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Fuzzy Supra Preopen sets

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Abstract

In this paper fuzzy supra preopen sets and fuzzy supra preclosed sets are introduced and certain properties and relations of fuzzy supra preopen and fuzzy supra preclosed sets are investigated.

Keywords - Fuzzy supra topological space, fuzzy supra preopen set, fuzzy supra preclosed set.

1. INTRODUCTION

In 1982, Mashhour et al. [9] introduced the notion of preopen sets in topological spaces. In 1991, Singal and Rajvanshi [14] introduced the concept of fuzzy preopen sets in fuzzy topological spaces.

In 1983, Mashhour et al. [8] introduced the concept of supra topological spaces. In 1987, Abd El-Monsef et al. [1] introduced fuzzy supra topological spaces. In 2010, Sayed [13] introduced supra preopen sets. In 2016, Othaman [6] introduced fuzzy supra preopen and fuzzy supra preclosed sets using the notion of fuzzy generalised open sets and fuzzy generalised closed sets. In this paper the concept of fuzzy supra preopen sets and fuzzy supra preclosed sets are introduced using fuzzy supra open and fuzzy supra closed sets.

2. PRELIMINARY DEFINITIONS

Throughout this paper X denotes a non empty set.

***Definition : 2.1* [15]**

A fuzzy set in X is a map f: X → I = [0,1] . The family of fuzzy sets of X is denoted by I^X.

Following are some basic operations of fuzzy sets in X. For the fuzzy sets f and g in X,

1) f = g if and only if f(x) = g(x) for all x ∈ X

2) f ≤ g if and only if f(x) ≤ g(x) for all x ∈ X

3) (f ∨ g)(x) = max { f(x),g(x) } for all x ∈ X

4) (f ∧ g)(x) = min { f(x),g(x) } for all x ∈ X

5) $f^c(x)$ = 1- f(x) for all x ∈ X here f^c denotes the complement of f.

6) f ≤ g if and only if $f^c \geq g^c$

7) For a family { f_λ/ λ∈Λ } of fuzzy sets defined on a set X

$$(\vee_{\lambda \in \Lambda} f_\lambda)(x) = \vee_{\lambda \in \Lambda}(f_\lambda(x))$$

$$(\wedge_{\lambda \in \Lambda} f_\lambda)(x) = \wedge_{\lambda \in \Lambda}(f_\lambda(x))$$

8) For any α ∈ I, the constant fuzzy set **α** in X is a fuzzy set in X defined by **α**(x) = α for all x ∈ X.

0 denotes null fuzzy set in X and **1** denotes universal fuzzy set in X.

***Definition :2.2* [4]**

A fuzzy topological space is a pair (X,δ) where X is a nonempty set and δ is a family of fuzzy set on X satisfying the following properties:

1) The constant functions **0** and **1** belong to δ.

2) f, g ∈ δ implies f ∧ g ∈ δ.

3) f_λ ∈ δ for each λ ∈ Λ implies $(\vee_{\lambda \in \Lambda} f_\lambda) \in \delta$.

Then δ is called a fuzzy topology on X. Every member of δ is called fuzzy open set. The complement of a fuzzy open set is called fuzzy closed set.

***Definition :2.3* [4]**

The closure and interior of a fuzzy set $f \in I^X$ are defined respectively as

$cl(f) = \wedge\{ g / g$ is a fuzzy closed set in X and $f \leq g \}$

$int(f) = \vee \{ g / g$ is a fuzzy open set in X and $g \leq f \}$

Clearly cl(f) is the smallest fuzzy closed set containing f and int(f) is the largest fuzzy open set contained in f.

***Remark : 2.4*[2]**

For a family $\{f_\lambda / \lambda \in \Lambda\}$ of fuzzy sets in a fuzzy topological space (X,δ)

1. $\vee cl(f_\lambda) \leq cl(\vee f_\lambda)$
2. $\vee int(f_\lambda) \leq int(\vee f_\lambda)$

If the collection is finite, then

$\vee cl(f_\lambda) = cl(\vee f_\lambda)$

***Definition :2.5* [11]**

A collection δ^* of fuzzy sets in a set X is called fuzzy supra topology on X if the following conditions are satisfied:

1) **0** and **1** belongs to δ^*.

2) $f_\lambda \in \delta^*$ for each $\lambda \in \Lambda$ implies $(\vee_{\lambda\in\Lambda} f_\lambda) \in \delta^*$.

The pair (X,δ^*) is called a fuzzy supra topological space. The elements of δ^* are called fuzzy supra open sets and the complement of a fuzzy supra open set is called fuzzy supra closed set.

***Definition :2.6* [11]**

Let (X,δ^*) be a fuzzy supra topological space and f be a fuzzy set in X, then the fuzzy supra closure and fuzzy supra interior of f defined respectively as

$cl^*(f) = \wedge\{ g / g$ is a fuzzy supra closed set in X and $f \leq g\}$

$int^*(f) = \vee \{ g / g$ is a fuzzy supra open set in X and $g \leq f\}$

***Definition: 2.7* [11]**

Let (X,δ) be a fuzzy topological space and δ^* be a fuzzy supra topology on X .We call δ^* a fuzzy supra topology associated with δ if $\delta \leq \delta^*$

***Remark: 2.8* [11]**

1) The fuzzy supra closure of a fuzzy set f in a fuzzy supra topological space is the smallest fuzzy supra closed set containing f.

2) The fuzzy supra interior of a fuzzy set f in a fuzzy supra topological space is the largest fuzzy supra open set contained in f.

3) If (X,δ^*) is an associated fuzzy supra topological space with the fuzzy topological space (X,δ) and f is any fuzzy set in X, then

$$\text{int}(f) \leq \text{int}^*(f) \leq f \leq \text{cl}^*(f) \leq \text{cl}(f)$$

***Theorem :2.9* [11]**

For any two fuzzy sets f and g in a fuzzy supra topological space (X,δ^*),

1) f is a fuzzy supra closed set if and only if $\text{cl}^*(f) = f$.

2) f is a fuzzy supra open set if and only if $\text{int}^*(f) = (f)$.

3) $f \leq g$ implies $\text{int}^*(f) \leq \text{int}^*(g)$ and $\text{cl}^*(f) \leq \text{cl}^*(g)$

4) $\text{cl}^*(\text{cl}^*(f)) = \text{cl}^*(f)$ and $\text{int}^*(\text{int}^*(f)) = \text{int}^*(f)$.

5) $\text{cl}^*(f \vee g) \geq \text{cl}^*(f) \vee \text{cl}^*(g)$

6) $\text{cl}^*(f \wedge g) \leq \text{cl}^*(f) \wedge \text{cl}^*(g)$

7) $\text{int}^*(f \vee g) \geq \text{int}^*(f) \vee \text{int}^*(g)$

8) $\text{int}^*(f \wedge g) \leq \text{int}^*(f) \wedge \text{int}^*(g)$

9) $\text{cl}^*(f^c) = [\text{int}^*(f)]^c$

10) $\text{int}^*(f^c) = [\text{cl}^*(f)]^c$

3. FUZZY SUPRA PREOPEN SETS

Definition :3.1

Let (X,δ^*) be a fuzzy supra topological space. A fuzzy set f is a called fuzzy supra preopen set if

$$f \leq \text{int}^*(\text{cl}^*(f))$$

and a fuzzy set f is called a fuzzy supra preclosed set if

$$\text{cl}^*(\text{int}^*(f)) \leq f$$

Theorem :3.2

Let f be a fuzzy set in a fuzzy supra topological space (X,δ^*),then f is a fuzzy supra preopen set if and only if f^c is a fuzzy supra preclosed set.

Proof:

Let f be a fuzzy supra preopen set

$f \leq int^*(cl^*(f))$

$\therefore \quad f^c \geq (int^*(cl^*(f)))^c$

$\geq cl^*((cl^*(f))^c)$

$\geq cl^*(int^*(f^c))$

$\therefore f^c$ is a fuzzy supra preclosed set

Conversely

Let f^c be a fuzzy supra preclosed set

$f^c \geq cl^*(int^*(f^c))$

$\therefore (f^c)^c \leq (cl^*(int^*(f^c)))^c$

$\therefore f \leq int^*((int^*(f^c))^c)$

$\leq int^*(cl^*((f^c)^c))$

$\leq int^*(cl^*((f))$

$\therefore f^c$ is a fuzzy supra preopen set

Remark : 3.3

For a family $\{f_\lambda / \lambda \in \wedge\}$ of fuzzy sets in a fuzzy supra topological space (X,δ^*)

1. $\vee (cl^*(f_\lambda)) \leq cl^*(\vee f_\lambda)$
2. $\vee (int^*(f_\lambda)) \leq int^*(\vee f_\lambda)$

If the collection is finite, then

$\vee cl^*(f_\lambda) = cl^* (\vee f_\lambda)$

Theorem: 3.4

1) Arbitrary union of fuzzy supra preopen sets is a fuzzy supra preopen set.

2) Arbitrary intersection of fuzzy supra preclosed sets is a fuzzy supra preclosed set.

Proof :

1) Let $\{f_\lambda / \lambda \in \wedge\}$ be a collection of fuzzy supra preopen sets.

Then for each λ,

$f_\lambda \leq int^*(cl^*(f_\lambda))$

$\therefore \bigvee f_\lambda \leq \bigvee\{int^*(cl^*(f_\lambda))\}$

$\leq int^*\{\bigvee (cl^*(f_\lambda))\}$

$\leq int^*(cl^*(\vee f_\lambda))$

∴ Arbitrary union of fuzzy supra preopen sets is a fuzzy supra preopen set.

2) From (1)

$\vee f_\lambda \leq int^*(cl^*(\vee f_\lambda))$

$\therefore [\vee f_\lambda]^c \geq (int^*(cl^*(\vee f_\lambda)))^c$

$\therefore \wedge f_\lambda^c \geq cl^*[(cl^*(\vee f_\lambda))^c]$

$\geq cl^*(int^*[(\vee f_\lambda)^c])$

$\geq cl^*(int^*(\wedge f_\lambda^c))$

∴ Arbitrary intersection of fuzzy supra preclosed sets is a fuzzy supra preclosed set.

Theorem: 3.5

Every fuzzy supra open set is fuzzy supra preopen set.

Proof :

Let f be a fuzzy supra open set in (X,δ^*)

$\therefore int^*(f) = f$

Since $f \leq cl^*(f)$

By the property of fuzzy supra interior,

$f \leq g$ implies $int^*(f) \leq int^*(g)$

$\therefore int^*(f) \leq int^*[cl^*(f)]$

$\therefore f \leq int^*[cl^*(f)]$

Theorem :3.6

Every fuzzy supra closed set is fuzzy supra preclosed set.

Proof :

Let f be a fuzzy supra closed set in (X,δ^*)

$\therefore cl^*(f) = f$

Since $int^*(f) \leq f$

By the property of fuzzy supra closure,

$f \leq g$ implies $cl^*(f) \leq cl^*(g)$

$\therefore cl^*(int^*(f)) \leq cl^*(f)$

$\therefore cl^*(int^*(f)) \leq f$

Definition :3.7

Let (X,δ^*) be a fuzzy supra topological space and f be a fuzzy set in X, then the fuzzy supra preclosure and fuzzy supra preinterior of f is defined respectively as

$pcl^*(f) = \wedge \{ g / g$ is a fuzzy supra preclosed set in X and $f \leq g\}$

$pint^*(f) = \vee \{ g / g$ is a fuzzy supra preopen set in X and $g \leq f\}$

Remark :3.8

It is obvious that $pint^*(f)$ is a fuzzy supra preopen set and $pcl^*(f)$ is a fuzzy supra preclosed set.

Theorem :3.9

For any fuzzy set f in a fuzzy supra topological space (X,δ^*),

1) $[pint^*(f)]^c = pcl^*(f^c)$

2) $[pcl^*(f)]^c = pint^*(f^c)$

Proof :

1) consider

$pint^*(f) = \vee \{ g / g$ is a fuzzy supra preopen set in X and $g \leq f\}$

$[pint^*(f)]^c = 1 - \vee \{ g / g$ is a fuzzy supra preopen set in X and $g \leq f\}$

$= \wedge \{ g^c / g^c$ is a fuzzy supra preclosed set in X and $g^c \geq f^c\}$

$= pcl^*(f^c)$

2) $pcl^*(f) = \wedge \{ g / g$ is a fuzzy supra preclosed set in X and $f \leq g\}$

$[pcl^*(f)]^c = 1 - \wedge \{ g / g$ is a fuzzy supra preclosed set in X and $f \leq g\}$

$= \vee \{ g^c / g^c$ is a fuzzy supra preopen set in X and $f^c \geq g^c \}$

$= pint^*(f^c)$

Theorem :3.10

For any two fuzzy sets f and g in fuzzy supra topological space (X,δ^*),then if $f \leq g$ implies

1) $pint^*(f) \leq pint^*(g)$

2) $pcl^*(f) \leq pcl^*(g)$

Proof:

1) $pint^*(f) = \vee \{ h / h$ is a fuzzy supra preopen set in X and $h \leq f\}$

Since $pint^*(f)$ is the largest fuzzy supra preopen set contained in f

$pint^*(f) \leq f$

$f \leq g$ implies $pint^*(f) \leq f \leq g$

implies $pint^*(f) \leq g$

$\therefore pint^*(f)$ is the fuzzy supra preopen set contained in g.

But $pint^*(g)$ is the largest fuzzy supra preopen set contained in g.

$\therefore pint^*(f) \leq pint^*(g)$

2) $f \leq g$ if and only if $f^c \geq g^c$

consider $g^c \leq f^c$

$pint^*(g^c) \leq pint^*(f^c)$

$(pcl^*(g))^c \leq (pcl^*(f))^c$

$\therefore pcl^*(f) \leq pcl^*(g)$

Theorem :3.11

For any two fuzzy sets f and g in fuzzy supra topological space (X,δ^*)

1) $pint^*(f) \vee pint^*(g) \leq pint^*(f \vee g)$

2) $pcl^*(f) \wedge pcl^*(g) \geq pcl^*(f \wedge g)$

3) $pcl^*(f) \vee pcl^*(g) \leq pcl^*(f \vee g)$

4) $pint^*(f) \wedge pint^*(g) \geq pint^*(f \wedge g)$

Proof :

1) $f \leq f \vee g$

$g \leq f \vee g$

then $pint^*(f) \leq pint^*(f \vee g)$

$pint^*(g) \leq pint^*(f \vee g)$

$\therefore pint^*(f) \vee pint^*(g) \leq pint^*(f \vee g)$

2) $f \wedge g \leq f$

$f \wedge g \leq g$

then $pcl^*(f \wedge g) \leq pcl^*(f)$

$pcl^*(f \wedge g) \leq pcl^*(g)$

$\therefore pcl^*(f \wedge g) \leq pcl^*(f) \wedge pcl^*(g)$

3)) $f \leq f \vee g$

$g \leq f \vee g$

then $pcl^*(f) \leq pcl^*(f \vee g)$

$pcl^*(g) \leq pcl^*(f \vee g)$

$\therefore pcl^*(f) \vee pcl^*(g) \leq pcl^*(f \vee g)$

4) $f \wedge g \leq f$

$f \wedge g \leq g$

then $pint^*(f \wedge g) \leq pint^*(f)$

$pint^*(f \wedge g) \leq pint^*(g)$

$\therefore pint^*(f \wedge g) \leq pint^*(f) \wedge pint^*(g)$

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