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Flow features of a conducting visco-elastic fluid past a vertical permeable plate

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Abstract

This paper is concerned with the mathematical model for studying the effects of heat and mass transfer of a visco-elastic fluid on a conducting magnetohydrodynamic boundary layer flow past a vertical permeable plate. A magnetic field of uniform strength is applied in the direction perpendicular to the plate. There is no external electric field imposed on the system and the magnetic Reynolds number is very small. The important properties of overall structure of the fluid motion are studied and the governing equations are solved analytically by regular perturbation technique. The influence of the magnetic field and the elasticity on the flow as well as on the skin friction are examined quantitatively and graphical illustrations are made in possible cases. Applications of this model have been noticed in various industrial and chemical processes.

Keywords: Visco-elastic, free convection, MHD, permeability, porous.

Subject classification number: 76A05, 76A10

1. INTRODUCTION

An extensive study has been performed in the last few decades to get an enhanced analysis of visco-elastic fluid flow due to many practical applications which can be approximated as transport phenomena in porous media. The mechanisms of visco-elastic boundary layer flow are capitalized in various manufacturing processes such as fabrication of adhesive tapes, extrusion of plastic sheets, coating layers in rigid surfaces etc. Various blood flow problems are also explained using the visco-elastic boundary layer theory.

The hydromagnetic convection with heat and mass transfer in porous medium has been studied due to its importance in the design of MHD generators and accelerators in geophysics, in design of underground water energy storage system, soil-sciences, astrophysics, nuclear power reactors and so on. Several authors have studied the forced, free and mixed convection flows of a viscous incompressible fluid along a vertical surface in absence of magnetic field, notable amongst them are Sparrow and Gregg[1], Merkin[2], Loyed and Sparrow[3]. Somess[4], Soundalgeker and Ganesan[5], Khair and Bejan[6], and Lin and Wu[7] have studied heat and mass transfer on flow past a vertical plate. The problem of combined heat and mass transfer in MHD free convective flow from a vertical surface with ohmic heating and viscous dissipation has been studied by Chen[8].. The effect of ohmic heating on the MHD free convection heat transfer has been examined for a Newtonian fluid by Hossain *et al*.[9]. Anjali Devi and Kandasamy[10] have studied the effects of chemical reaction, heat and mass transfer on non-linear MHD laminar boundary layer flow over a wedge with suction or injection. Raptis *et al*.[11] have investigated the viscous flow over a non-linearly stretching sheet in presence of chemical reaction and magnetic field. Nayak *et al*.[12] have studied the heat and mass transfer effects on MHD of visco-elastic fluid over a stretching sheet through porous medium in presence of chemical reaction.

Again, in many rheological models the mechanical behaviour have been proposed through visco-elastic fluid flow. The analysis of chemical reaction, heat and mass transfer in MHD visco-elastic fluid flows have attracted numerous scientists and engineers for the last several decades because of its fascination and importance in various technological devices and understanding the diverse cosmic phenomena and power generation of energy. This study helps to solve many biological problems. Considering the model of visco-elastic fluid, many scientists have solved problems of engineering interests viz Saxena and Dubey[13], Choudhury and Dey[14], Choudhury and Dhar[15] etc. Ahmed and Sarkar[16] have analyzed the MHD natural convection flow of viscous incompressible fluid from a vertical plate. The specific objective of this paper is to extend the work done in [16]. We have analyzed the heat and mass transfer by free-convection steady flow of a visco-elastic fluid past a vertical permeable plate under the action of transverse magnetic field and the shearing stress at the plate are obtained and illustrated graphically to observe the visco-elastic effects in combination with other flow parameters. The visco-elastic fluid behaviour is characterized by Walters liquid (Model B') as many fluids of pragmatic significance exhibit visco-elastic behaviour.

2. MATHEMATICAL FORMULATION:

We consider a steady two-dimensional free-convective boundary layer flow of an incompressible and electrically conducting visco-elastic fluid with heat and mass transfer past a vertical porous plate. A uniform magnetic field of strength B_o is applied along the normal to the plate. The x'-axis is taken along the vertical plate and y'-axis is taken normal to the plate. Let u' and v' be the components of the velocity in x' and y' directions respectively, taken along and perpendicular to the plate. Also, we consider that the interaction of induced magnetic field with the flow is of negligible order in comparision with the interaction of the imposed magnetic field. The electrical conductivity of the fluid is also assumed to be of smaller order in magnitude.

The governing equations are:

$$\frac{\partial v'}{\partial y'} = 0 \Rightarrow v' = -v_0 \tag{2.1}$$

$$v'\frac{\partial u'}{\partial y'} = \upsilon\frac{\partial^2 u'}{\partial y'^2} - \frac{k_0}{\rho}\left(v'\frac{\partial^3 u'}{\partial y'^3}\right) + g\beta\left(T' - T_\infty\right) + g\beta^*\left(C - C_\infty\right) - \frac{\sigma B_0^2}{\rho}u'^2 - \frac{\upsilon}{k'}u' \tag{2.2}$$

$$v'\frac{\partial T'}{\partial y'} = \frac{\bar{\kappa}}{\rho C_P}\frac{\partial^2 T'}{\partial y'^2} + \frac{\upsilon}{C_P}\left(\frac{\partial u'}{\partial y'}\right)^2 - \frac{k_0}{\rho C_P}v'\frac{\partial u'}{\partial y'}\frac{\partial^2 u'}{\partial y'^2} + \frac{\sigma B_0^2}{\rho C_P}u'^2 \tag{2.3}$$

$$v'\frac{\partial C'}{\partial y'} = D\frac{\partial^2 C'}{\partial y'^2} \tag{2.4}$$

where the symbols have their usual meanings.

The relevant boundary conditions are:

$$\left.\begin{array}{l} u' = v' = 0, T' = T_0, C' = C_0 \ \ at \ \ y' = 0 \\ u' \to 0, T' \to T_\infty, C' \to C_\infty \ \ as \ \ y' \to \infty \end{array}\right\} \tag{2.5}$$

We introduce the following non –dimensional parameters:

$$y = \frac{v_0 y'}{\upsilon} \qquad u = \frac{u'}{v_0} \qquad \theta = \frac{T' - T_\infty}{T_0 - T_\infty} \qquad P_r = \frac{\mu C_P}{\bar{\kappa}} \qquad k = \frac{k_0 v_0^2}{\rho\upsilon^2}$$

$$C = \frac{C' - C_\infty}{C_0 - C_\infty} \qquad S_C = \frac{\upsilon}{D} \qquad K = \frac{v_o^2 k'}{\upsilon^2} \qquad G_r = \frac{g\beta\left(T_0 - T_\infty\right)\upsilon^2}{v_0^3}$$

$$M = \frac{\sigma B_0^2 \upsilon}{\rho v_0^2} \qquad Gm = \frac{g\beta\left(C_0 - C_\infty\right)\upsilon}{v_0^3} \qquad E = \frac{v_0^2}{C_P\left(T_0 - T_\infty\right)}$$

where k is the non-dimensional elastico-viscous parameter, K is the permeable parameter, M is the magnetic parameter, G_r is the Grashof number for heat transfer, G_m is the Grashof number for mass transfer, P_r is the Prandtl number, S_c is the Schmidt number.

The non-dimensional forms of the equations (2.2) to (2.4) are of the forms:

$$k\,\frac{d^3u}{dy^3}+\frac{d^2u}{dy^2}+\frac{du}{dy}+(M+\frac{1}{k})u=-G_r\theta-G_mC \tag{2.6}$$

$$\frac{d^2\theta}{dy^2}+P_r\frac{d\theta}{dy}+kP_rE\,\frac{du}{dy}\frac{d^2u}{dy^2}+P_rMEu^2+P_rE\left(\frac{du}{dy}\right)^2=0 \tag{2.7}$$

$$\frac{d^2C}{dy^2}+S_C\frac{dC}{dy}=0 \tag{2.8}$$

The corresponding boundary conditions are

$$\left.\begin{aligned} &u=0,\theta=1,C=1\ at\ y=0\\ &u\to 0,\theta\to 0,C\to 0\ as\ y\to\infty \end{aligned}\right\} \tag{2.9}$$

3. METHOD OF SOLUTION:

The solution of equation (2.8) subject to the boundary condition (2.9) given by

$$C=e^{-S_Cy} \tag{3.1}$$

To solve the coupled non-linear equation (2.6) and (2.7) , we expand velocity and temperature in powers of Eckert number(as for incompressible fluids E<< 1) as follows:

$$\left.\begin{aligned} &u=u_0+Eu_1+0(E^2)\\ &\theta=\theta_0+E\theta_1+0(E^2) \end{aligned}\right\} \tag{3.2}$$

Applying (3.2) in equations (2.6) and (2.7), equating the like powers of E, neglecting $0(E^2)$ and higher powers we get the following equations:

Zeroth-order equations:

$$ku_0^{///}+u_0^{//}+u_0^{/}-(M+K^{-1})u_0=-G_r\theta_0-G_mC \tag{3.3}$$

$$\theta_0^{//}+P_r\theta_0^{/}=0 \tag{3.4}$$

First-order equations:

$$ku_1^{///} + u_1^{//} + u_1^{/} - (M + K^{-1})u_1 = -G_r\theta_1 \tag{3.5}$$

$$\theta_1^{//} + P_r\theta_1^{/} + P_rMu_0^{2} + kP_ru_0u_o^{//} + P_ru_0^{/2} = 0 \tag{3.6}$$

The modified boundary conditions are given by

$$\left.\begin{array}{l} u_0 = 0, u_1 = 0, \theta_0 = 1, \theta_1 = 0 \;\; at\; y = 0 \\ u_0 \to 0, u_1 \to 0, \theta_0 \to 0, \theta_1 \to 0 \;\; as\; y \to \infty \end{array}\right\} \tag{3.7}$$

The solution of equation (3.4) is

$$\theta_0 = e^{-\Pr y} \tag{3.8}$$

To solve equations (3.3) , (3.5) and (3.6), we use multi-parameter perturbation scheme for k following Nowinski and Ismail(18) (k<< 1, for small shear rate) as follows:

$$\left.\begin{array}{l} u_0 = u_{00} + ku_{01} + 0(k^2) \\ u_1 = u_{10} + ku_{11} + 0(k^2) \\ \theta_1 = \theta_{10} + k\theta_{11} + 0(k^2) \end{array}\right\} \tag{3.9}$$

Using (3.9) in the equation (3.3), (3.5) and (3.6), equating the like powers of k and neglecting the co-efficients of $0(k^2)$ and higher-order terms we get

Zeroth-order equations:

$$u_{00}^{//} + u_{00}^{/} - (M + K^{-1})u_{00} = -G_r\theta_0 - G_mC \tag{3.10}$$

$$u_{10}^{/} + u_{10}^{/} - (M + K^{-1})u_{10} = -G_r\theta_{10} \tag{3.11}$$

$$P_r\theta_{10}^{/} + \theta_{10}^{//} + P_ru_{00}^{/2} + P_rMu_{00}^{2} = 0 \tag{3.12}$$

First-order equations:

$$u_{00}^{///} + u_{01}^{//} + u_{01}^{/} - (M + K^{-1})u_{01} = 0 \tag{3.13}$$

$$u_{10}^{///} + u_{11}^{//} + u_{11}^{/} + u_{10}^{///} - (M + K^{-1})u_{11} = -G_r\theta_{11} \tag{3.14}$$

$$P_r\theta_{11}^{/} + \theta_{11}^{//} + P_ru_{00}^{/}u_{00}^{//} + 2P_ru_{00}^{/}u_{01}^{/} + 2P_rMu_{00}u_{01} = 0 \tag{3.15}$$

with transformed boundary conditions:

$$\left.\begin{array}{l} u_{00} = 0, u_{01} = 0, u_{10} = 0, u_{11} = 0, \theta_{10} = 0, \theta_{11} = 0 \; at\; y = 0 \\ u_{00} \to 0, u_{01} \to 0, u_{10} \to 0, u_{11} \to 0, \theta_{10} \to 0, \theta_{11} \to 0 \; as\; y \to \infty \end{array}\right\} \tag{3.16}$$

Solving equations (3.10) to (3.15) relevant to the boundary conditions (3.17),
the expressions for velocity and temperature are obtained as follows:

$$u = a_{83}e^{-a_5 y} + a_{84}e^{-Scy} + a_{85}e^{-\Pr y} + E[a_{86}e^{-a5y} + a_{87}e^{\Pr y} + a_{88}e^{-2a5y} + a_{89}e^{-2Scy} + a_{90}e^{-2\Pr y} + a_{91}e^{-(a5+Sc)y} + a_{92}e^{-(\Pr+Sc)y} + a_{93}e^{-(a5+\Pr)y} \tag{3.18}$$

$$\theta = e^{-P_r y} + E[a_{94}e^{-P_r y} + a_{95}e^{-2a_5 y} + a_{96}e^{-2Scy} + a_{97}e^{-2P_r y} + a_{98}e^{-(a5+Sc)y} + a_{99}e^{-(\Pr+Sc)y} + a_{100}e^{-(a5+\Pr)y}] \tag{3.19}$$

The non-dimensional shearing stress is given by

$$\sigma = \left(\frac{\sigma_{xy}}{\rho v_0^2}\right)_{y=0} = u'(0) + ku''(0)$$

The non-dimensional heat flux in terms of Nusselt number at the plate is given as

$$N_u = \left(\frac{d\theta}{dy}\right)_{y=0}$$

The rate of mass diffusion in the form of Sherwood number is given as

$$S_h = \left(\frac{dC}{dy}\right)_{y=0}$$

4. RESULTS AND DISCUSSIONS

The effects of heat and mass transfer of a visco-elastic fluid on a conducting magnetohydrodynamic boundary layer flow past a vertical permeable plate have been analysed. The pertinent flow characteristics are illustrated graphically in figures 1 to 13. In this problem, we have considered E=0.001, while the other parameters vary over a range which are listed in figure captions and chosen arbitrarily. The zero value of the visco-elastic parameter k indicates the Newtonian fluid motion, on the other hand the non-zero values (k=0.01,0.02) represent the visco-elastic fluid flow phenomenon. The variations of fluid velocity u against the distance y are exhibited in the figures 1 to 7 to illustrate the effect of visco-elasticity on the velocity field in presence of other flow parameters involved in the problem. It is noticed that the fluid velocity accelerates near the plate rapidly to a considerable extent and then drops as one moves away from the plate. Also, the growth of the visco-elastic parameter decelerates the speed of the fluid velocity as compared to the simple flow fluid. In this problem, the flow structure is discussed for $G_r > 0$ which corresponds to the externally cooled plate. The ratio of buoyancy force to viscous force characterizes the Grashof number for heat transfer (G_r) which is very useful in many technological applications. It is noticed from the figures 1 and 2 that the rising value of G_r enhances the speed of the fluid velocity in visco-elastic and Newtonian fluid flow phenomenon. Again, in most engineering fields, it is observed that the heat transfer and mass transfer occur simultaneously though their characteristics are different. The mass is deported due to movement of the fluid in connection of mass transfer. G_m represents the free convection parameter for mass transfer. In fluid motion, $G_m > 0$ means that the free stream concentrations is less than the concentration at the boundary surface. Figures 1 and 3 reveal that the growth of G_m subdues the viscosity of the fluid flow and hence the fluid velocity takes diminishing trend. The simultaneous effect of momentum and thermal diffusion in fluid flows is exhibited by the Prandtl number and the role of Prandtl number is significant in heat transfer mechanism. Figures 1 and 4 indicate that the increase of Prandtl number helps to increase the fluid viscosity in case of cooling problems and subsequently the fluid becomes thick. As a result, the speed of fluid velocity is reduced.The combined effect of momentum and mass diffusion characterises the Schmidt number and in mass transfer problems the presence of this parameter shows an effective result. Figures 1 and 5 reveal the effect of Schmidt number in this study. The viscosity of the fluid rises with the rise of Schmidt number S_c and consequently the fluid velocity takes an decelerating trend in both types of fluid. Figures 1 and 6 illustrate the effect of permeability parameter K on the velocity distribution. It is clear from the figures that the descending value of permeability parameter K raises in both types. Physically as K increases the degree of porosity of the porous medium increases. Figures 1 and 7 display the results of fluid velocity for various values of magnetic field parameter M. The fluid velocity decreases in both Newtonian and non-Newtonian fluids with the increase of M, this is due to the fact that the effect of a transverse magnetic field gives rise to a resistive type force called Lorentz force. Figures 8 to 12 illustrate the effects of different flow parameters on the skin friction of the fluid under consideration. And from the graphically representation it is observed that the shearing stress declines with the increasing value of Prandtl number and Schmidt number and shows opposite trend with increase of Grashof number for mass and heat transfer in both Newtonian and non-Newtonian flows. But with the enhancement of magnetic parameter the shearing stress decelerates for Newtonian fluid but an reverse pattern is noticed for visco-elastic fluid. Figures 13 exhibits that the visco-elastic parameter has no significant effect on the temperature field. Both the temperature and concentration fields are significantly affected by Newtonian and non-Newtonian fluid only due to restraining effect of elasticity of the visco-elastic fluid.

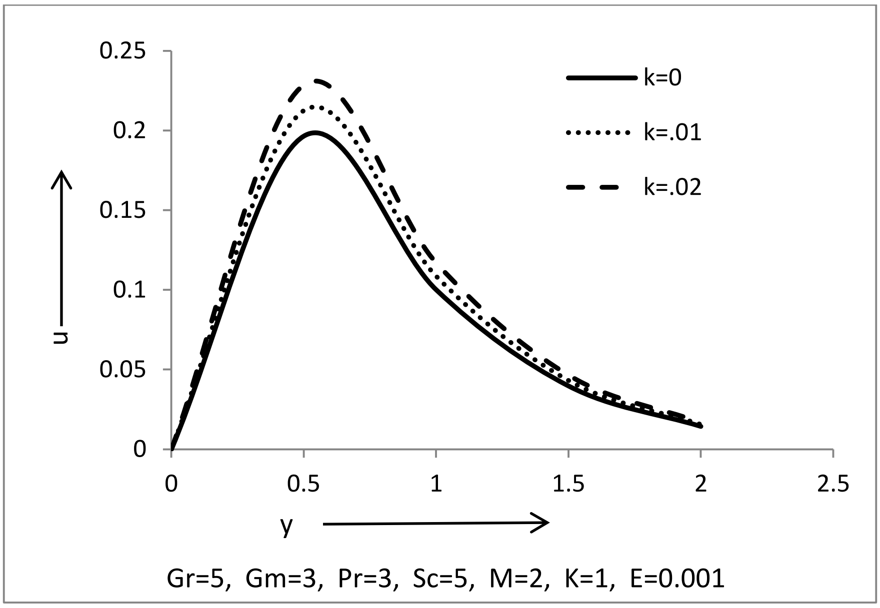

Figure-1. Variation of u against y.

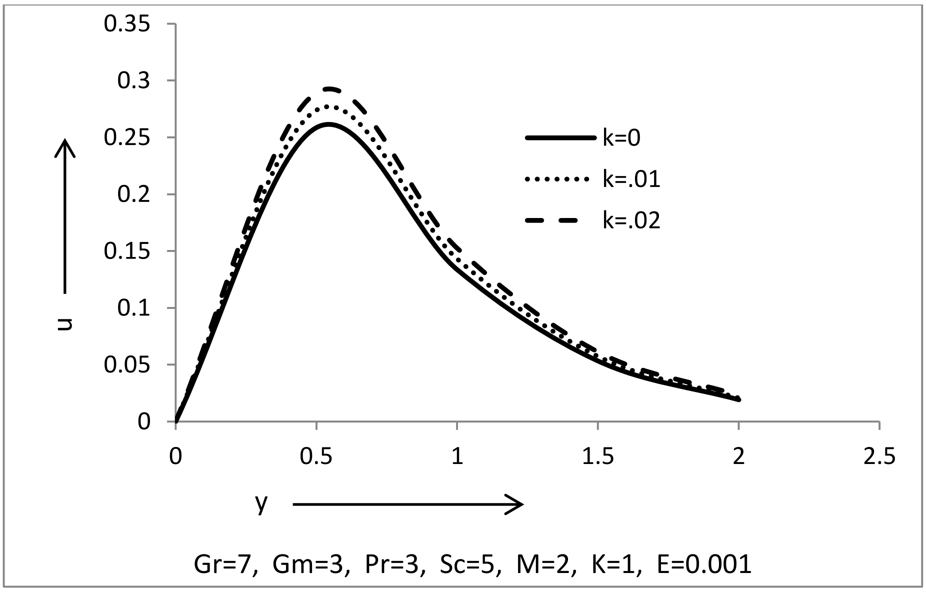

Figure-2. Variation of u against y.

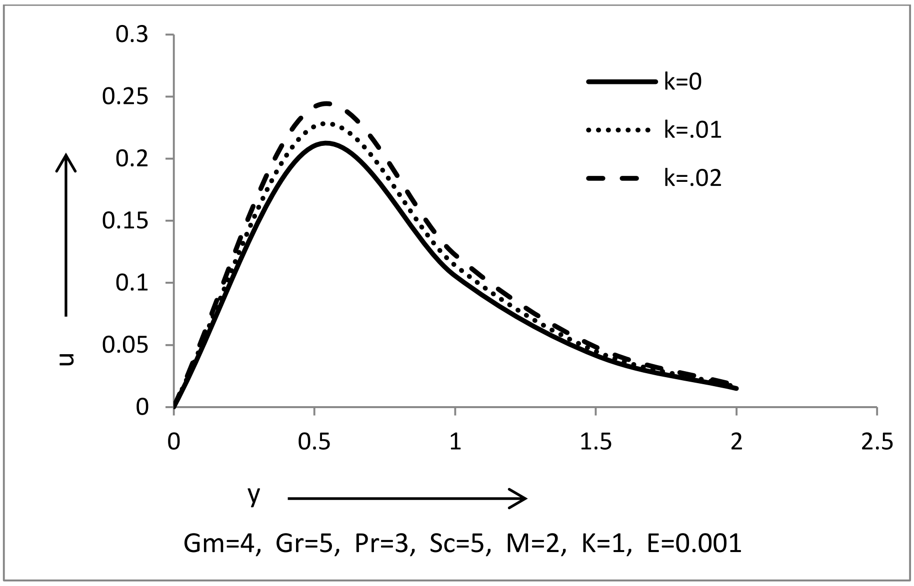

Figure-3. Variation of u against y.

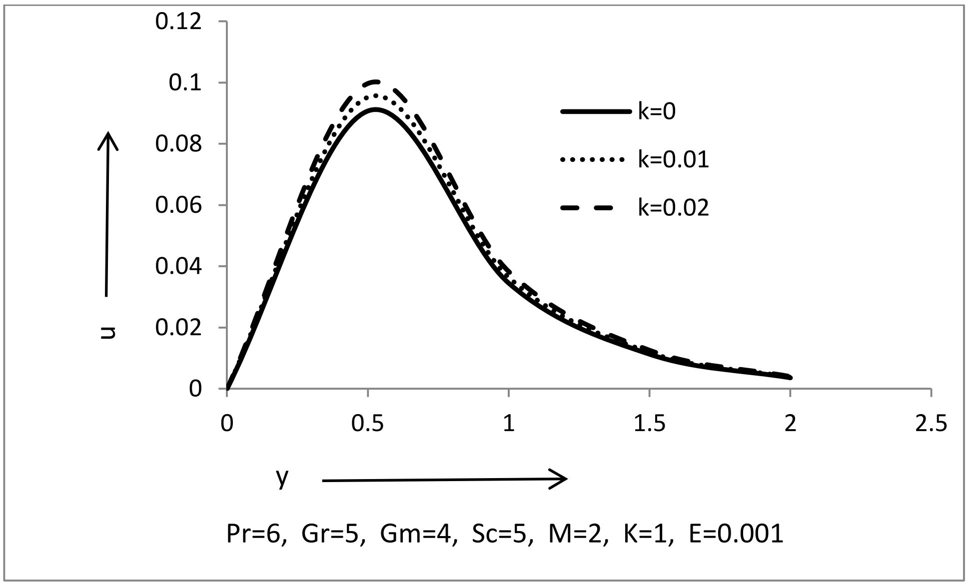

Figure-4. Variation of u against y.

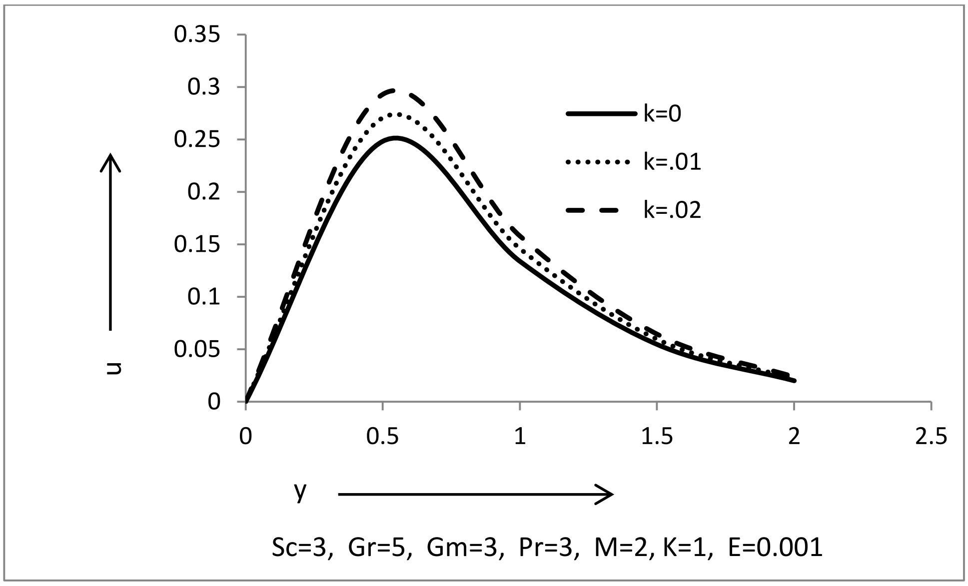

Figure-5. Variation of u against y.

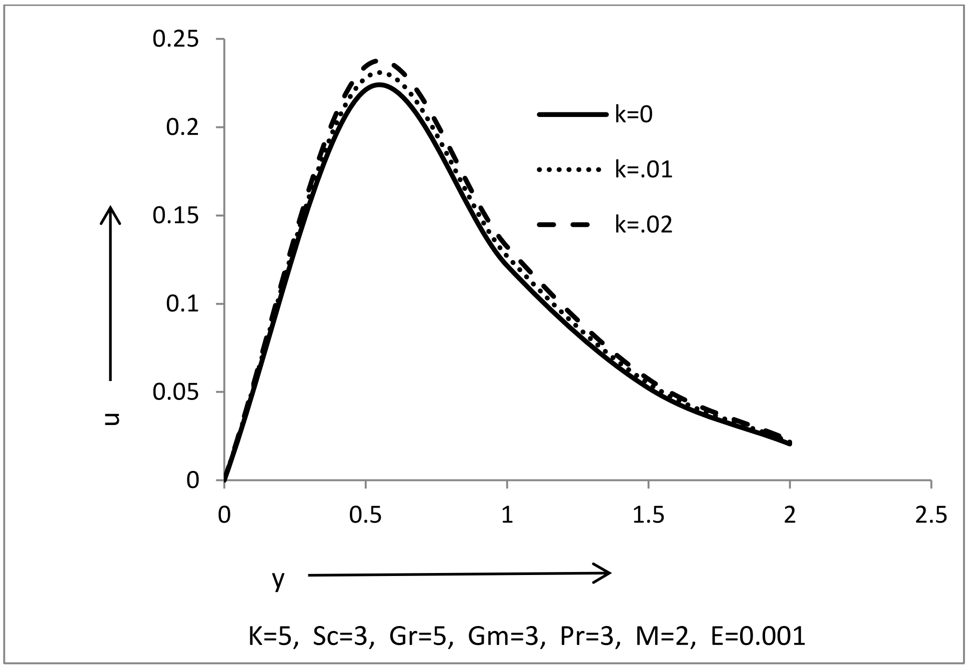

Figure-6. Variation of u against y.

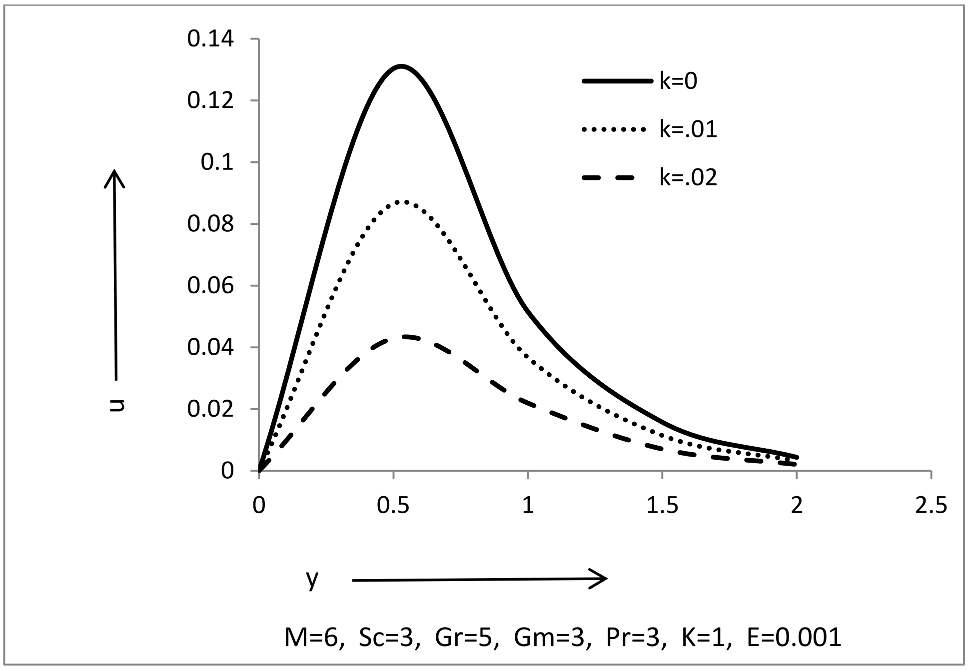

Figure-7. Variation of u against y.

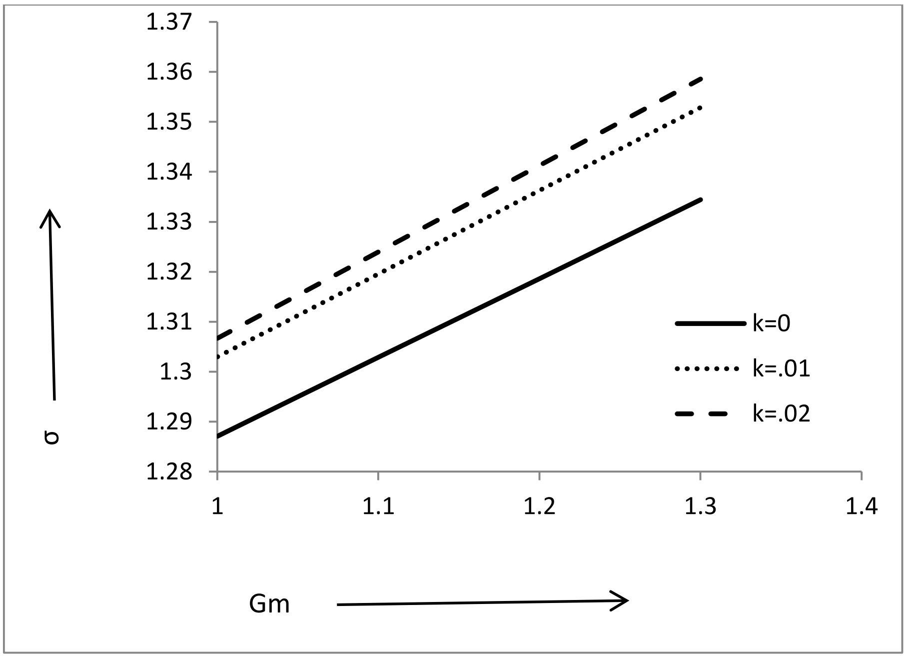

Figure-8. Variation of σ against Gm.

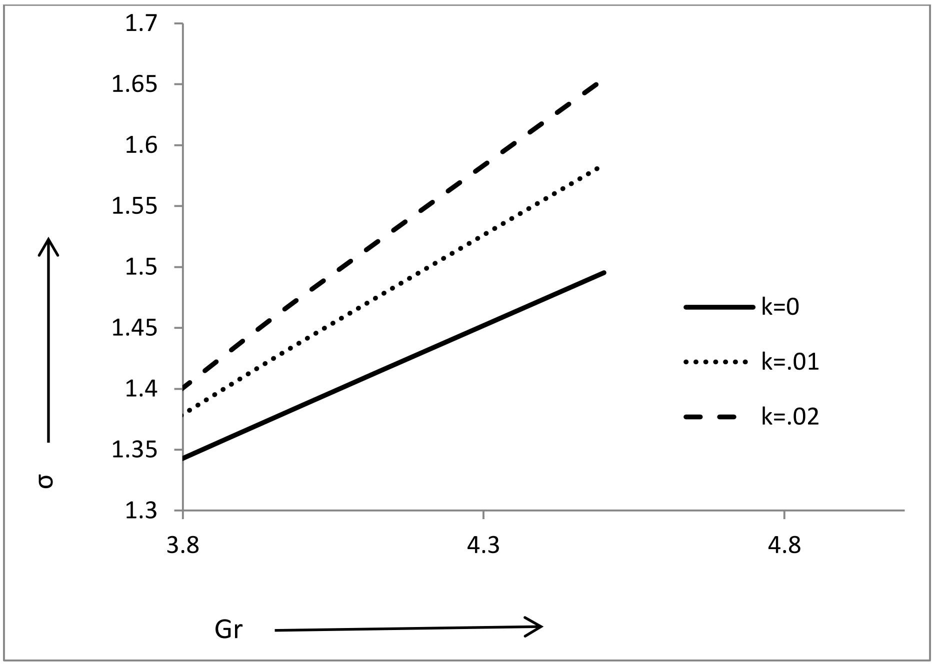

Figure-9. Variation of σ against Gr.

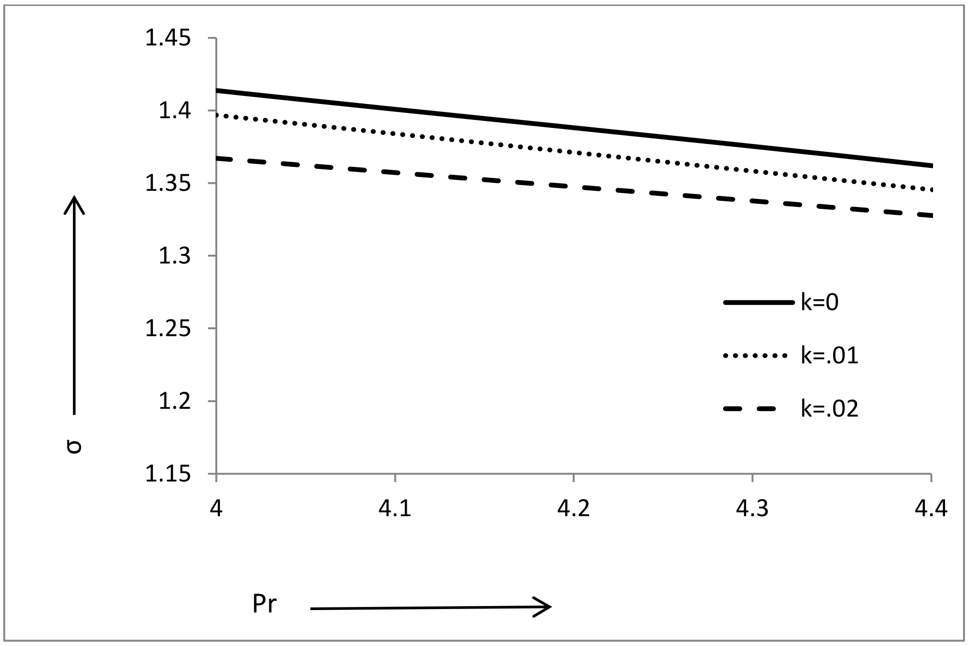

Figure-10. Variation of σ against Pr.

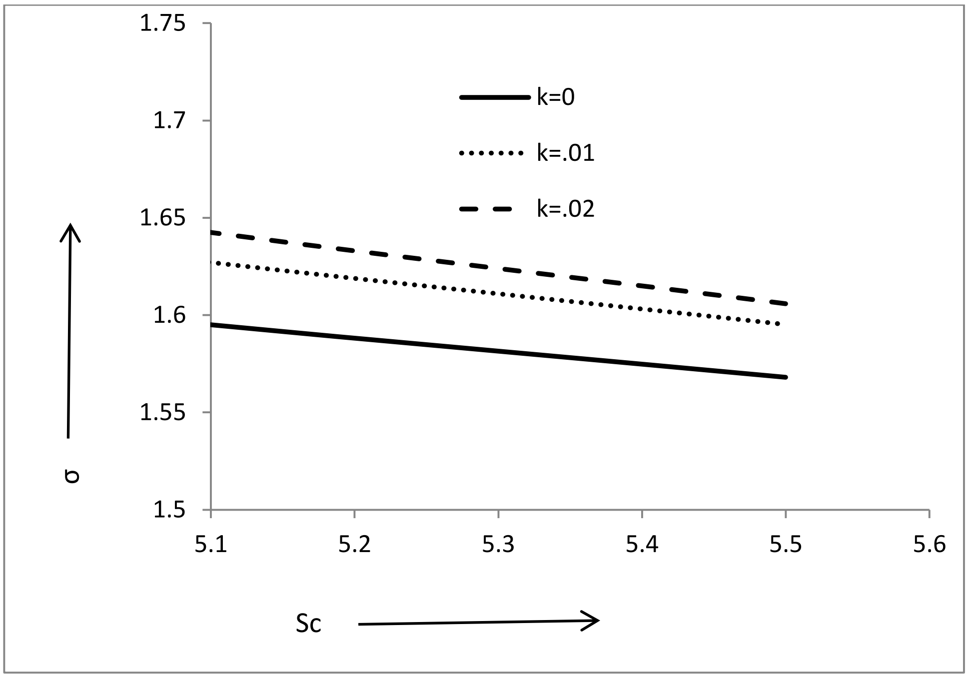

Figure-11. Variation of σ against Sc.

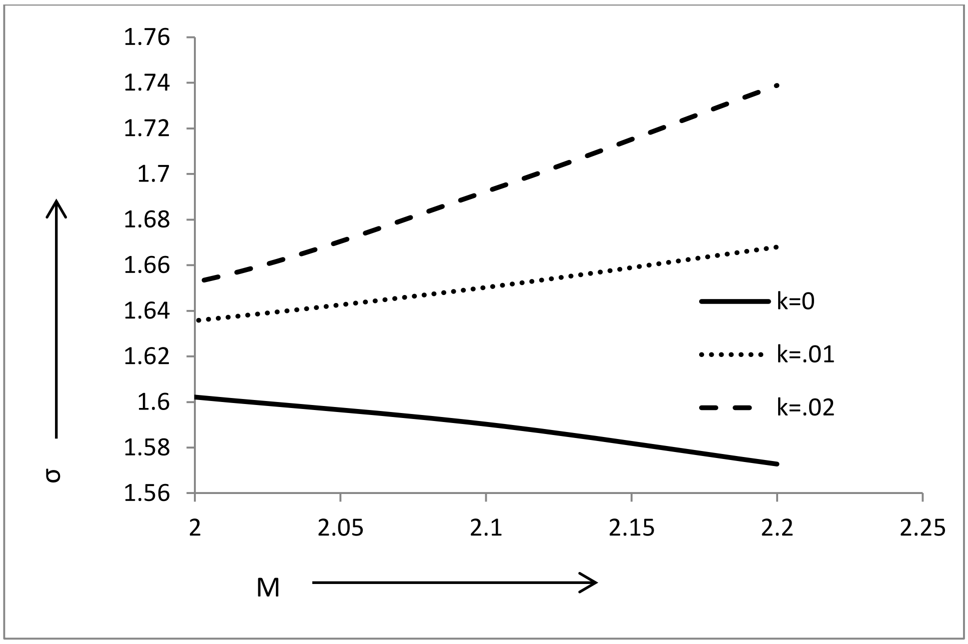

Figure-12. Variation of σ against M.

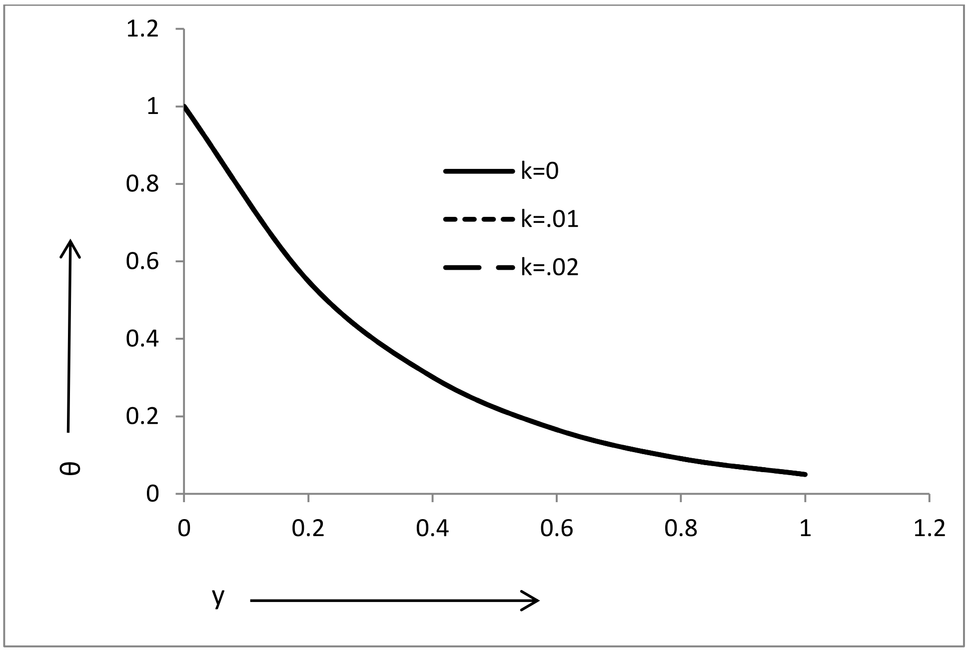

Figure-13. Variation of θ against y.

5. CONCLUSION:

From the present investigation, we make the following conclusions

(i) The fluid velocity distribution is parabolic in nature in both Newtonian and visco-elastic fluids.

(ii) The fluid velocity accelerates near the plate but decelerates far away from the plate in both Newtonian and visco-elastic fluid flow regions.

(iii) The fluctuations of shearing stress with the enhancement of significant flow parameters are prominent in both Newtonian and visco-elastic fluid flow phenomenon.

(iv) The shearing stress diminishes with the increase of Prandtl number and Schmidth number but reverse trend is niticed in case of Grashof number for heat transfer and Grashof number for mass transfer.

(v) Due to enhancement of magnetic parameter the variation of shearing stress takes an decelerating trend in case of Newtonian fluid but an opposite pattern is observed for visco-elastic fluid.

(vi) The temperature and concentration fields are not significantly affected by the visco-elastic parameter due to restraining effect played by the elasticity of the fluid.

(vii) The Nusselt and Sherwood number are not considerably affected by visco-elastic parameter.

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