} (K, E)$ and $(H, E) \tilde{\cap} (K, E) = \tilde{\phi}$. Since $(H, E) \tilde{\cap} (K, E) = \tilde{\phi}$, $(H, E) \tilde{\subseteq} (K, E)^c$. But $(K, E)^c$ is binary soft closed, so that $(F, E) \tilde{\subseteq} (H, E) \tilde{\subseteq} \overline{(H, E)} \tilde{\subseteq} (K, E)^c \tilde{\subseteq} (G, E)$.

Hence $(F, E) \tilde{\subseteq} (H, E) \tilde{\subseteq} \overline{(H, E)} \tilde{\subseteq} (G, E)$.

Conversely, suppose that for every binary soft closed set (F, E) and a binary soft open set (G, E) such that $(F, E) \tilde{\subseteq} (G, E)$, there is a binary soft open set (H, E) such that $(F, E) \tilde{\subseteq} (H, E) \tilde{\subseteq} \overline{(H, E)} \tilde{\subseteq} (G, E)$. Let (F_1, E) , (F_2, E) be any two binary soft disjoint closed sets, then $(F_1, E) \tilde{\subseteq} (F_2, E)^c$ where $(F_2, E)^c$ is binary soft open. Hence there is a binary soft open set (H, E) such that $(F_1, E) \tilde{\subseteq} (H, E) \tilde{\subseteq} \overline{(H, E)} \tilde{\subseteq} (F_2, E)^c$. But then $(F_2, E) \tilde{\subseteq} \overline{((H, E))^c}$ and $(H, E) \tilde{\cap} \overline{((H, E))^c} \neq \tilde{\phi}$.

Hence $(F_1, E) \tilde{\subseteq} (H, E)$ and $(F_2, E) \tilde{\subseteq} \overline{((H, E))^c}$ with $(H, E) \tilde{\cap} \overline{((H, E))^c} = \tilde{\phi}$. Hence $(U_1, U_2, \tau_{\Delta}, E)$ is binary soft normal space.

Proposition 4.11. Let $(U_1, U_2, \tau_{\Delta_y}, E)$ be binary soft subspace of a binary soft topological space $(U_1, U_2, \tau_{\Delta}, E)$ and (F, E) be a binary soft open(closed) in

$(U_1, U_2, \tau_{\Delta_y}, E)$. If \tilde{Y} is a binary soft open(closed) in $(U_1, U_2, \tau_{\Delta}, E)$, then (F, E) is binary soft open(closed) in $(U_1, U_2, \tau_{\Delta}, E)$.

Theorem 4.12. A binary soft closed subspace of a binary soft normal is binary soft normal.

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Competing Interests

The authors declare that no competing interests exist.

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