

Algorithm: Three Dimensional Finite Volume Numerical Grid Technique

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Abstract

In this paper, we present an efficient computational algorithms for solving three dimensional steady state heat transfer in cube using finite volume grid technique. The implementation of the algorithm for steady state finite volume structured grids tri-diagonal linear system using MS Excel is presented. An example is given in order to illustrate the algorithms.

Keywords: Computational algorithm; Discretized linear system; Finite volume method; MS Excel; Structured grids; Steady state; Tri-diagonal system.

1. INTRODUCTION

In the last few decades, revolution in the computer technology has led to development of numerous computational grid techniques for solving many engineering problems [1]. As mathematical modelling became an integral part of analysis of engineering

problems, a variety of numerical grid techniques have been developed. A commonly used numerical technique is the finite difference method (FDM), described in references [2], [3] and [4]. The another numerical technique called the finite element method (FEM) developed originally for the solution of structural problem, has been applied to the solution of heat conduction problems and other details about this technique can be seen in the papers [3],[5],[6] and [7]. The next popular numerical technique is finite volume method (FVM) was originally developed as a special finite difference formulation; for more detailed the reader may consult [8]. Each of these methods has its own merits and demerits depending on the problem to be solved. Out of the available numerical grid techniques, the finite volume technique is one of the most flexible and versatile technique for solving the problems in computational fluid dynamics.

The remainder of the paper is organised as follows. In Section 2, a short review of finite volume (FV) techniques with the help of TDMA (Tri-Diagonal Matrix Algorithm) solver is given. In Section 3, formulation of three dimensional heat flow problems with dirichlet boundary conditions. In Section 4, the proposed computational algorithm for 3D finite volume method. An illustrative example and the implementation of algorithm using MS excel are presented in section 5. Finally, section 6 concludes the paper.

2. FINITE VOLUME METHOD

The Finite Volume Method is an increasing popular numerical technique for the approximate solution of partial differential equations. For more detailed the reader may consult [8]. The Finite Volume analysis involves three basic steps.

- In The problem domain is defined and divided the solution domain into discrete control volume. Let us place a numbers of nodal points in the given space and domain is divided in such way that, each node is surrounded by the control volume or grid and the physical boundaries coincide with the control volume boundaries.
- The integration of the governing equation over the control volume to yield a discretized equation at its nodal point.
- Solve the set of discretized equations using TDMA solver.

2.1. FINITE VOLUME DISCRETIZATIONS

The General form of discretised equations for three dimensional steady state heat flow problems are given by equation:

$$a_p \theta_p = \sum a_i \theta_i + S_\theta \quad (1)$$

$$a_p = \sum a_i - S_p \tag{2}$$

$$a_i = \frac{kA}{\Delta} \tag{3}$$

Where a_i are the neighbouring coefficients a_W, a_E and a_W, a_E, a_N, a_S in one and two dimensional respectively, θ_i are the values of the function θ at the neighbouring nodes, S_θ and S_p are the values obtained from the linear source term $S_\theta + S_p\theta_p$ which is the function of the dependent variable. Note that, to obtain the values S_θ and S_p from the linear source term $S_\theta + S_p\theta_p$ with boundary B.

For Fixed value θ_B ,

$$S_\theta = \frac{2kA}{\Delta} \theta_B \text{ and } S_p = -\frac{2kA}{\Delta}$$

For Fixed Flux q ,

$$S_\theta = q \times A \text{ and } S_p = 0$$

2.2. TDMA

Linear systems also arise in numerous other engineering and scientific applications and a large number of techniques have been developed for their solution. This kind of linear system was solved by using software, see [9-10] and [11]. The tri diagonal matrix algorithm (TDMA), also known also Thomas algorithm, is a simplified form of Gaussian elimination that can be used to solve tri diagonal system of equations

$$-a_i\theta_{i-1} + b_i\theta_i - c_i\theta_{i+1} = d_i \quad i = 1, \dots, n \tag{4}$$

The TDMA is based on the Gaussian elimination procedure and consist of two parts - a forward elimination phase and a backward substitution phase. The TDMA is actually a direct method for one dimensional situation, but it can be applied iteratively in a line-by-line fashion, to solve multidimensional problems and is widely used in CFD programs. Let us consider the system for $i = 1, \dots, n$ and we use the general form of the TDMA solver is given by

$$\theta_i = A_i\theta_{i+1} + B_i \tag{5}$$

Where

$$A_i = \frac{c_i}{b_i - a_iA_{i-1}} \quad \text{and} \quad B_i = \frac{a_iB_{i-1} + d_i}{b_i - a_iA_{i-1}}$$

To solve the above system TDMA is applied along the north-south lines for three dimensional problems, the discretised equation is re-arranged in the form

$$-a_S\theta_S + a_P\theta_P - a_N\theta_N = a_W\theta_W + a_E\theta_E + a_B\theta_B + a_T\theta_T + S_\theta \quad (6)$$

3. PROBLEM FORMULATION

Consider three dimensional steady state heat transfer in the cube with boundary conditions, the mathematical formulation of this problems is given by

$$\frac{\partial}{\partial x} \left(k \frac{\partial \theta}{\partial x} \right) + \frac{\partial}{\partial y} \left(k \frac{\partial \theta}{\partial y} \right) + \frac{\partial}{\partial z} \left(k \frac{\partial \theta}{\partial z} \right) = 0, \quad 0 \leq x, y, z \leq 1 \quad (7)$$

Subject to the boundary conditions

$$\begin{aligned} \theta &= f_1(y, z) && \text{at } x = 0, 0 \leq y, z \leq 1 \\ \theta &= f_2(y, z) && \text{at } x = 1, 0 \leq y, z \leq 1 \\ \theta &= g_1(x, z) && \text{at } y = 0, 0 \leq x, z \leq 1 \\ \theta &= g_2(x, z) && \text{at } y = 1, 0 \leq x, z \leq 1 \\ \theta &= h_1(x, y) && \text{at } z = 0, 0 \leq x, y \leq 1 \\ \theta &= h_2(x, y) && \text{at } z = 1, 0 \leq x, y \leq 1 \end{aligned}$$

Where f_1, f_2, g_1, g_2, h_1 and h_2 are the function of x, y, z or may be constant and the solution region with boundary sides as shown in Figure 1.

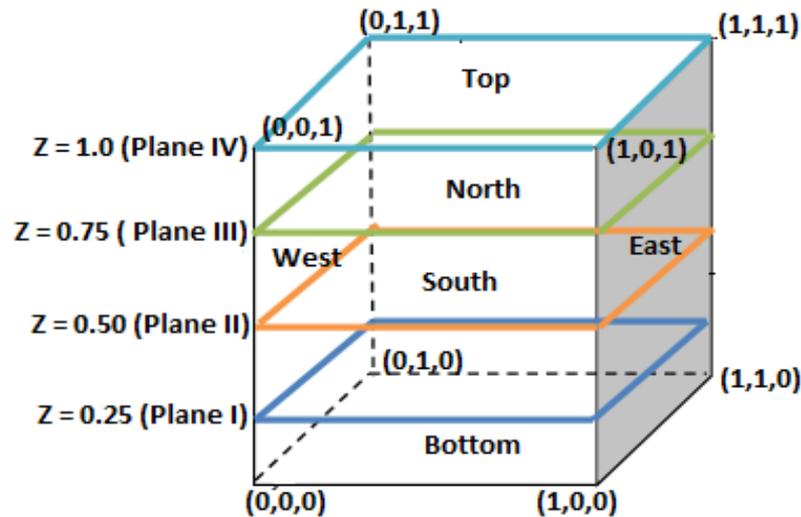


Figure 1: Three dimensional solution region with boundary sides to illustrative 3DFVGA

In this case, the three dimensional solution region is divided into the four XY plane and there are 64 nodes of Figure 2. There nodes are divided into different categories

according to their positions are shown in Table 1. Note that a_{bn} is the coefficient of boundary node and a_{nbn} is the coefficient of no boundary node.

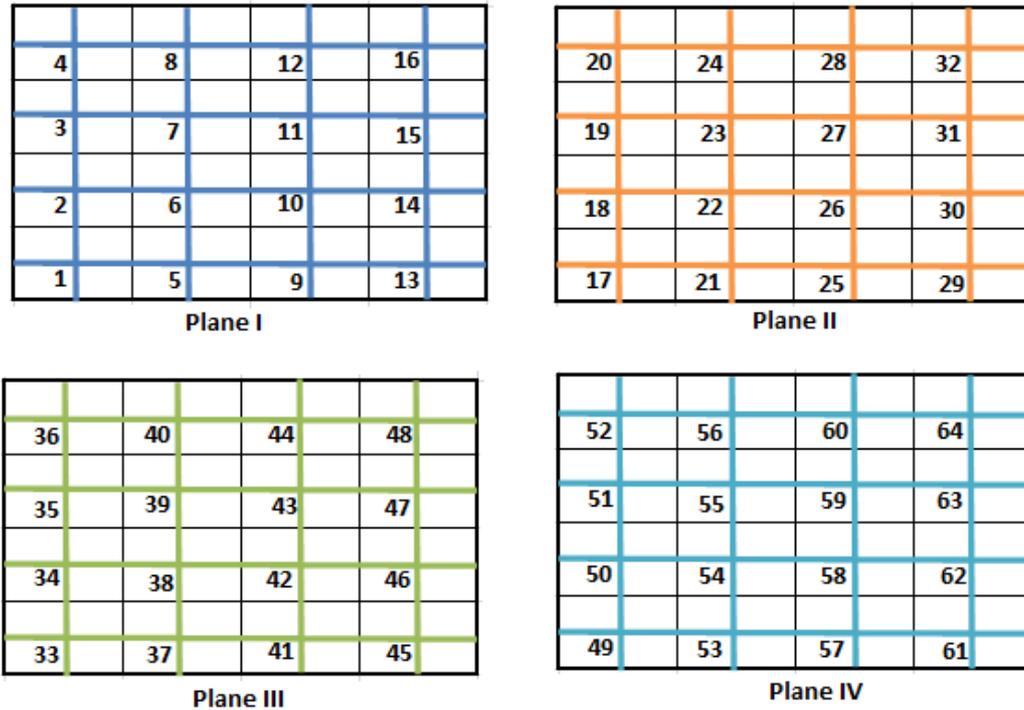


Figure 2: The Four XY planes with grids and nodes to illustrative 3D FVGA

Table 1: The positions of the nodes for the 3D FVGA

Planes	Boundary Nodes			No Boundary Nodes
	One boundary	Two boundary	Three boundary	
I	6,7,10,11	2,3,5,8,9,12,14,15	1,4,13,16	
II	18,19,21,24,25,28,30,31	17,20,29,32	----	22,23,26,27
III	34,35,37,40,41,44,46,47	33,36,45,48	----	38,39,42,43
IV	54,55,58,59	50,51,53,56,57,60,62,63	49,52,61,64	

4. COMPUTATIONAL ALGORITHM

To solve the three dimensional steady state heat transfers in the cube represent the

equation (7) using finite volume grid technique, proceed as follows.

Algorithm 4.1. Finite Volume Grid Algorithm for Three Dimensional Problems.

If the temperature θ is the function of x, y, z and may be constant then

Step 1: For the one boundary nodes

$$\left\{ \begin{array}{l} a_{bn} = 0 \\ a_{nbn} = \frac{kA}{\Delta} \\ S = \frac{2kA}{\Delta} \theta \\ S_p = -\frac{2kA}{\Delta} \\ a_p = a_{bn} + \sum a_{nbn} - S_p \end{array} \right.$$

Step 2: For the Two boundary nodes.

$$\left\{ \begin{array}{l} a_{bn} = 0 \\ a_{nbn} = \frac{kA}{\Delta} \\ S = \frac{2kA}{\Delta} \theta_1 + \frac{2kA}{\Delta} \theta_2 \\ S_p = -\frac{2kA}{\Delta} - \frac{2kA}{\Delta} \\ a_p = \sum a_{bn} + \sum a_{nbn} - S_p \end{array} \right.$$

Step 3: For the three boundary nodes

$$\left\{ \begin{array}{l} a_{bn} = 0 \\ a_{nbn} = \frac{kA}{\Delta} \\ S = \frac{2kA}{\Delta} \theta_1 + \frac{2kA}{\Delta} \theta_2 + \frac{2kA}{\Delta} \theta_3 \\ S_p = -\frac{2kA}{\Delta} - \frac{2kA}{\Delta} - \frac{2kA}{\Delta} \\ a_p = \sum a_{bn} + \sum a_{nbn} - S_p \end{array} \right.$$

Step 4: For the No boundary nodes

$$\begin{cases} a_{nbn} = \frac{kA}{\Delta} \\ S_{\theta} = S_p = 0 \\ a_p = \sum a_{nbn} - S_p \end{cases}$$

Iteration 1

Step 5: Calculation for Plane I

- Set the given vector $a_N = a, a_P = b, a_S = c, \theta_W = \theta_E = \theta_B = \theta_T = 0$ and $a_W\theta_W + a_E\theta_E + a_B\theta_B + a_T\theta_T + S = d$

5.1. For $i = 1$, set $A_0 = 0, B_0 = 0$ and compute

$$A_1 = \frac{c_1}{b_1 - a_1A_0} \text{ and } B_1 = \frac{a_1B_0 + d_1}{b_1 - a_1A_0}$$

5.2. For $i = 2, \dots, n$ and compute

$$A_i = \frac{c_i}{b_i - a_iA_{i-1}} \text{ and } B_i = \frac{a_iB_{i-1} + d_i}{b_i - a_iA_{i-1}}$$

5.3. For $i = n - \dots - 1$, Set $\theta_{n+1} = 0$ and compute $\theta_i = A_i\theta_{i+1} + B_i$

The End of the First Line

- Set the given vectors $a_N = a, a_P = b, a_S = c, \theta_E = \theta_B = \theta_T = 0, \theta_W = \theta$ (Result of first line) and $a_W\theta_W + a_E\theta_E + a_B\theta_B + a_T\theta_T + S = d$.
- Repeated steps 5.1, 5.2 and 5.3 then get the result of second line.

The End of the Second Line

- Set the given vectors $a_N = a, a_P = b, a_S = c, \theta_E = \theta_B = \theta_T = 0, \theta_W = \theta$ (Result of second line) and $a_W\theta_W + a_E\theta_E + a_B\theta_B + a_T\theta_T + S = d$.
- Repeated steps 5.1, 5.2 and 5.3 then get the result of third line.

The End of the Third Line

- Set the given vectors $a_N = a, a_P = b, a_S = c, \theta_E = \theta_B = \theta_T = 0, \theta_W = \theta$ (Result of third line) and $a_W\theta_W + a_E\theta_E + a_B\theta_B + a_T\theta_T + S = d$.
- Repeated steps 5.1, 5.2 and 5.3 then get the result of fourth line.

The End of the Fourth Line

Step 6: Calculation for Plane II

- Set the given vectors $a_N = a, a_P = b, a_S = c, \theta_E = \theta_W = \theta_T = 0, \theta_B = \theta(\text{Result of first line-Plane I-Iteration 1})$ and $a_W\theta_W + a_E\theta_E + a_B\theta_B + a_T\theta_T + S = d$.
- Repeated steps 5.1, 5.2 and 5.3 then get the result of first line.

The End of the First Line

- Set the given vectors $a_N = a, a_P = b, a_S = c, \theta_E = \theta_T = 0, \theta_W = \theta(\text{Result of first line-Plane II-Iteration 1}), \theta_B = \theta(\text{Result of second line-Plane I-Iteration 1})$, and $a_W\theta_W + a_E\theta_E + a_B\theta_B + a_T\theta_T + S = d$.
- Repeated steps 5.1, 5.2 and 5.3 then get the result of second line.

The End of the Second Line

- Set the given vectors $a_N = a, a_P = b, a_S = c, \theta_E = \theta_T = 0, \theta_W = \theta(\text{Result of second line-Plane II-Iteration 1}), \theta_B = \theta(\text{Result of third line-Plane I-Iteration 1})$ and $a_W\theta_W + a_E\theta_E + a_B\theta_B + a_T\theta_T + S = d$.
- Repeated steps 5.1, 5.2 and 5.3 then get the result of third line.

The End of the Third Line

- Set the given vectors $a_N = a, a_P = b, a_S = c, \theta_E = \theta_T = 0, \theta_W = \theta(\text{Result of third line-Plane II-Iteration 1}), \theta_B = \theta(\text{Result of fourth line-Plane I-Iteration 1})$ and $a_W\theta_W + a_E\theta_E + a_B\theta_B + a_T\theta_T + S = d$.
- Repeated steps 5.1, 5.2 and 5.3 then get the result of fourth line.

The End of the Fourth Line**Step 7: Calculation for Plane III**

- Set the given vectors $a_N = a, a_P = b, a_S = c, \theta_E = \theta_W = \theta_T = 0, \theta_B = \theta(\text{Result of first line-Plane II-Iteration 1})$ and $a_W\theta_W + a_E\theta_E + a_B\theta_B + a_T\theta_T + S = d$.
- Repeated steps 5.1, 5.2 and 5.3 then get the result of first line.

The End of the First Line

- Set the given vectors $a_N = a, a_P = b, a_S = c, \theta_E = \theta_T = 0, \theta_W = \theta(\text{Result of first line-Plane III-Iteration 1}), \theta_B = \theta(\text{Result of second line-Plane II-Iteration 1})$, and $a_W\theta_W + a_E\theta_E + a_B\theta_B + a_T\theta_T + S = d$.

- Repeated steps 5.1, 5.2 and 5.3 then get the result of second line.

The End of the Second Line

- Set the given vectors $a_N = a, a_P = b, a_S = c, \theta_E = \theta_T = 0, \theta_W = \theta$ (Result of second line-Plane III-Iteration 1), $\theta_B = \theta$ (Result of third line-Plane II-Iteration 1) and $a_W\theta_W + a_E\theta_E + a_B\theta_B + a_T\theta_T + S = d$.
- Repeated steps 5.1, 5.2 and 5.3 then get the result of third line.

The End of the Third Line

- Set the given vectors $a_N = a, a_P = b, a_S = c, \theta_E = \theta_T = 0, \theta_W = \theta$ (Result of third line-Plane III-Iteration 2), $\theta_B = \theta$ (Result of fourth line-Plane II-Iteration 1) and $a_W\theta_W + a_E\theta_E + a_B\theta_B + a_T\theta_T + S = d$.
- Repeated steps 5.1, 5.2 and 5.3 then get the result of fourth line.

The End of the Fourth Line

Step 8: Calculation for Plane IV

- Set the given vectors $a_N = a, a_P = b, a_S = c, \theta_E = \theta_W = \theta_T = 0, \theta_B = \theta$ (Result of first line-Plane III-Iteration 1) and $a_W\theta_W + a_E\theta_E + a_B\theta_B + a_T\theta_T + S = d$.
- Repeated steps 5.1, 5.2 and 5.3 then get the result of first line.

The End of the First Line

- Set the given vectors $a_N = a, a_P = b, a_S = c, \theta_E = \theta_T = 0, \theta_W = \theta$ (Result of first line-Plane IV-Iteration 1), $\theta_B = \theta$ (Result of second line-Plane III-Iteration 1), and $a_W\theta_W + a_E\theta_E + a_B\theta_B + a_T\theta_T + S = d$.
- Repeated steps 5.1, 5.2 and 5.3 then get the result of second line.

The End of the Second Line

- Set the given vectors $a_N = a, a_P = b, a_S = c, \theta_E = \theta_T = 0, \theta_W = \theta$ (Result of second line-Plane IV-Iteration 1), $\theta_B = \theta$ (Result of third line-Plane III-Iteration 1) and $a_W\theta_W + a_E\theta_E + a_B\theta_B + a_T\theta_T + S = d$.
- Repeated steps 5.1, 5.2 and 5.3 then get the result of third line.

The End of the Third Line

- Set the given vectors $a_N = a, a_P = b, a_S = c, \theta_E = \theta_T = 0, \theta_W = \theta$ (Result of third line-Plane IV-Iteration 2), $\theta_B = \theta$ (Result of fourth line-Plane III-

Iteration 1) and $a_W\theta_W + a_E\theta_E + a_B\theta_B + a_T\theta_T + S = d$.

- Repeated steps 5.1, 5.2 and 5.3 then get the result of fourth line.

The End of the Fourth Line

Iteration 2

Step 9: Calculation for Plane I

- Set the given vector $a_N = a, a_P = b, a_S = c, \theta_W = \theta_B = 0, \theta_E = \theta$ (Result of Second line-Plane I Iteration 1) $\theta_T = \theta$ (Result of first line-Plane II Iteration 1) ,and $a_W\theta_W + a_E\theta_E + a_B\theta_B + a_T\theta_T + S = d$.
- Repeated steps 5.1, 5.2 and 5.3 then get the result of first line.

The End of the First Line.

- Set the given vectors $a_N = a, a_P = b, a_S = c, \theta_B = 0, \theta_W = \theta$ (Result of first line-Plane I Iteration 2), $\theta_E = \theta$ (Result of third line-Plane I Iteration 1), $\theta_T = \theta$ (Result of second line-Plane II Iteration 1), and $a_W\theta_W + a_E\theta_E + a_B\theta_B + a_T\theta_T + S = d$.
- Repeated steps 5.1, 5.2 and 5.3 then get the result of second line.

The End of the Second Line

- Set the given vectors $a_N = a, a_P = b, a_S = c, \theta_B = 0, \theta_W = \theta$ (Result of second line-Plane I Iteration 2), $\theta_E = \theta$ (Result of fourth line-Plane I Iteration 1), $\theta_T = \theta$ (Result of third line-Plane II Iteration 1), and $a_W\theta_W + a_E\theta_E + a_B\theta_B + a_T\theta_T + S = d$.
- Repeated steps 5.1, 5.2 and 5.3 then get the result of third line.

The End of the Third Line.

- Set the given vectors $a_N = a, a_P = b, a_S = c, \theta_E = \theta_B = 0, \theta_W = \theta$ (Result of third line-Plane I Iteration 2), $\theta_T = \theta$ (Result of fourth line-Plane II Iteration 1), and $a_W\theta_W + a_E\theta_E + a_B\theta_B + a_T\theta_T + S = d$.
- Repeated steps 5.1, 5.2 and 5.3 then get the result of fourth line.

The End of the Fourth Line

Step 10: Calculation for Plane II

- Set the given vectors $a_N = a, a_P = b, a_S = c, \theta_W = 0, \theta_E = \theta$ (Result of

second line-Plane II-Iteration 1), $\theta_B = \theta(\text{Result of first line-Plane I Iteration 2})$, $\theta_T = \theta(\text{Result of first line-Plane III Iteration 1})$ and $a_W\theta_W + a_E\theta_E + a_B\theta_B + a_T\theta_T + S = d$.

- Repeated steps 5.1, 5.2 and 5.3 then get the result of first line.

The End of the First Line

- Set the given vectors $a_N = a, a_P = b, a_S = c$, , $\theta_W = \theta(\text{Result of first line-Plane II-Iteration 2})$, $\theta_E = \theta(\text{Result of third line-Plane II-Iteration 1})$, $\theta_B = \theta(\text{Result of second line-Plane I Iteration 2})$, $\theta_T = \theta(\text{Result of second line-Plane III Iteration 1})$, and $a_W\theta_W + a_E\theta_E + a_B\theta_B + a_T\theta_T + S = d$.

- Repeated steps 5.1, 5.2 and 5.3 then get the result of second line.

The End of the Second Line

- Set the given vectors $a_N = a, a_P = b, a_S = c$, $\theta_W = \theta(\text{Result of second line-Plane II-Iteration 2})$, $\theta_E = \theta(\text{Result of fourth line-Plane II-Iteration 1})$, $\theta_B = \theta(\text{Result of third line-Plane I Iteration 2})$, $\theta_T = \theta(\text{Result of third line-Plane III Iteration 1})$, and $a_W\theta_W + a_E\theta_E + a_B\theta_B + a_T\theta_T + S = d$.

- Repeated steps 5.1, 5.2 and 5.3 then get the result of third line.

The End of the Third Line

- Set the given vectors $a_N = a, a_P = b, a_S = c, \theta_E = 0$, $\theta_W = \theta(\text{Result of third line –Plane II-Iteration 2})$, $\theta_B = \theta(\text{Result of fourth line-Plane I-Iteration 2})$, $\theta_T = \theta(\text{Result of fourth line-Plane III Iteration 1})$, and $a_W\theta_W + a_E\theta_E + a_B\theta_B + a_T\theta_T + S = d$.

- Repeated steps 5.1, 5.2 and 5.3 then get the result of fourth line.

The End of the Fourth Line

Step 11: Calculation for Plane III

- Set the given vectors $a_N = a, a_P = b, a_S = c, \theta_W = 0$, $\theta_E = \theta(\text{Result of second line-Plane III-Iteration 1})$, $\theta_B = \theta(\text{Result of first line-Plane II-Iteration 2})$, $\theta_T = \theta(\text{Result of first line-Plane IV-Iteration 1})$ and $a_W\theta_W + a_E\theta_E + a_B\theta_B + a_T\theta_T + S = d$.

- Repeated steps 5.1, 5.2 and 5.3 then get the result of first line.

The End of the First Line

- Set the given vectors $a_N = a, a_P = b, a_S = c, \theta_W = \theta(\text{Result of first line-Plane$

III-Iteration 2), $\theta_E = \theta(\text{Result of third line-Plane III-Iteration 1})$, $\theta_B = \theta(\text{Result of second line-Plane II-Iteration 2})$, $\theta_T = \theta(\text{Result of second line-Plane IV-Iteration 1})$ and $a_W\theta_W + a_E\theta_E + a_B\theta_B + a_T\theta_T + S = d$.

- Repeated steps 5.1, 5.2 and 5.3 then get the result of second line.

The End of the Second Line

- Set the given vectors $a_N = a, a_P = b, a_S = c, \theta_W = \theta(\text{Result of second line-Plane III-Iteration 2})$, $\theta_E = \theta(\text{Result of fourth line-Plane III-Iteration 1})$, $\theta_B = \theta(\text{Result of third line-Plane II-Iteration 2})$, $\theta_T = \theta(\text{Result of third line-Plane IV-Iteration 1})$ and $a_W\theta_W + a_E\theta_E + a_B\theta_B + a_T\theta_T + S = d$.
- Repeated steps 5.1, 5.2 and 5.3 then get the result of third line.

The End of the Third Line

- Set the given vectors $a_N = a, a_P = b, a_S = c, \theta_E = 0, \theta_W = \theta(\text{Result of third line-Plane III-Iteration 2})$, $\theta_B = \theta(\text{Result of fourth line-Plane II-Iteration 2})$, $\theta_T = \theta(\text{Result of fourth line-Plane IV-Iteration 1})$, and $a_W\theta_W + a_E\theta_E + a_B\theta_B + a_T\theta_T + S = d$.
- Repeated steps 5.1, 5.2 and 5.3 then get the result of fourth line.

The End of the Fourth Line

Step 12: Calculation for Plane IV

- Set the given vectors $a_N = a, a_P = b, a_S = c, \theta_W = \theta_T = 0, \theta_E = \theta(\text{Result of second line-Plane IV-Iteration 1})$, $\theta_B = \theta(\text{Result of first line-Plane III-Iteration 2})$, and $a_W\theta_W + a_E\theta_E + a_B\theta_B + a_T\theta_T + S = d$.
- Repeated steps 5.1, 5.2 and 5.3 then get the result of first line.

The End of the First Line.

- Set the given vectors $a_N = a, a_P = b, a_S = c, \theta_T = 0, \theta_W = \theta(\text{Result of first line-Plane IV-Iteration 2})$, $\theta_E = \theta(\text{Result of third line-Plane IV-Iteration 1})$, $\theta_B = \theta(\text{Result of second line-Plane III-Iteration 2})$, and $a_W\theta_W + a_E\theta_E + a_B\theta_B + a_T\theta_T + S = d$.
- Repeated steps 5.1, 5.2 and 5.3 then get the result of second line.

The End of the Second Line

- Set the given vectors $a_N = a, a_P = b, a_S = c, \theta_T = 0, \theta_W = \theta(\text{Result of second line-Plane IV-Iteration 2})$, $\theta_E = \theta(\text{Result of fourth line-Plane IV-}$

Iteration 1), $\theta_B = \theta$ (Result of third line-Plane III-Iteration 2), and $a_W\theta_W + a_E\theta_E + a_B\theta_B + a_T\theta_T + S = d$.

- Repeated steps 5.1, 5.2 and 5.3 then get the result of third line.

The End of the Third Line

- Set the given vectors $a_N = a, a_P = b, a_S = c, \theta_E = \theta_T = 0, \theta_W = \theta$ (Result of third line –Plane IV-Iteration 2), $\theta_B = \theta$ (Result of fourth line-Plane III-Iteration 2) and $a_W\theta_W + a_E\theta_E + a_B\theta_B + a_T\theta_T + S = d$.
- Repeated steps 5.1, 5.2 and 5.3 then get the result of fourth line.

The End of the Fourth Line.

The entire iteration 2 procedure is repeated until a converged solution is obtained. For the three dimensional problem, this algorithm proposed name as Finite Volume Grid Algorithm (3DFVGA).

5. AN ILLUSTRATIVE EXAMPLE

Let us consider the three dimensional steady state heat equation as shown by equation (7) with dirichlet boundary conditions

$$\theta(0, y, z) = \theta(x, 0, y) = \theta(x, y, 0) = 0$$

$$\theta(1, y, z) = yz$$

$$\theta(x, 1, z) = zx$$

$$\theta(x, y, 1) = xy$$

The analytic solution of this problem is given by $\theta(x, y, z) = xyz$ and its converged numerical solution is obtained after seventh iterations as shown in Table 2. In figure 1, three dimensional solution region cube is shown and is divided into four plane as shown in figure 2. The thermal conductivity of the cube material is $k = 1000$ w/m/k. Use a uniform grid size $\nabla x = \nabla y = \nabla z = 0.25$ m.

The input value with the boundary conditions of Figure 3. The Figure 4 shows that the input boundary data for the spreadsheet implementation to the known nodes potentials at the boundary of the three dimensional solution region. The coefficient and the source terms of the finite volume discretized equations for all the nodes are calculated plane by plane along the north-south lines as same to the two dimension case [12] and the orange, green, blue and yellow colour indicates that one, two, three and no boundary nodes of the figure 4.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
82																			
83																			
84																			
85																			
86																			
87																			
88																			
89																			
90																			
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100																			
101																			
102																			
103																			
104																			

Figure 4- The Finite Volume discretization with respective Plane I-IV to illustrate the 3DFVGA

The entire iteration plane by plane procedure is now repeated until a converged solution is obtained. The next all steps of this algorithm should be followed as per two dimensional case [12]. The numerical solution of such type of linear system has been obtained after seven iterations and which shows in Table 2.

Table 2: A Comparison between Finite Volume Numerical Solution after 7th iteration and Analytical solutions

Planes	I		II		III		IV	
	FVT	Analytic	FVT	Analytic	FVT	Analytic	FVT	Analytic
1	0.0019	0.0020	0.0064	0.0059	0.0097	0.0098	0.0136	0.0137
2	0.0055	0.0059	0.0176	0.0176	0.0288	0.0293	0.0408	0.0410
3	0.0101	0.0098	0.0283	0.0293	0.0480	0.0488	0.0680	0.0684
4	0.0123	0.0137	0.0395	0.0410	0.0677	0.0684	0.0954	0.0957
5	0.0055	0.0059	0.0197	0.0176	0.0295	0.0293	0.0409	0.0410
6	0.0174	0.0176	0.0546	0.0527	0.0877	0.0879	0.1225	0.1230
7	0.0305	0.0293	0.0867	0.0879	0.1453	0.1465	0.2041	0.2051
8	0.0400	0.0410	0.1186	0.1230	0.2033	0.2051	0.2858	0.2871
9	0.0090	0.0098	0.0329	0.0293	0.0495	0.0488	0.0684	0.0684
10	0.0305	0.0293	0.0920	0.0879	0.1470	0.1465	0.2050	0.2051
11	0.0529	0.0488	0.1463	0.1465	0.2435	0.2441	0.3415	0.3418

12	0.0600	0.0684	0.1985	0.2051	0.3403	0.3418	0.4781	0.4785
13	0.0165	0.0137	0.0435	0.0410	0.0687	0.0684	0.0957	0.0957
14	0.0400	0.0410	0.1260	0.1230	0.2052	0.2051	0.2867	0.2871
15	0.0700	0.0684	0.2057	0.2051	0.3417	0.3418	0.4784	0.4785
16	0.0901	0.0957	0.2819	0.2871	0.4776	0.4785	0.6698	0.6699

6. CONCLUSION

In this paper, finite volume numerical grid technique for steady state heat flow problems have been studied and obtained the numerical solution using proposed algorithm, via 3DFVGA. The proposed algorithm described here is a very effective, gratuitous and easy to implement in high-priced mathematical software such as MAPLE, MATHEMATICS and MATLAB for solving the finite volume structured grid of discretized equations which appear in CFD problems and many engineering application.

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