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Study on Axially Symmetric Vibrations of Poroelastic Solid Cylinder in Biot/Squirt (BISQ) Media

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Abstract

This paper deals with the axially symmetric vibrations of a poroelastic solid cylinder. The study here considers both the effects of Biot and Squirt flows in the pores and Pochhammer's method of analysis for waves in cylinder is extended to Biot's/Squirt media. The stress free boundary conditions used to derive the frequency equation. Phase velocity and attenuation are computed as a function of frequency for different squirt flow lengths. For the numerical results, the sandstone solids are employed, and the results are presented graphically.

Keywords: Poroelasticity, Biot/squirt (BISQ) media, Cylinder, Axially Symmetric vibrations, Frequency equation, Attenuation.

INTRODUCTION

The theory of poroelasticity is widely used today in many domains such as Seismology, Biomechanics, Civil Engineering, and Mechanical Engineering. Employing the (Biot's theory, 1956), Plane-strain vibrations of thick-walled hollow poroelastic cylinder is investigated by (Malla Reddy and Tajuddin 2000). Axially symmetric vibrations of fluid filled poroelastic circular shells is studied (Ahmed Shah 2008). Axially symmetric vibrations of finite composite poroelastic cylinders is investigated (Shah and Tajuddin 2009). Extensional waves in fluid saturated porous cylinders is investigated (Gardener 1962). In this paper, Pochammer method of analysis for waves in circular cylinders is extended to Biot's theory for an elastic porous solid saturated with a compressible viscous fluid. In all the above papers, pores are assumed to have uniform geometry, which are not realistic. In the squirt flow, pore geometry is considered individually. The Biot flow and squirt flow are two important mechanisms of fluid flow in porous media. In Biot mechanism, the fluid is forced to participate in the solid motion by viscous friction and inertial coupling, whereas in the squirt flow mechanism, the fluid is squeezed out of thin pores deformed by a passing wave. (Dvorkin and Nur 1993) proposed a consistent model dealing simultaneously with these two mechanisms of solid/fluid interaction as coupled processes. This model abbreviated as BISQ combines the two mechanisms by considering the fluid's motion both parallel (Biot mechanism) and transverse (Squirt-flow mechanism) to the direction of a wave propagation. The BISQ model relates the dynamic poroelastic behavior of a saturated solid to traditional poroelastic constants such as porosity, permeability, fluid compressibility and viscosity, and the characterisitic squirt-flow length. (Parra 1997, 2000) extends this BISQ model to include the transversely isotropic poroelastic medium. Effects of the Biot and squirt flow coupling interaction on anisotropic elastic waves is investigated (Yang and Zhang 2000). They concluded that the attenuation of quasi P-wave and quasi SV-wave strongly depend on the permeability anisotropy and the attenuation behaviors at low and high frequencies is contrary each other. Poroelastic wave equation including the Biot/squirt mechanism and the solid/fluid coupling anisotropy is studied (Yang and Zhang 2002). BISQ model for fluid filled porous medium is investigated (Yang and Chen 2001). The generalized BISQ wave equation based on the solid/fluid coupling anisotropy is studied by (Yang DH and Yang KD 2003). Acoustic wave propagation in saturated porous media is investigated (Diallo and Appel 2000). Seismic modeling in isotropic porous media based on BISQ model is studied by (Meng Qing Sheng et al. 2003). Differential form and numerical implementation of Biot's poroelasticity equations with squirt dissipation is investigated by (Carcione and Gurevich 2011).

In the present paper, axially symmetric vibrations of a poroelastic solid cylinder in the context of BISQ model is investigated. In particular case the vibration of slender cylinder, for which the ratio of diameter to wavelength is small is examined. In all the cases phase velocity and attenuation against the frequency for different characteristic squirt flow length is computed.

2. GOVERNING EQUATIONS

The equations of motion of a poroelastic solid (Biot 1956, Dvorkin and Nur 1993) in presence of dissipation (*b*) on the account of squirt flow takes the following form

$$N\nabla^2\vec{u} + \nabla((A+N)e + Q\varepsilon) = \frac{\partial^2}{\partial t^2}(\rho_{11}\vec{u} + \rho_{12}\vec{U}) + b\frac{\partial}{\partial t}(\vec{u} - \vec{U}),$$

$$\nabla P_1 = \frac{\partial^2}{\partial t^2}(\rho_{12}\vec{u} + \rho_{22}\vec{U}) - b\frac{\partial}{\partial t}(\vec{u} - \vec{U}). \tag{1}$$

Where ∇^2 is the Laplace operator, $\vec{u} = (u,v,w)$ and $\vec{U} = (U,V,W)$ are solid and fluid displacements, e and ε are the dilatations of solid and fluid respectively; the symbols

A, N, Q are all poroelastic constants; ρ_{ij} are mass coefficients. P_1 is the fluid pressure. The constitutive relations in the cylindrical coordinate system are

$$\sigma_{ij} = 2Ne_{ij} + (Ae + Q\varepsilon)\delta_{ij} \quad (i, j = r, \theta, z)$$
$$P_1 = -FS(Me + \varepsilon) \tag{2}$$

In eq. (2), e_{ij}'s, are the strain displacements, σ_{ij}'s are the solid stresses, δ_{ij} is the well known Kroneckar delta function. F, S are the Biot flow and squirt flow. $M = \frac{\alpha - \phi}{\phi}$, ϕ is the porosity and $\alpha = (1 - (\frac{2}{3}\mu + \lambda)\frac{1}{K_s})$, $N = \mu$, $A - \frac{Q^2}{R_1} = \lambda$, where λ, μ are lame constants, K_s is the bulk modulus of solid, and F, S are given by (Dvorkin and Nur 1993)

$$F = (\frac{1}{K_f} + \frac{\alpha - \phi}{\phi}\frac{1}{K_s})^{-1}, \; S = 1 - \frac{2J_1(\lambda_1 R)}{\lambda_1 R J_0(\lambda_1 R)}$$

and $\lambda_1^2 = \frac{\rho_f \omega^2}{F}(\frac{\phi + \rho_a / \rho_f}{\phi} + \frac{i\eta\phi}{\rho_f \omega k_{11}})$ (3)

In eq. (3), K_f is the bulk modulus of fluid, the sums $\rho_{11} + \rho_{12}$ and $\rho_{12} + \rho_{22}$ represents the mass of solid and fluid per unit volume of bulk material following (Biot, 1956) Furthermore the mass coefficients obey the inequalities $\rho_{11} > 0, \rho_{12} \le 0, \rho_{22} > 0, (\rho_{11}\rho_{22} - \rho_{12}^2) > 0$. The coefficient ρ_{12} is the mass coupling parameter between solid and fluid phases, while ρ_{11}, ρ_{22} satisfy the relations $\rho_{11} = (1-\phi)\rho_s - \rho_{12}$, $\rho_{22} = \phi\rho_f - \rho_{12}$, $\rho_{12} = -\rho_a$. ρ_a is the additional coupling density[9], η is the viscosity, k_{11} is the permeability, ρ_f, and ρ_s are the densities of fluid and solid, J_0, J_1 are the Bessel functions of zero and first order respectively. The parameter R represents the average characteristic squirt flow length.

Consider an isotropic poroelastic solid cylinderof radius a in cylindrical polar coordinate system (r,θ,z) with squirt flow length R as given in the figure-1.

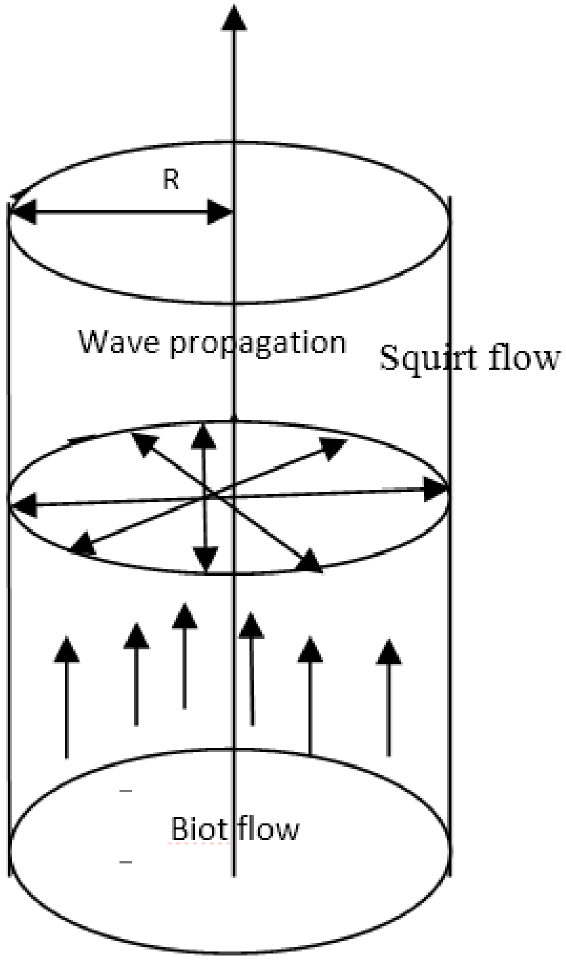

Fig1. The BISQ model, A cylindrical representative volume of a rock, R is the squirt flow length

For the axially symmetric vibrations, $\vec{u} = (u,0,w)$ and fluid $\vec{U} = (U,0,W)$ be the displacement vectors of solid and fluid respectively. The displacement potentials ϕ's and ψ's are introduced as follows

$$u = \frac{\partial \phi_1}{\partial r} - \frac{\partial \psi_1}{\partial z} \qquad w = \frac{\partial \phi_1}{\partial z} + \frac{\partial \psi_1}{\partial r} + \frac{\psi_1}{r}$$
$$U = \frac{\partial \phi_2}{\partial r} - \frac{\partial \psi_2}{\partial z} \qquad W = \frac{\partial \phi_2}{\partial z} + \frac{\partial \psi_2}{\partial r} + \frac{\psi_2}{r}. \tag{4}$$

Substitution of eq. (4) into eq. (1) gives

$$\begin{aligned} P\nabla^2\phi_1 + Q\nabla^2\phi_2 &= (\rho_{11}\ddot{\phi}_1 + \rho_{12}\ddot{\phi}_2) + b(\dot{\phi}_1 - \dot{\phi}_2), \\ -(FSM\nabla^2\phi_1 + FS\nabla^2\phi_2) &= (\rho_{12}\ddot{\phi}_1 + \rho_{22}\ddot{\phi}_2) - b(\dot{\phi}_1 - \dot{\phi}_2), \\ N\nabla^2\psi_1 &= (\rho_{11}\ddot{\psi}_1 + \rho_{12}\ddot{\psi}_2) + b(\dot{\psi}_1 - \dot{\psi}_2), \\ 0 &= (\rho_{12}\ddot{\psi}_1 + \rho_{22}\ddot{\psi}_2) - b(\dot{\psi}_1 - \dot{\psi}_2). \end{aligned} \tag{5}$$

In the above $\nabla^2 = \dfrac{\partial^2}{\partial r^2} + \dfrac{1}{r}\dfrac{\partial}{\partial r} + \dfrac{\partial^2}{\partial z^2}$, $P = A + 2N$, and dot over the quantity represents the partial differentiation with respect to time t. For free harmonic waves travelling in the z-direction, one can have

$$\begin{aligned} \varphi_1 &= F_1(r)\cos kz e^{i\omega t} \qquad \varphi_2 = F_2(r)\cos kz e^{i\omega t} \\ \psi_1 &= G_1(r)\sin kz e^{i\omega t} \qquad \psi_2 = G_2(r)\sin kz e^{i\omega t} \end{aligned} \tag{6}$$

In eq. (5), k is the wavenumber. ω is the frequency of wave, i is the complex unity. Substitution of $\phi_1, \phi_2, \psi_1, \psi_2$ into eqs. (5), gives

$$\begin{aligned} P\Delta F_1 + Q\Delta F_2 &= -\omega^2(M_{11}F_1 + M_{12}F_2), \\ -(FSM\Delta F_1 + FS\Delta F_2) &= -\omega^2(M_{12}F_1 + M_{22}F_2), \\ N\Delta G_1 &= -\omega^2(M_{11}G_1 + M_{12}G_2), \\ 0 &= \omega^2(M_{12}G_1 + M_{22}G_2). \end{aligned} \tag{7}$$

Solving the first two equations of eq. (7), one obtains

$$F_2 = E_1^2\Delta F_1 - E_2^2 F_1 \tag{8}$$

where $E_1^2 = \dfrac{-(PFS - FSMQ)}{\omega^2(FSM_{12} + QM_{22})}$, $E_2^2 = \dfrac{FSM_{11} + QM_{12}}{FSM_{12} + QM_{22}}$

Substituting eq. (8) into the first equation of eq. (7), one obtains

$$\Delta^2 F_1 + E_3^2 F_1 + E_4^2 F_1 = 0 \tag{9}$$

where

$$E_3^2 = \frac{P - QE_2^2 + \omega^2 M_{12}E_1^2}{QE_1^2}, \quad E_4^2 = \frac{\omega^2(M_{11} - M_{12}E_2^2)}{QE_1^2}$$

Equation (9) also can be written as

$$\Delta^2 F_1 + (\xi_1^2 + \xi_2^2)\Delta F_1 + (\xi_1^2 \xi_2^2) F_1 = 0 \tag{10}$$

where ξ_1^2 and ξ_2^2 are given by $\xi_1^2 + \xi_2^2 = E_3^2$ and $\xi_1^2 \xi_2^2 = E_4^2$. Equation (10) can be written as

$$(\Delta + \xi_1^2) F_1 = 0, \quad (\Delta + \xi_2^2) F_1 = 0, \tag{11}$$

where $\Delta = \frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} - k^2$ and $M_{11} = \rho_{11} - \frac{ib}{\omega}, \quad M_{12} = \rho_{12} + \frac{ib}{\omega}, \quad M_{22} = \rho_{22} - \frac{ib}{\omega},$

Solving the eq. (10), gives F_1, F_2. Similarly, G_1, G_2 from the last two equations of eq. (7). Substituting these into eq. (6) one obtains

$$\begin{aligned} \phi_1 &= (C_1 K_0(pr) + C_2 K_0(qr)) \cos kz e^{i\omega t}, \\ \phi_2 &= (C_1 \delta_1^2 K_0(pr) + C_2 \delta_2^2 K_0(qr)) \cos kz e^{i\omega t}, \\ \psi_1 &= A_1 K_1(dr) \sin kz e^{i\omega t}, \\ \psi_2 &= -\frac{M_{12}}{M_{22}} A_1 K_1(dr) \sin kz e^{i\omega t}, \end{aligned} \tag{12}$$

In eq. (12), C_1, C_2, A_1 are all arbitrary constants, $K_n(x)$ is the modified Bessel function of second kind of order n, δ_1^2 and δ_2^2 are given by

$$\delta_1^2 = \frac{(PFS - FSMQ)V_1^{-2} - (FSM_{11} + QM_{12})}{(FSM_{12} + QM_{22})},$$

$\delta_2^2 =$ similar expression as δ_1^2 with V_1^{-2} replaced by V_2^{-2} and

$$p, q, d = k(1 - \xi_i^2), \quad \xi_i = \frac{\omega}{kV_i}, \quad i = 1,2,3. \tag{13}$$

In eq. (13), V_i $(i = 1,2,3)$ are the dilatational waves of first and second kind and shear wave velocities, respectively, which are given by

$$\begin{aligned} \frac{1}{V_1^2} + \frac{1}{V_2^2} &= \frac{PM_{22} - QM_{12} - FSM_{11} + FSMQM_{12}}{PFS - FSMQ} \\ \frac{1}{V_1^2 V_2^2} &= \frac{M_{11}M_{22} - M_{12}^2}{PFS - FSMQ} \\ V_3^2 &= \frac{NM_{22}}{M_{11}M_{22} - M_{12}^2} \end{aligned} \tag{14}$$

From the eq. (14), it is clear that squirt flow mechanism effects the velocity of dilatational waves whereas shear velocity is not effected. By substituting eq. (12) in eq. (3), one can obtain solid displacement components as given below.

$$\begin{aligned} u &= -(C_1 pK_1(pr) + C_2 qK_1(qr) + A_1 kK_1(dr))\cos kz e^{i\omega t} \\ w &= -(C_1 kK_0(pr) + C_2 kK_0(qr) + A_1 dK_0(dr))\sin kz e^{i\omega t} \end{aligned} \tag{15}$$

Substituting the displacements into eq. (2), the relevant stresses and fluid pressure are, given by

$$\begin{aligned} \sigma_{rr} + P_1 &= (A_{11}(r)C_1 + A_{12}(r)C_2 + A_{13}(r)A_1)\cos kz e^{i\omega t}, \\ \sigma_{rz} &= (A_{21}(r)C_1 + A_{22}(r)C_2 + A_{23}(r)A_1)\sin kz e^{i\omega t}, \\ P_1 &= (A_{31}(r)C_1 + A_{32}(r)C_2)\cos kz e^{i\omega t} \end{aligned} \tag{13}$$

3. BOUNDARY CONDITIONS AND FREQUENCY EQUATIONS

The boundary conditions at $r = a$ are given by

$$\sigma_{rr} + P_1 = \sigma_{rz} = P_1 = 0 \tag{17}$$

Eq. (17), gives a system of three homogeneous equations in three arbitrary constants C_1, C_2, A_1.

For the nontrivial solution, the determinant of coefficient matrix must be zero and one arrives at the frequency equation.

$$\begin{vmatrix} 0 & FSM + FS\delta_1^2 & FSM + FS\delta_2^2 \\ -V^2\Theta(da) & 2V_1(V_1^2 - V^2)^{1/2}\Theta(pa) & 2V_2(V_2^3 - V^2)^{1/2}\Theta(qa) \\ 2V_3^2\Theta(da) + 2(V_3^2 - V^2) & \begin{array}{c} 2V_1(V_1^2 - V^2)^{1/2}\Theta(pa) \\ + 2V_1^2 - (V^2 H_1 / N) \end{array} & \begin{array}{c} 2V_2(V_2^2 - V^2)^{1/2}\Theta(qa) \\ + 2V_2^2 - (V^2 H_2 / N) \end{array} \end{vmatrix} = 0. \tag{18}$$

where

$$\begin{aligned} \Theta(x) &= K_1(x)/K_0(x), \\ V &= \omega / k, \\ H_1 &= P - FSM - (FS - Q)\delta_1^2, \\ H_2 &= P - FSM - (FS - Q)\delta_2^2. \end{aligned} \tag{19}$$

4. LIMITING CASES

Case 4.1 Elastic Solid

Eq. (18) is implicit in nature involving the frequency and complex velocities V of extensional waves. If the fluid effects are ignored the eq. (18) will be analogous to Pochhammar frequency equation of elastic solid cylinder. This condition implies that $V_2 \to 0$, consequently $qa \to \infty$ and $\Theta(qa) \to 0$. Further-more, it may be seen that $H_2 \to 0$ then the frequency equation (18) takes the following form

$$\begin{vmatrix} -V^2\Theta(da) & 2V_1(V_1^2 - V^2)^{1/2}\Theta(pa) \\ 2V_3^2\Theta(da) + 2(V_3^2 - V^2) & 2V_1(V_1^2 - V^2)^{1/2}\Theta(pa) + 2V_1^2 - V^2\dfrac{V_1^2}{V_3^2} \end{vmatrix} = 0. \tag{20}$$

where V_1 and V_3 denote the velocity of longitudinal and shear waves respectively, in elastic solid.

Case 4. 2 Low frequency range for slender rods

A reference frequency f_c is defined in Biot [1] by

$$f_c = \frac{\eta\phi^2}{2\pi\rho_2 k_{11}}. \tag{21}$$

If the frequency of vibration is less than 0.15 f_c it may be assumed that Poiseulle flow takes place in all the pores and hence the dissipative coefficient b may be taken equal to $\dfrac{\eta\phi^2}{k_{11}}$ independent of the frequency. This defines the low frequency range. Consider the function Θ which occurs in the frequency equation. As the frequency tends to zero, V_3^2 and V_1^2 tends toward the real numbers N/ρ and H/ρ, respectively. Consequently the ratio of diameter to wavelength is small enough, the arguments of $\Theta(da)$ and $\Theta(pa)$ are small and the functions can be replaced by the unity. On the other hand V_2 is given by

$$V_2^2 = (i\omega/b)(PFS - FSMQ)/H. \tag{22}$$

When the frequency is low, in the argument of $\Theta(qa)$, the value of V^2 can be ignored compared with V_2^2 and the function can be replaced by $\Theta(a\omega/V_2)$. This is a function of complex variable and may be resolved into its real and imaginary parts.

$$\Theta(i^{3/2}a(c\omega)^{1/2}) = \Theta_r(a(c\omega)^{1/2}) - i\Theta_i(a(c\omega)^{1/2}), \tag{23}$$

where c is defined by

$$c = bH/(PFS - FSMQ). \tag{24}$$

The graphical representation of Θ_r and Θ_i are depicted in figure-2. From the figure it is clear

that Θ_r is greater when its argument is close to the value 0.5. In the low frequency range, the frequency equation may be simplified by the following substitutions

$$\begin{aligned} &\Theta(da) \to 1 \\ &\Theta(pa) \to 1 \\ &\Theta(qa) \to \Theta_r - i\Theta_i \end{aligned} \tag{22}$$

Using the above approximation in the frequency equation, one obtains the following equation is obtained.

$$\begin{aligned} &V^4[(FSM + FS\delta_2^2)\frac{V_1^2}{V_3^2} - (FSM + FS\delta_1^2)\frac{V_1^2}{V_3^2}] + V^2[-2V_1^2(FSM + FS\delta_2^2) + \\ &6V_1(FSM + FS\delta_2^2)\times(V_1^2 - V^2)^{1/2} + 2V_2^2(FSM + FS\delta_1^2) - \\ &6V_2(FSM + FS\delta_2^2)(V_2^2 - V^2)^{1/2}\Theta(qa)] + [8V_3^2V_2(FSM + FS\delta_1^2)\times \\ &(V_2^2 - V^2)^{1/2}\Theta(qa) - 8V_3^2V_1(FSM + FS\delta_2^2)(V_1^2 - V^2)^{1/2}] = 0. \end{aligned} \tag{26}$$

From this equation, it is clear that two types of waves exist namely the dilatational waves of first and second kind.

5. NUMERICAL RESULTS

Numerical results are computed for the eq. (26) for two poroelastic solids namely material-1 is sandstone saturated with kerosene while material-2 is sandstone saturated with water are given by (Jose and Gurevich 2011), (Yew and Jogi 1976), (Fatt 1957). The parameter values are given in Table-1. The additional coupling parameter value $420kg/m^3$ is given by (Dovrkin and Nur 1993). Eq. (26) is biquadratic polynomial with complex valid and can be separated into real and imaginary parts. Real part gives phase velocity C_r and imaginary part gives phase velocity C_i. The attenuation coefficient is given by (Solorza 2004)

$$Q^{-1} = \frac{2C_i}{C_r} \tag{27}$$

For a given material, an implicit relation between phase velocity, attenuation and frequency at fixed squirt flow lengths is obtained. Phase velocity and attenuation is computed as a function of frequency at fixed squirt flow length and the results are depicted in figure 3-6. Figure-3 shows the plot of phase velocity against the frequency for material-1. From thi figure, it is observed that as the frequency increases, phase velocity decreases. Figure-4 show the plot of attenuation against the frequency for material-1. From this figure, it is clear that as the frequency increases attenuation increases and the values are greater when R=0.001 than that of R=0.005. Figure-5 shows the plot of phase velocity against frequency for material-2. From this figure, it is clear that as the frequency increases phase velocity is maximum when frequency is 6. Figure-6 show the plot of attenuation against frequency for material-2. From this figure it is seen that as the frequency increases attenuation decreases.

Table 1

Kerosene saturated sandstone	**Water saturated sandstone**
$A = 0.306\times10^{10} N/m^2$	$A = 0.4436\times10^{10} N/m^2$
$N = 0.2765\times10^{10} N/m^2$	$N = 0.922\times10^{10} N/m^2$
$Q = 0.07635\times10^{10} N/m^2$	$Q = 0.013\times10^{10} N/m^2$
$R = 0.0326\times10^{10} N/m^2$	$R = 0.0637\times10^{10} N/m^2$
$\rho_{11} = 1.926137\times10^3 kg/m^3$	$\rho_{11} = 1.90302\times10^3 kg/m^3$
$\rho_{12} = -0.00213\times10^3 kg/m^3$	$\rho_{12} = 0$
$\rho_{22} = 0.21537\times10^3 kg/m^3$	$\rho_{22} = 0.268\times10^3 kg/m^3$
Bulk modulus, K_s = 6Gpa	Bulk modulus, K_s = 50Gpa
Bulk modulus, K_f = 2.14Gpa	Bulk modulus, K_f =2.25Gpa
Porosity, $\phi = 0.26$	Porosity, $\phi = 0.2$
Permeability, k_{11} = 290mD	Permeability, k_{11} = 200mD
Density, $\rho_f = 1000 kg/m^3$	Density, $\rho_f = 1040 kg/m^3$
Density, $\rho_s = 2100 kg/m^3$	Density, $\rho_s = 2650 kg/m^3$
Viscocity, η = 1.53 mPa	Viscocity, η =0.001Pa s

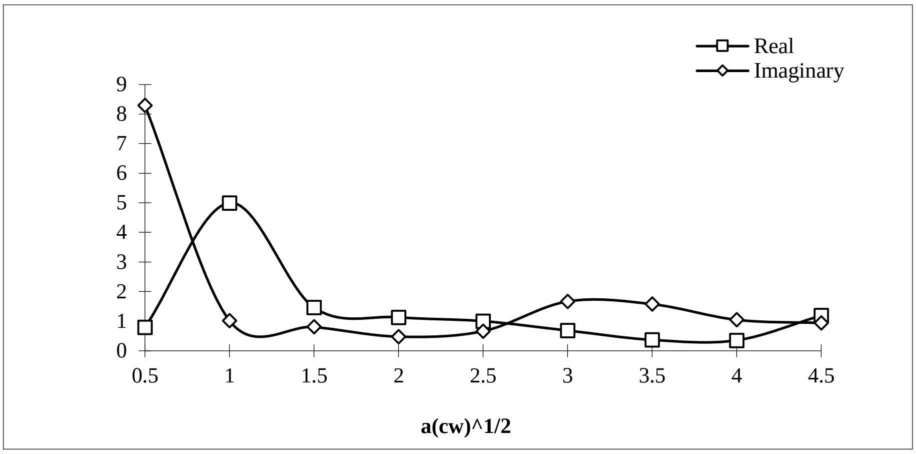

Fig-2 Graph of the function of real and imaginary

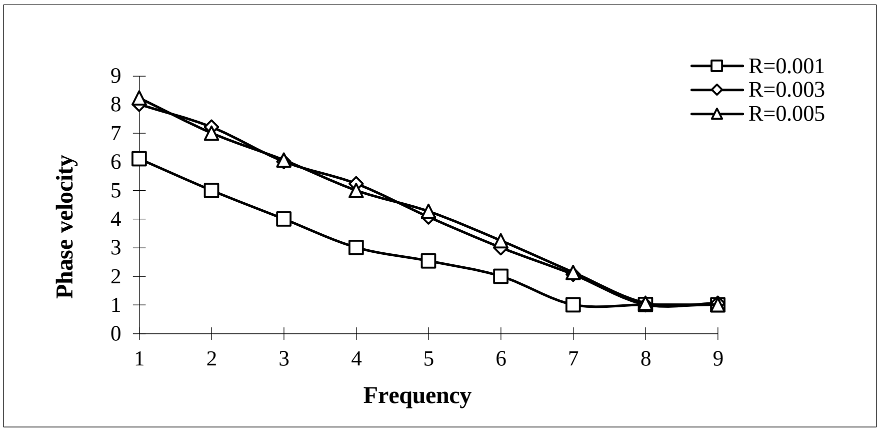

Fig 3: Variation of phase velocity with frequency (Mat-1)

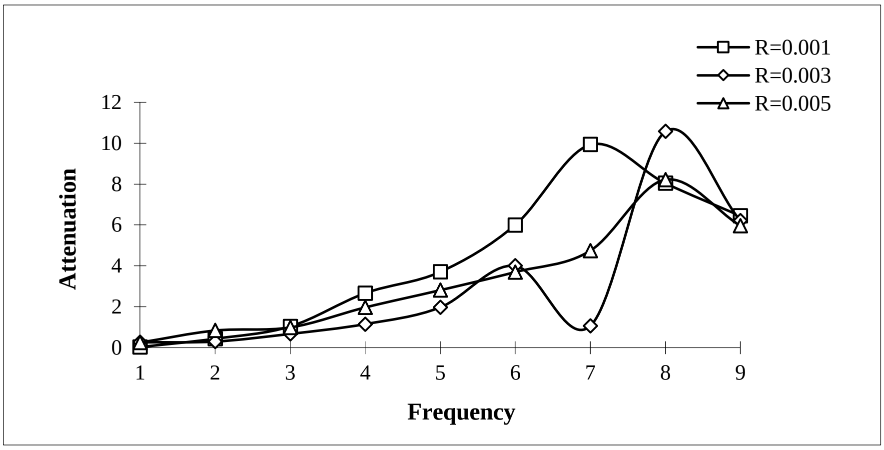

Fig 4: Variation of phase velocity with attenuation (Mat-1)

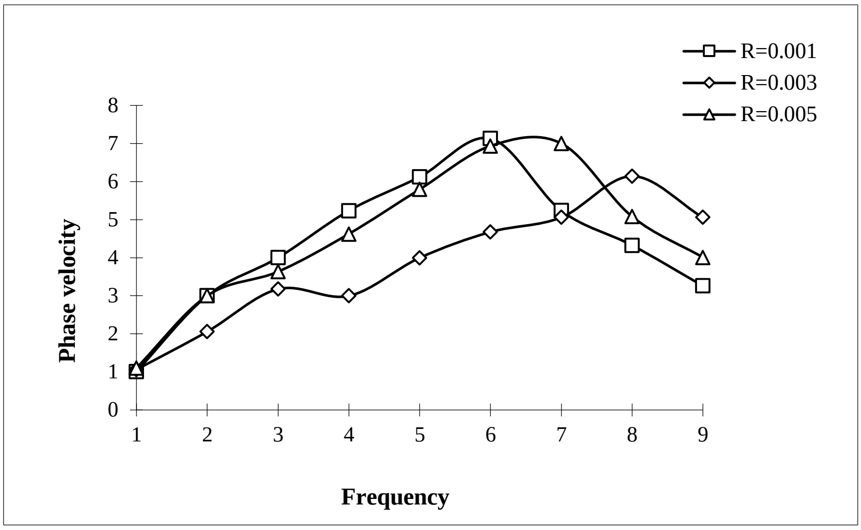

Fig 5: Variation of phase velocity with frequency (Mat-2)

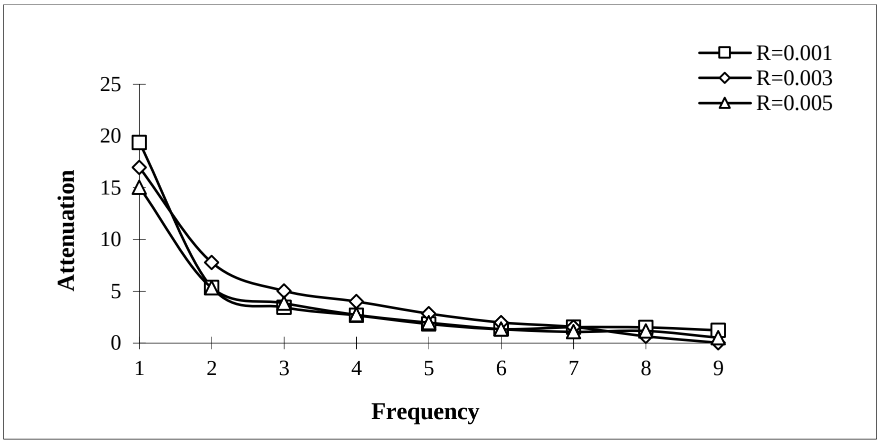

Fig 6: Variation of attenuation with wavenumber (Mat-2)

6. CONCLUSION

Employing Biot's theory, axially symmetric vibrations of a poroelastic solid cylinder in Biot/squirt media is investigated. Pochhammer's method of analysis for waves in cylinder is extended to Biot's/Squirt media. Phase velocity and attenuation against is computed for fixed squirt flow lengths. From the results it is clear that as the frequency increases phase velocity increases for material-2 and decreases for material-1. Attenuation values decreases for material-2 and increases for material-1. This is due to the influence of additional coupling parameter. This kind of analysis can be made for any poroelastic solid cylinder if the values of parameters are available.

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