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Fractional Complex Transform for Solving the Fractional Differential Equations

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Abstract

In this paper, fractional complex transform (FCT) with help of New Iterative Method (NIM) is used to obtain numerical and analytical solutions for the fractional Fokker-planck equation, Fractional Nonlinear Gas Dynamics equation and the nonlinear time-fractional Fisher's equation and fractional telgraph equation. Fractional complex transform (FCT) is proposed to convert fractional differential equations to its differential partner and then applied NIM to the new obtained equations. Several examples are given and the results are compared to exact solutions. The results reveal that the method is very effective and simple.

Keywords: Fractional complex transform, New iterative method, fractional Fokker-planck equation, fractional nonlinear Gas Dynamics equation and the nonlinear time-fractional Fisher's equation and fractional telgraph equation.

1. INTRODUCTION

Fractional models have been shown by many scientists to adequately describe the operation of variety of physical and biological processes and systems. Con sequently, considerable attention has been given to the solution of fractional ordinary differential equations, integral equations and fractional partial differential equations of physical interest. Since most fractional differential equations do not have exact analytic solutions, approximation and numerical techniques, therefore, are used extensively ([9]-[11], [14]-[15]). Numerical and analytical methods have included finte difference method ([7], [17)], Adomian decomposition method ([23]-[24]).

Transform is an important method to solve mathematical problems. Many useful transforms for solving various problems were appeared in open literature, such as the travelling wave transform [1] , the Laplace transform [16], the Fourier transform [8], the Bücklund transformation [18], the integral transform [19], and the local fractional integral transforms [29].

Very recently the fractional complex transform ([12]-[13], [31]-[36]) was suggested to convert fractional order differentialequations with modified Riemann-Liouville derivatives into integer order differential equations, and the resultant equations can be solved by advanced calculus.

The time-fractional Fokker-Planck equation serves as a mathematical model for a number of problems in physical and biological sciences. It arisesfrom a diffusion approximation of some stochastic processes regarcted as Markovian and continuous. It is a beneralized diffusion equation governing the evolution of the probability density in time. For the two-variable case, to whilch attention is restricted here [6].

It is commonly known that the equation of Gas dynamics is the mathematical expressions of conservation laws which exist in engineering practices such as conservation of mass, conservation of momentum, conservation of energy etc. The nonlinear equations of ideal gas dynamics are applicable for three types of nonlinear waves like shock fronts, rarefactions, and contact discontinuities. In 1981, Steger and Warming addressed that the conservation-law form of the inviscid gas dynamic equation possesses a remarkable property by virtue of which the nonlinear flux vectors are homogeneous functions of degree one which permits the splitting of flux vectors into subvectors by similarity transformations [5].

The time-fractional Fisher's equation (TFFE), which is a mathematical model for a wide range of important physical phenomena, is a partial differential equation obtained from the classical Fisher equation by replacing the time derivative with a fractional derivative of order α .

The telegraph equation developed by Oliver Heaviside in 1880 is widely used in Science and Engineering. Its applications arise in signal analysis for transmission and propagation of electrical signals and also modelling reaction diffusion. In recent years, great interest has been developed in fractional differential equation because of its frequent appearance in fluid mechanics, mathematical biology, electrochemistry, and physics. A space-time fractional telegraph equation is obtained from the classical telegraph equation by replacing the time and space derivative terms by fractional derivatives and complex transform method ([33]-[36]).

The paper is organized as follows: In section 2, we provide the Basic Complex Transform. Section 3, Basic Idea of New Iterative Method (NIM). Sections 4,Applications.Sections 5, Conclusion.

2. FRACTIONAL COMPLEX TRANSFORM (FCT)[20]

Consider the following general fractional defferential equation

$$f(u,u_t^{(\alpha)},u_x^{(\beta)},u_y^{(\gamma)},u_z^{(\lambda)},u_t^{(2\alpha)},u_x^{(2\beta)},u_y^{(2\gamma)},u_z^{(2\lambda)},...)=0 \tag{2.1}$$

where $u_t^{(\alpha)} = \partial^\alpha u(x,y,z,t)/\partial^\alpha$ denotes the modified Riemann-Liouville derivative. $0 < \alpha \le 1$, $0 < \beta \le 1$, $0 < \gamma \le 1$, $0 < \lambda \le 1$.

Introducing the following transforms

$$T = \frac{qt^\alpha}{\Gamma(1+\alpha)}, \tag{2.2}$$

$$X = \frac{pt^\beta}{\Gamma(1+\beta)}, \tag{2.3}$$

$$Y = \frac{kt^\gamma}{\Gamma(1+\gamma)}, \tag{2.4}$$

$$Z = \frac{lt^\lambda}{\Gamma(1+\lambda)}, \tag{2.5}$$

where $p;q;k$ and l are constants.

Using the above transforms, we can convert fractional derivatives into classical derivatives:

$$\frac{\partial^\alpha u}{\partial t^\alpha} = q\frac{\partial u}{\partial T} \tag{2.6}$$

$$\frac{\partial^\beta u}{\partial x^\beta} = p\frac{\partial u}{\partial X} \tag{2.7}$$

$$\frac{\partial^{\gamma} u}{\partial y^{\gamma}} = k\frac{\partial u}{\partial Y} \tag{2.8}$$

$$\frac{\partial^{\lambda} u}{\partial z^{\lambda}} = l\frac{\partial u}{\partial Z} \tag{2.8}$$

Therefore, we can easily covert the fractional differential equations into partial differential equations, so that everyone familiar with advanced calculus can deal with fractional calculus without any diffculty. For example, consider a fractional differential equation

$$\frac{\partial^{\alpha} u}{\partial t^{\alpha}} + 2u\frac{\partial^{\beta} u}{\partial x^{\beta}} + 4\frac{\partial^{\gamma} u}{\partial y^{\gamma}} + 5\frac{\partial^{\lambda} u}{\partial z^{\lambda}} = 0. \tag{2.9}$$

By using the above transformations we get:

$$q\frac{\partial u}{\partial T} + 2up\frac{\partial u}{\partial X} + 2k\frac{\partial u}{\partial Y} + 5k\frac{\partial u}{\partial Z} = 0. \tag{2.10}$$

which can be solved by New Iterative method.

3. NEW ITERATIVE METHOD(NIM)[4]

To describe the idea of the NIM, consider the following general functional equation ([2]-[3], [22]-[23], [27]-[28], [30]):

$$u(x) = f(x) + N(u(x)), \tag{3.1}$$

where N is a nonlinear operator from a Banach space $B \to B$ and f is a known function. We are looking for a solution u of (3.1) having the series form

$$u(x) = \sum_{i=0}^{\infty} u_i \tag{3.2}$$

The nonlinear operator N can be decomposed as follows

$$N\left(\sum_{i=0}^{\infty} u_i\right) = N(u_0) + \sum_{j=0}^{\infty}\left\{N\left(\sum_{j=0}^{\infty} u_j - N\left(\sum_{j=0}^{\infty} u_j\right)\right)\right\} \tag{3.3}$$

From Eqs. (3.2) and (3.3), Eq. (3.1) is equivalent to

$$\sum_{i=0}^{\infty} u_i = f + N(u_0) + \sum_{j=0}^{\infty}\left\{N\left(\sum_{j=0}^{\infty} u_j\right) - N\left(\sum_{j=0}^{\infty} u_j\right)\right\} \tag{3.4}$$

We define the recurrence relation:

$$u_0 = f, \quad (3.5a)$$

$$u_1 = N(u_0), \tag{3.5b}$$

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$$u_{n+1} = N(u_0 + u_1 + ... + u_n) - N(u_0 + u_1 + ... + u_{n-1}), \qquad n = 1,2,3,... \tag{3.5c}$$

Then:

$$(u_0 + u_1 + ... + u_{n+1}) = N(u_0 + u_1 + ... + u_n),\ n = 1,2,3,..,$$

$$u = \sum_{i=0}^{\infty} u_i = f + N\left(\sum_{i=0}^{\infty} u_i\right). \tag{3.6}$$

If N is a contraction, i.e.

$$\|N(x) - N(y)\| \le k\|x - y\|,\ 0 < k < 1,$$

$$\|u_{n+1}\| = \|N(u_0 + u_1 + ... + u_n) - N(u_0 + u_1 + ... + u_{n-1})\| \tag{3.7}$$

$$\le k\|u_n\| \le ...k^n\|u_0\| \quad n = 1,2,3,...,,$$

and the series $\sum_{i=0}^{\infty} u_i$ absolutely and uniformly converges to a solution of (2.1), which is unique, in view of the Banach fixed point theorem [3].

The k-term approximate solution of (2.1) and (2.2) is given by $\sum_{i=0}^{k-1} u_i$

3.1 Reliable Algorithm

After the above presentation of the NIM, we introduce a reliable algorithm for solving nonlinear partial differential equations using the NIM. Consider the following nonlinear partial differential equation of arbitrary order:

$$D_t^n = A(u, \partial u) + B(x,t), \qquad n \in N \tag{3.8a}$$

with the initial conditions

$$\frac{\partial^\alpha}{\partial t^\alpha} u(x,0) = h_m(x), \qquad m = 0,1,2,...,n-1, \tag{3.8b}$$

where A is a nonlinear function of u and ∂u (partial derivatives of u with respect to x and t) and B is the source function. In view of the integral operators, the initial value problem (3.8a) and (3.8b) is equivalent to the following integral equation

$$u(x,t)-\sum_{m=0}^{n-1} h_m(x)\frac{t^m}{m!}+I_t^n B(x,t)+I_t^n A = f+N(u), \tag{3.9}$$

Where

$$f=\sum_{m=0}^{n-1} h_m(x)\frac{t^m}{m!}+I_t^n B(x,t), \tag{3.10}$$

and

$$N(u)=I_t^n A \tag{3.11}$$

where $I_t^n\ t$ is an integral operator of n fold.
We get the solution of (3.9) by employing the algorithm(3.5).

4. APPLICATIONS

In this section, We apply the new iterative method approach to study five examples

4.1 Example[6]

We consider the Fractional Fokker-planck equation:

$$\frac{\partial^\alpha u}{\partial t^\alpha}-(1+x)\frac{\partial u}{\partial x}-(x^2e^t)\frac{\partial^2 u}{\partial x^2}=0 \tag{4.1}$$

subject to the initial condition

$$u(x,0)=1+x, \tag{4.2}$$

To apply FCT to Eq.(4.1), we use the above transformations:

$$\frac{\partial^\alpha u}{\partial t^\alpha}=q\frac{\partial u}{\partial T}$$

So we have the following partial differential equation:

$$q\frac{\partial u}{\partial T}=(1+x)\frac{\partial u}{\partial x}+(x^2e^t)\frac{\partial^2 u}{\partial x^2} \tag{4.3}$$

For simplicity we set $q=1$, so we get

$$\frac{\partial u}{\partial T}=(1+x)\frac{\partial u}{\partial x}+(x^2e^t)\frac{\partial^2 u}{\partial x^2} \tag{4.4}$$

Now, we solve Eq. (4.4) by means of NIM. To apply NIM to (4.4), we construct the correction functional as follows:

$$u_n(x,t) = u_{n-1} + I_T[(1+x)\frac{\partial u_{n-1}}{\partial x} + (x^2e^t)\frac{\partial^2 u_{n-1}}{\partial x^2}]$$

$$u_0(x,t) = u_0 = 1+x$$

$$u_1(x,t) = I_T[(1+x)\frac{\partial u_0}{\partial x} + (x^2e^t)\frac{\partial^2 u_0}{\partial x^2}]$$

$$I_T[(1+x)(1)]$$

$$u_1 = (1+x)T$$

$$u_2(x,t) = I_T[(1+x)\frac{\partial u_1}{\partial x} + (x^2e^t)\frac{\partial^2 u_1}{\partial x^2}]$$
$$= I_T[(1+x)T]$$

$$u_2 = (1+x)\frac{T^2}{2}$$

$$u_3(x,t) = I_T[(1+x)\frac{\partial u_2}{\partial x} + (x^2e^t)\frac{\partial^2 u_2}{\partial x^2}]$$
$$= I_T[(1+x)\frac{T^2}{2}]$$

$$u_2 = (1+x)\frac{T^3}{6}$$

$$\vdots$$

$$u_n = (1+x)\frac{T^n}{n!}$$

$$u(x,t) = \sum_{n=0}^{n-1}(1+x)\frac{T^n}{n!}$$

By the fractional complex transform

$$T = \frac{t^\alpha}{\Gamma(1+\alpha)}$$

We have

$$u_1 = (1+x)\frac{t^\alpha}{\Gamma(1+\alpha)}$$

$$\vdots$$

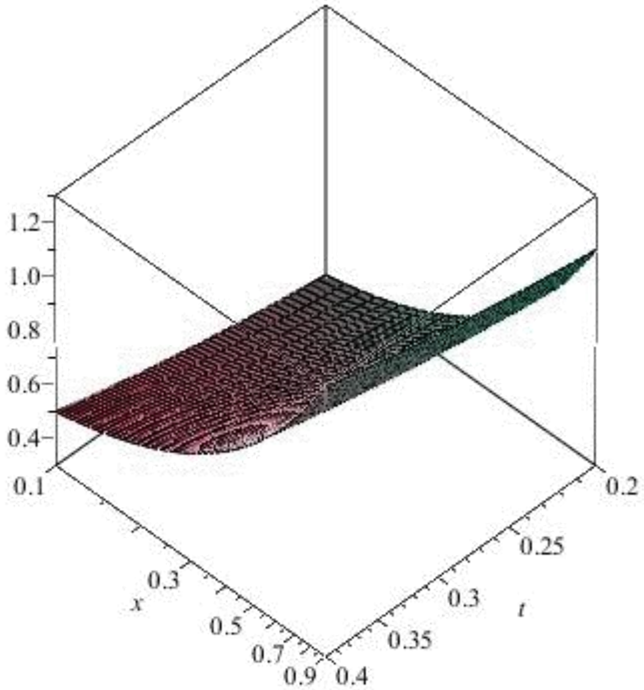
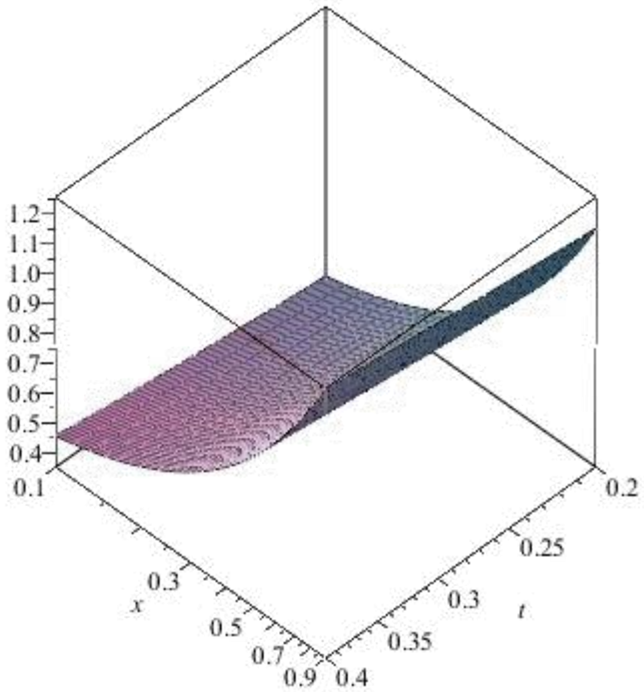


Fig. 1. *the exact solution of* $u(x,t)$ *at* $\alpha = 1$



Fige .2.the approximate solution of $u(x,t)$ *at* $\alpha = 0.5$

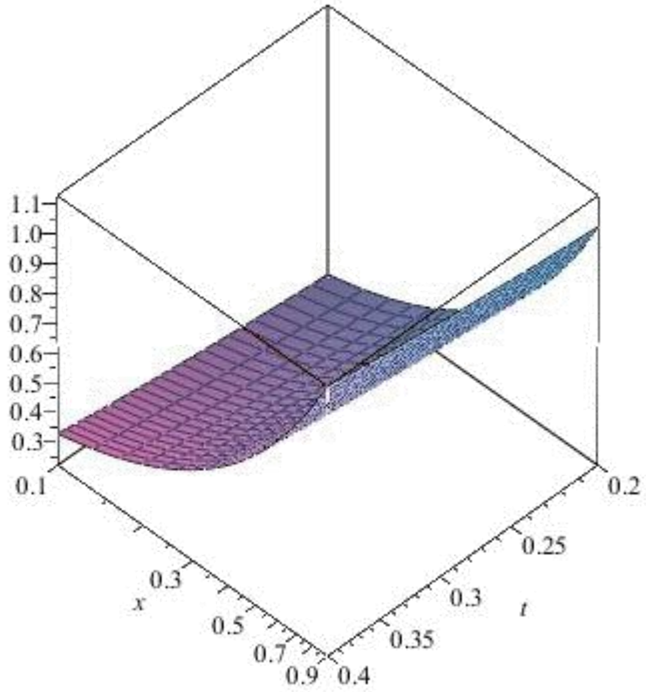


Fig. 3. the approximate solution of $u(x,t)$ *at* $\alpha = 0.9$

4.2 Example [25]

We consider the following nonlinear time- fractional gas dynamics equation of the form:

$$D_t^{\alpha} + \frac{1}{2}(u^2)_x - u(1-u) = 0 \qquad t > 0 \qquad 0 < \alpha \le 1$$

$$\frac{\partial^{\alpha} u}{\partial t^{\alpha}} = u - u^2 - \frac{1}{2}(u^2)_x \tag{4.5}$$

subject to the initial condition

$$u(x,0) = e^{-x} \tag{4.6}$$

To apply FCT to Eq.(4.5), we use the above transformations:

$$\frac{\partial^{\alpha} u}{\partial t^{\alpha}} = q\frac{\partial u}{\partial T},$$

so we have the following partial differential equation:

$$q\frac{\partial u}{\partial T} = u - u^2 - \frac{1}{2}(u^2)_x \tag{4.7}$$

For simplicity we set $q = 1$, so we get

$$\frac{\partial u}{\partial T} = u - u^2 - \frac{1}{2}(u^2)_x \tag{4.8}$$

Now, we solve Eq. (4.8) by means of NIM. To apply NIM to (4.8), we construct the correction functional as follows:

$$u_n(x,t) = u_{n-1} + I_T[(u_n - u_n^2 - \frac{1}{2}(u_n^2)_x] \tag{4.9}$$

$$u_0(x,t) = u_0 = e^{-x}$$

$$u_1(x,t) = I_T[(u_0 - u_0^2 - \frac{1}{2}(u_0^2)_x]$$

$$I_T[e^{-x} - e^{-2x} - \frac{1}{2}(-2e^{-2x})]$$

$$u_1 = e^{-x}T$$

$$u_2(x,t) = I_T[u_1 - u_1^2 - \frac{1}{2}(u_1^2)_x]$$

$$= I_T[e^{-x}T - e^{-2x}T^2 - \frac{1}{2}(-2e^{-2x}]$$

$$u_2 = e^{-x}\frac{T^2}{2}$$

$$u_3(x,t) = I_T[u_2 - u_2^2 - \frac{1}{2}(u_2^2)_x]$$

$$= I_T[(1+x)\frac{T^2}{2}]$$

$$u_3 = e^{-x}\frac{T^3}{6}$$

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$$u_n = e^{-x}\frac{T^n}{n!}$$

$$u(x,t) = \sum_{n=0}^{n-1}\frac{e^{-x}T^n}{n!}$$

By the fractional complex tranform

$$T = \frac{t^{\alpha}}{\Gamma(1+\alpha)}$$

We have

$$u_1 = e^{-x}\frac{t^{\alpha}}{\Gamma(1+\alpha)}$$

$$\vdots$$

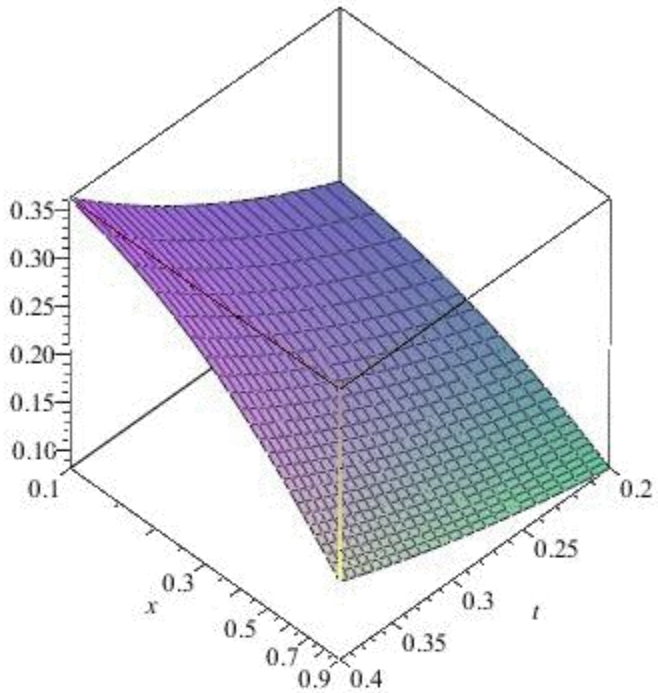

Fig. 4. the exact solution of $u(x,t)$ *at* $\alpha = 1$

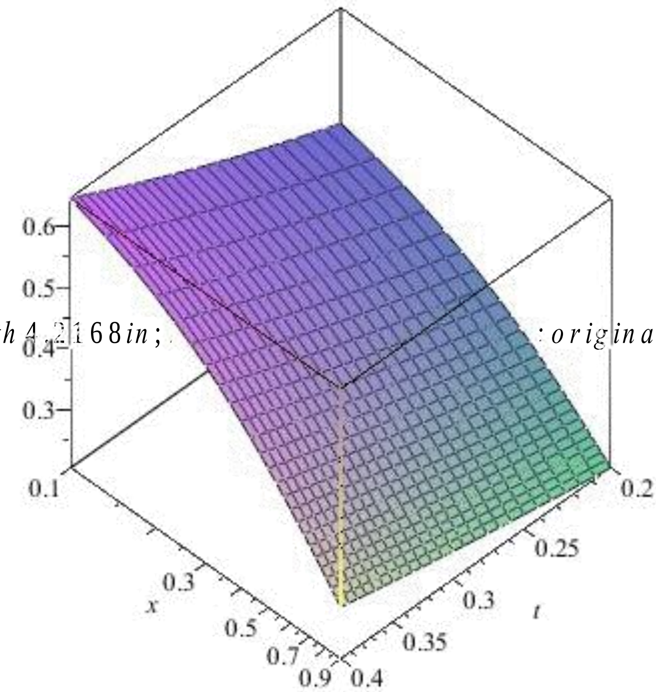

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Fig. 5. the approximate solution of $u(x,t)$ at $\alpha = 0.5$

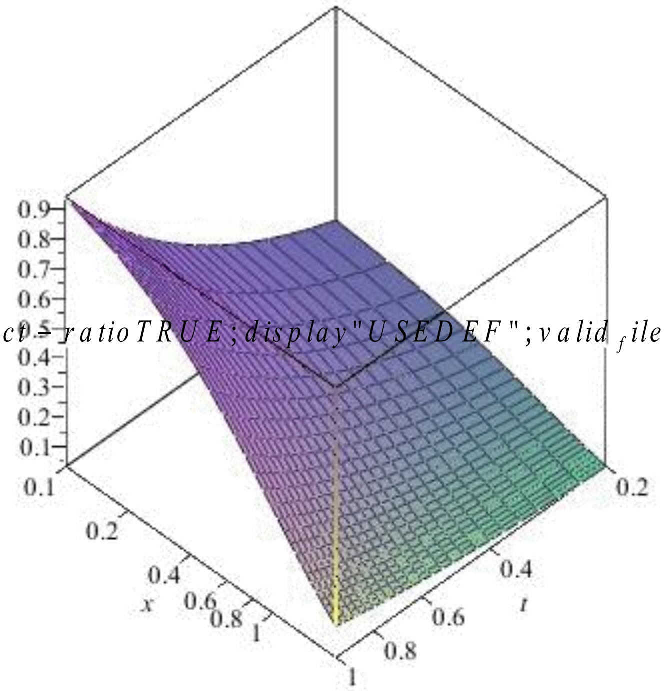

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Fig. 6. the approximate solution of $u(x,t)$ at $\alpha = 0.9$

4.3 Example[26]

We consider the followingThe time-fractional Fisher's equation (TFFE) of the form:

$$D_t^{\alpha}u(x,t)=u_{xx}+6u(1-u) \qquad t>0 \qquad 0<\alpha\le 1 \tag{4.10}$$

subject to the initial condition

$$u(x,0)=\frac{1}{(1+e^x)^2} \tag{4.11}$$

To apply FCT to Eq.(4.10), we use the above transformations:

$$\frac{\partial^{\alpha}u}{\partial t^{\alpha}}=q\frac{\partial u}{\partial T}$$

so we have the following partial differential equation:

$$q\frac{\partial u}{\partial T}=u_{xx}+6u-6u^2 \tag{4.11}$$

For simplicity we set $q=1$, so we get

$$\frac{\partial u}{\partial T}=u_{xx}+6u-6u^2 \tag{4.12}$$

Now, we solve Eq. (4.12) by means of NIM. To apply NIM to (4.12), we construct the correction functional as follows:

$$u_n(x,t)=u_{n-1}+I_T[u_{nxx}+6u_n-6u_n^2] \tag{4.13}$$

$$u_0(x,t)=u_0=\frac{1}{(1+e^x)^2}$$

$$u_1(x,t)=I_T[u_{0xx}+6u_0-6u_0^2]$$

$$I_T[-2e^x(1+e^x)^{-3}+6e^{2x}(1+e^x)^{-4}+6(1+e^x)^{-2}-6(1+e^x)^{-4}]$$

$$u_1=[-2e^x(1+e^x)^{-3}+6e^{2x}(1+e^x)^{-4}+6(1+e^x)^{-2}-6(1+e^x)^{-4}]T$$

$$u_2(x,t)=I_T[u_{1xx}+6u_1-6u_1^2]$$

$$=I_T[(-144e^{3x}(1+e^x)^{-5}+42e^{2x}(1+e^x)^{-4}-2e^x(1+e^x)^{-3}+120e^{4x}(1+e^x)^{-6})T)$$

$$+(6(-2e^x(1+e^x)^{-3}+6e^{2x}(1+e^x)^{-4}+6(1+e^x)^{-2}-6(1+e^x)^{-4})T)-6(76e^{2x}(1+e^x)^{-6}$$

$$-24e^{3x}(1+e^x)^{-7}+24e^x(1+e^x)^{-7}-24e^x(1+e^x)^{-5}-72e^{2x}(1+e^x)^{-8}+36e^{4x}(1+e^x)^{-8}$$
$$+36(1+e^x)^{-4}-72(1+e^x)^{-6}+36(1+e^x)^{-8})T^2]$$

$$u_2 = [(-144e^{3x}(1+e^x)^{-5} + 42e^{2x}(1+e^x)^{-4} - 2e^x(1+e^x)^{-3} + 120e^{4x}(1+e^x)^{-6})\frac{T^2}{2}]$$
$$+3[(-2e^x(1+e^x)^{-3} + 6e^{2x}(1+e^x)^{-4} + 6(1+e^x)^{-2} - 6(1+e^x)^{-4})T^2]$$

$$-[2(76e^{2x}(1+e^x)^{-6} - 24e^{3x}(1+e^x)^{-7} + 24e^x(1+e^x)^{-7} - 24e^x(1+e^x)^{-5} - 72e^{2x}(1+e^x)^{-8}$$
$$+36e^{4x}(1+e^x)^{-8} + 36(1+e^x)^{-4} - 72(1+e^x)^{-6} + 36(1+e^x)^{-8})T^3]$$

By the fractional complex tranform

$$T = \frac{t^\alpha}{\Gamma(1+\alpha)}$$

we have

$$u_1 = [-2e^x(1+e^x)^{-3} + 6e^{2x}(1+e^x)^{-4} + 6(1+e^x)^{-2} - 6(1+e^x)^{-4}]\frac{t^\alpha}{\Gamma(1+\alpha)}$$

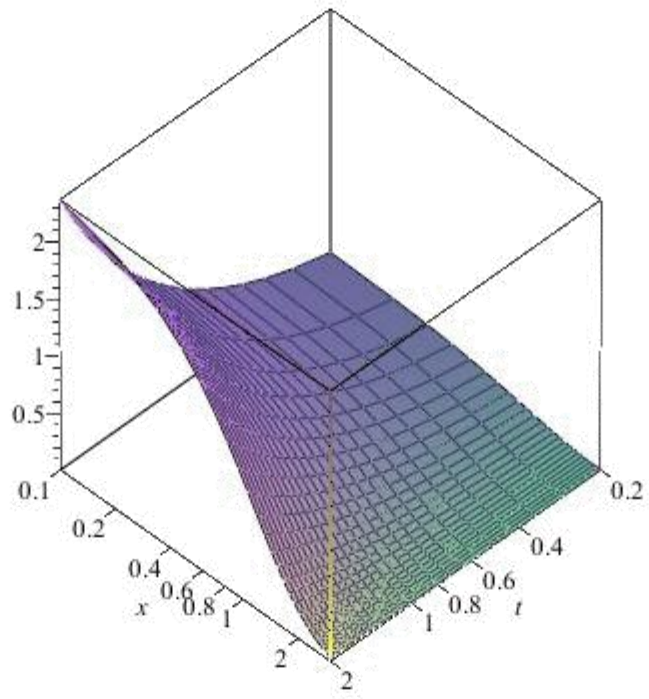

Fig. 7. the exact solution of $u(x,t)$ at $\alpha = 1$

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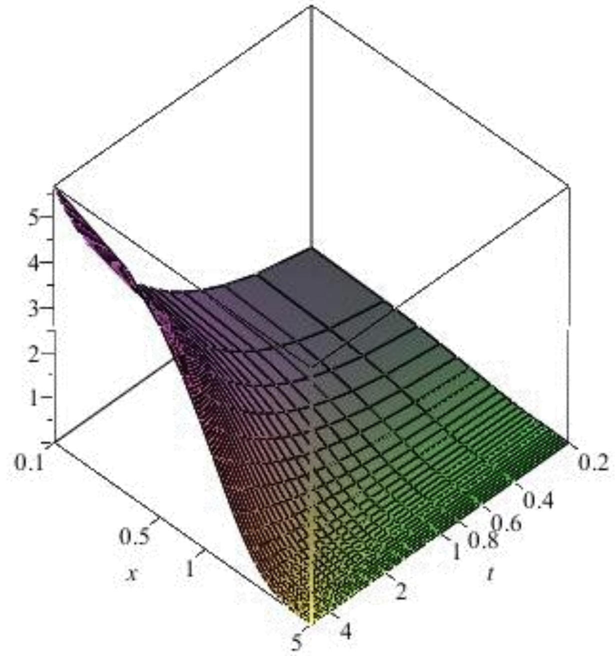
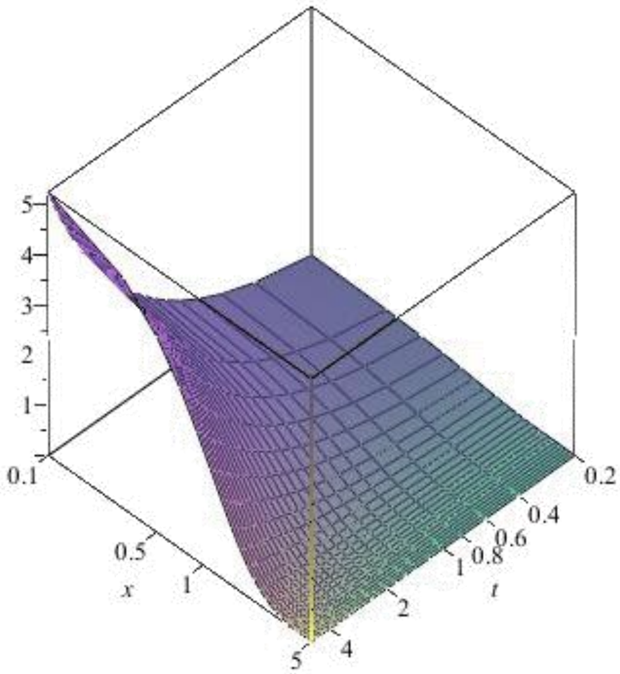


$Fig. 8. the\ approximate\ solution\ of\ u(x,t)\ at\ \alpha = 0.5$



$Fig. 9. the\ approximate\ solution\ of\ u(x,t)\ at\ \alpha = 0.9$

4.4 Example[21]

We consider the time-fractional telegraph equation:

$$D_x^{\beta} u = \frac{\partial^2 u}{\partial t^2} + \frac{\partial u}{\partial t} + u \tag{4.14}$$

subject to the initial condition

$$u(x,0) = e^{-t} \tag{4.15}$$

To apply FCT to Eq.(4.1), we use the above transformations:

$$\frac{\partial^{\beta} u}{\partial x^{\beta}} = p\frac{\partial u}{\partial X}$$

so we have the following partial differential equation:

$$p\frac{\partial u}{\partial X} = \frac{\partial^2 u}{\partial t^2} + \frac{\partial u}{\partial t} + u \tag{4.16}$$

For simplicity we set $p = 1$, so we get

$$\frac{\partial u}{\partial X} = \frac{\partial^2 u}{\partial t^2} + \frac{\partial u}{\partial t} + u \tag{4.17}$$

Now, we solve Eq. (4.17) by means of NIM. To apply NIM to (4.17), we construct the correction functional as follows:

$$u_n(x,t) = u_{n-1} + I_X[\frac{\partial^2 u_n}{\partial t^2} + \frac{\partial u_n}{\partial t} + u]$$

$$u_0(x,t) = u_0 = e^{-t}$$

$$u_1(x,t) = I_X[\frac{\partial^2 u_0}{\partial t^2} + \frac{\partial u_0}{\partial t} + u_0]$$
$$= I_X[e^{-t} - e^{-t} + e^{-t}]$$

$$u_1 = e^{-t} X$$

$$u_2(x,t) = I_X[\frac{\partial^2 u_1}{\partial t^2} + \frac{\partial u_1}{\partial t} + u_1]$$
$$= I_X[e^{-t} X - e^{-t} X + e^{-t} X]$$

$$u_2 = e^{-t}\frac{X^2}{2}$$

$$u_3(x,t) = I_X[\frac{\partial^2 u_2}{\partial t^2} + \frac{\partial u_2}{\partial t} + u_2]$$
$$= I_X[e^{-t}\frac{X^2}{2} - e^{-t}\frac{X^2}{2} + e^{-t}\frac{X^2}{2}]$$

$$u_3 = e^{-t}\frac{X^3}{6}$$

By the fractional complex tranform

$$X = \frac{x^\alpha}{\Gamma(1+\alpha)}$$

we have

$$u_1 = e^{-t}\frac{x^\alpha}{\Gamma(1+\alpha)}$$
$$\vdots$$

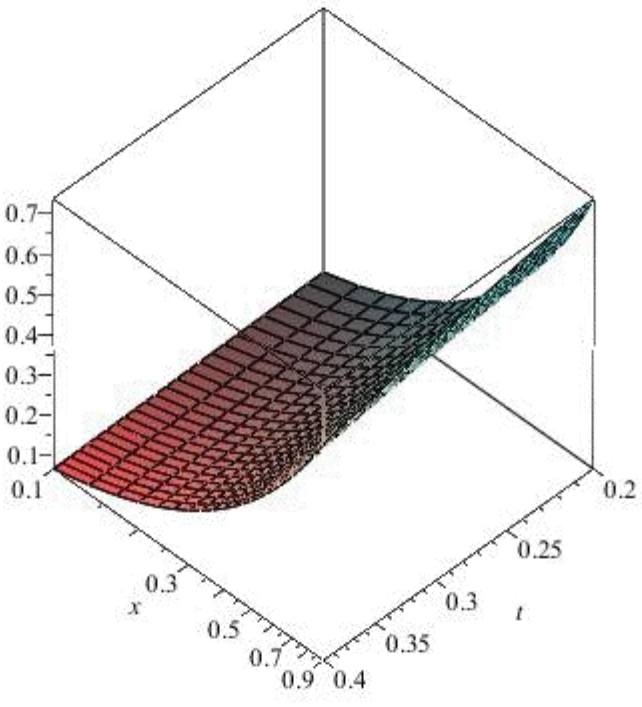


Fig. 10. the exact solution of $u(x,t)$ *at* $\alpha = 1$

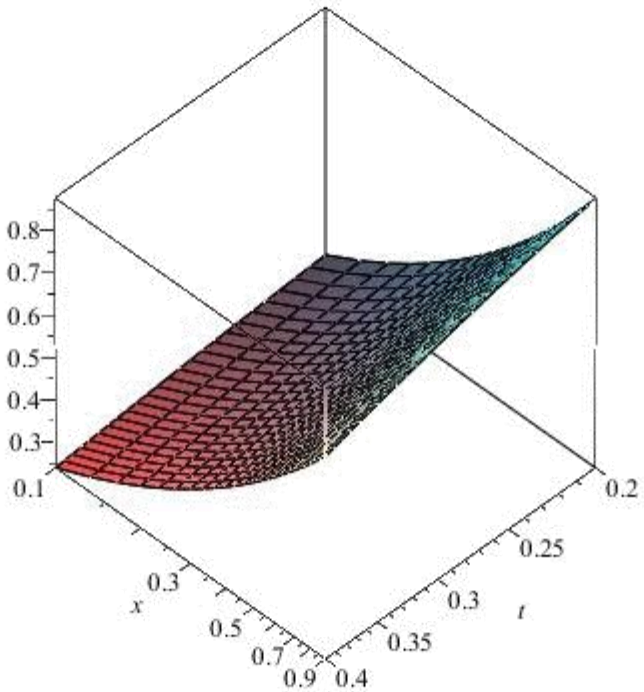


*Fig.*11. *the approximate solution of* $u(x,t)$ *at* $\alpha = 0.5$

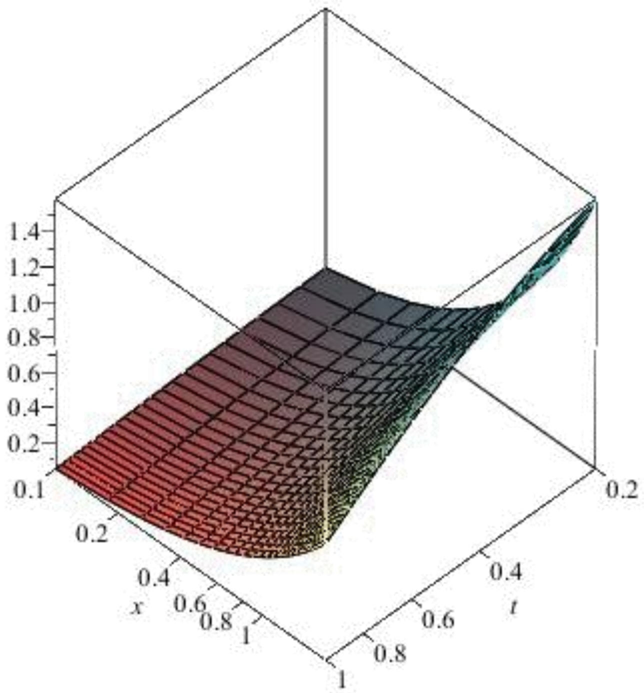


*Fig.*12.*the approximate solution of* $u(x,t)$ *at* $\alpha = 0.9$

5. CONCLUSION

In this paper, we have successfully developed FCT with help of NIM to obtain approximate solution of the fractional differential equations. The fractional complex transform can easily convert a fractional differential equations to its differential partner, so that its New Iterative algorithm can be simply constructed. The fractional complex transform is extremely simple but effectivee for solving fractional differential equations. The method is accessible to all with basic knowledge of Advanced Calculus and with little fractional calculus. It may be concluded that FCT NIM is very powerful and efficient in finding analytical as well as numerical solutions for wide classes of fractional differential equations.

REFERENCES

[1] A. Bekir, (2009), New exact travelling wave solutions of some complex nonlinear equations, Communications in Nonlinear Science and Numerical Simulation, 14(4), 1069-1077.

[2] A. Hemeda, (2012), Formulation and solution of nth-order derivative fuzzy integro-differential equation using new iterative method with a reliable algorithm. Journal of Applied Mathematics, Article ID 325473, 17 pages.

[3] A. Jerri , (1999), Introduction to Integral Equations with Applications. Second Ed. Wiley. Interscience.

[4] A. R. Mohamed and S. A. Mohamed , (2014), New Iterative Method for Solving the Fornberg-Whitham Equation and Comparison with Homotopy Perturbation Transform Method,British Journal of Mathematics & Computer Science 4(9): 1213-1227.

[5] A. S. Abedl-Rady, S. Z. Rida, A. A. M. Arafa, H. R. Abedl-Rahim, (2014), Variational Iteration Sumudu Transform Method for Solving Fractional Nonlinear Gas Dynamics Equation, International Journal of Research Studies in Science, Engineering and Technology, 1(9), 82-90.

[6] A. S. Mohamed, A. M. S. Mahdy and A. A. H. Mtawa, (2015), Aproximate analytical solution to a time-fractional Fokker-Planck equation, BOTHALA Journal, 45(4), 57-69.

[7] C. Tadjeran, M. M. Meerschaert, (2007), A second-order accurate numerical method for the two-dimensional fractional diffusion equation, J. Comput. Phys., 220 , 813-823.

[8] E. Sejdi, I Djurovi, (2011), Fractional Fourier transform as a signal processing tool: An overview of recent developments, Signal Processing, 91(6) ,1351-1369.

[9] F. Mainardi, (1997), Fractional calculus: Some basic problems in continuum and statistical mechanics, Springer-Verlag, New York, pp. 291-348.

[10] I. Podlubny, (1999), Fractional Differntial Equations, Academic Press, New York.

[11] I. Podlubny, (2002), Geometric and physical interpretation of fractional integration and fractional differentiation, Fract. Calc. Appl. Anal., 5367- 386.

[12] J. He and Z. Li, Converting Fractional differentia equations into partial differential equations, Thermal Science, DOI REFERENCE: 10.2298/TSCI110503068H

[13] J. He, (2011), A Short Remark on Fractional Variational Iteration Method, Physics Letters A, DOI: 10.1016/j.physleta.07.033

[14] J. H. He, (1998), Nonlinear oscillation with fractional derivative and its applications. In: International Conference on Vibrating Engineering, 98, Dalian, China, 288-291.

[15] J. H. He, (1999) Some applications of nonlinear fractional differential equations and their approximations, Bull. Sci. Technol., 15 (2) 86-90.

[16] M. Cassol, S. Wortmann and U. Rizza, (2009), Analytic modeling of two-dimensional transient atmospheric pollutant dispersion by double GITT and Laplace Transform techniques, Environmental Modelling & Software, 24(1), 144-151.

[17] M. M. Meerschaert and C. Tadjeran, (2006), Finite difference approximations for two sided space-fractional partial differential equations, Appl. Numer. Math., 56.80-90.

[18] P. Gordoa, A. Pickering and Z. Zhu, (2010), Bcklund transformations for a matrix second Painlev equation, Physics Letters A, 374 (34), 3422-3424.

[19] R. Cotta and M. Mikhailov, (1993), Integral transform method, Applied Mathematical Modelling, 17 (3), 156-161.

[20] R. Yulita Molliq and B. Batiha, (2012), Approximate analytic solutions of fractional Zakharov Kuznetsov equations by fractional complex transform, International Journal of Engineering and Technology, 1 (1), 1-13.

[21] R. Cui and Y. Hu, (2016), Fractional power series method for solving fractional differemtial equation, Journal of Advances in Mathematics, 12(4).

[22] S. Bhalekar and V. Daftardar-Gejji, (2008), New iterative method: application to partial differential equations. Applied Mathematics and Computation, 203(2), 778-783.

[23] S. Bhalekar and V. Daftardar-Gejji, (2010), Solving evolution equations using a new iterative method Numerical Methods for Partial Differential Equations, 26(4), 906-916.

[24] S. Momani and K. Al-Khaled, (2005), Numerical solutions for systems of fractional differential equations by the decomposition method, Appl. Math. Comput. 162 (3), 1351-1365.

[25] S. Momani, (2005), An explicit and numerical solutions of the fractional KdV equation, Math. Comput. Simul., 70 (2) ,110-118.

[26] S. Momani and Z. Odibat, (2007), Homotopy perturbation method for nonlinear partial differential equations of fractional order, Physics Letters A,

365, 345–350.

[27] V. Daftardar-Gejji and H. Jafari, (2006), An iterative method for solving nonlinear functional equations, Journal of Mathematical Analysis and Applications, 316(2), 753-763.

[28] V. Daftardar-Gejji and S. Bhalekar, (2010), Solving fractional boundary value problems with Dirichlet boundary conditions using a new iterative method, Computers & Mathematics with Applications, 59(5), 1801-1809.

[29] X. Yang , (2011), Local Fractional Integral Transforms, Progress in Nonlinear Science, 4, 1-225.

[30] Y. Cherruault , (1989) Convergence of Adomian's method. Kybernetes,18(2), 31-38.

[31] Z. Li and J.He ,(2010), Fractional Complex Transform for Fractional Differential Equations, Mathematical and Computational Applications, 15 (5), 970-973.

[32] Z. Li, (2010), An Extended Fractional Complex Transform, Journal of Nonlinear Science and Numerical Simulation, 11, 0335-0337.

[33] A. Saravanan and N. Magesh, (2016), An efficient computational technique for solving the Fokker–Planck equation with space and time fractional derivatives , Journal of King Saud University Science, 28(2), 161-166.

[34] Y. C. Enesiz and A. Kurt, (2016), New Fractional Complex Transform for Conformable Fractional Partial Differential Equations, JAMSI, 12(2), 41-47.

[35] O. Ozkan, (2017), Approximate Analytical Solutions of Systems of Fractional Partial DifferentialEquations, Karaelmas Fen ve Müh. Derg. 7(1), 63-67.

[36] O. Guner and A. Bekir, (2018), Solving nonlinear space-time fractional differential equations via ansatz method, Computational Methods for Differential Equations, 6(1), 1-11.