Global Journal of Pure and Applied Mathematics.
ISSN 0973-1768 Volume 14, Number 1 (2018), pp. 181-187

http://www.ripublication.com/gjpam.htm

On nano $\pi g\beta$-closed sets

M. Sajan Joseph

Department of Mathematics,
Arul Anandar College, Karumathur,
Madurai Dt, Tamil Nadu, India.

I. Rajasekaran

Department of Mathematics,
Tirunelveli Dakshina Mara Nadar Sangam College,
T. Kallikulam-627 113, Tirunelveli District, Tamil Nadu, India.

O. Nethaji

2/71, West Street, Sangampatti - 625 514,
Madurai District, Tamil Nadu, India.

Abstract

In this paper, a new class of sets called $\pi g\beta$-closed sets in nano topological spaces is introduced and its properties are studied and studied of nano $\pi g\beta$-closed sets which is implied by that of nano $g\beta$-closed sets.

AMS subject classification: 54A05, 54C10, 54B05.
Keywords: nano π-open set, nano πg-open set, nano πgp-open sets, nano πgs-open set and nano $\pi g\beta$-open set.

1. Introduction

Lellis Thivagar et al. [2] introduced a nano topological space with respect to a subset X of an universe which is defined in terms of lower approximation and upper approximation and boundary region. The classical nano topological space is based on an equivalence relation on a set, but in some situation, equivalence relations are nor suitable for coping with granularity, instead the classical nano topology is extend to general binary relation based covering nano topological space

Rajasekaran et al. [5, 6, 7, 8] initiated the study nano πg-closed sets, nano $g\beta$-closed sets, nano πgp-closed sets, nano πgs-closed sets. In this paper, a new class of sets called $\pi g\beta$-closed sets in nano topological spaces is introduced and its properties are studied and studied of nano $\pi g\beta$-closed sets which is implied by that of nano $g\beta$-closed sets.

2. Preliminaries

Throughout this paper $(U, \tau_R(X))$ (or X) represent nano topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset H of a space $(U, \tau_R(X))$, $Ncl(H)$ and $Nint(H)$ denote the nano closure of H and the nano interior of H respectively. We recall the following definitions which are useful in the sequel.

Definition 2.1. [3] Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$.

1. The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is, $L_R(X) = \bigcup_{x \in U} \{R(x) : R(x) \subseteq X\}$, where R(x) denotes the equivalence class determined by x.

2. The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is, $U_R(X) = \bigcup_{x \in U} \{R(x) : R(x) \cap X \neq \phi\}$.

3. The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not - X with respect to R and it is denoted by $B_R(X)$. That is, $B_R(X) = U_R(X) - L_R(X)$.

Proposition 2.2. [2] If (U, R) is an approximation space and $X, Y \subseteq U$; then

1. $L_R(X) \subseteq X \subseteq U_R(X)$;

2. $L_R(\phi) = U_R(\phi) = \phi$ and $L_R(U) = U_R(U) = U$;

3. $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$;

4. $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$;

5. $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$;

6. $L_R(X \cap Y) \subseteq L_R(X) \cap L_R(Y)$;
7. $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$ whenever $X \subseteq Y$;
8. $U_R(X^c) = [L_R(X)]^c$ and $L_R(X^c) = [U_R(X)]^c$;
9. $U_R U_R(X) = L_R U_R(X) = U_R(X)$;
10. $L_R L_R(X) = U_R L_R(X) = L_R(X)$.

Definition 2.3. **[2]** Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then by the Property 2.2, R(X) satisfies the following axioms:

1. U and $\phi \in \tau_R(X)$,
2. The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$,
3. The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.

That is, $\tau_R(X)$ is a topology on U called the nano topology on U with respect to X. We call $(U, \tau_R(X))$ as the nano topological space. The elements of $\tau_R(X)$ are called as nano open sets and $[\tau_R(X)]^c$ is called as the dual nano topology of $[\tau_R(X)]$.

Remark 2.4. **[2]** If $[\tau_R(X)]$ is the nano topology on U with respect to X, then the set $B = \{U, \phi, L_R(X), B_R(X)\}$ is the basis for $\tau_R(X)$.

Definition 2.5. **[2]** If $(U, \tau_R(X))$ is a nano topological space with respect to X and if $H \subseteq U$, then the nano interior of H is defined as the union of all nano open subsets of H and it is denoted by $Nint(H)$.

That is, $Nint(H)$ is the largest nano open subset of H. The nano closure of H is defined as the intersection of all nano closed sets containing H and it is denoted by $Ncl(H)$.

That is, $Ncl(H)$ is the smallest nano closed set containing H.

Definition 2.6. A subset H of a nano topological space $(U, \tau_R(X))$ is called

1. nano α-open [2] if $H \subseteq Nint(Ncl(Nint(H)))$.
2. nano semi open [2] if $H \subseteq Ncl(Nint(H))$.
3. nano regular-open [2] if $H = Nint(Ncl(H))$.
4. nano π-open [1] if the finite union of nano regular-open sets.
5. nano pre-open [2] if $H \subseteq Nint(Ncl(H))$.
6. nano β-open [4] if $H \subseteq Ncl(Nint(Ncl(H)))$

The complements of the above mentioned sets is called their respective closed sets.

Definition 2.7. A subset H of a nano topological space $(U, \tau_R(X))$ is called;

1. nano πg-closed [5] if $Ncl(H) \subseteq G$, whenever $H \subseteq G$ and G is nano π-open.
2. nano $g\beta$-closed set [8] if $N\beta cl(H) \subseteq G$, whenever $H \subseteq G$ and G is nano open.
3. nano πgp-closed set [6] if $Npcl(H) \subseteq G$, whenever $H \subseteq G$ and G is nano π-open.
4. nano πgs-closed set [7] if $Nscl(H) \subseteq G$, whenever $H \subseteq G$ and G is nano π-open.

The complements of the above mentioned sets is called their respective open sets.

Definition 2.8. [7] A subset H of a space $(U, \tau_R(X))$ is called a nano strong $\mathcal{B}_Q$-set if $Nint(Ncl(H)) = Ncl(Nint(H))$.

3. On nano $\pi g\beta$-closed sets

Definition 3.1. A subset H of a space $(U, \tau_R(X))$ is nano $\pi g\beta$-closed if $N\beta cl(H) \subseteq G$ whenever $H \subseteq G$ and G is nano π-open.

The complement of nano $\pi g\beta$-open if $H^c = U - H$ is nano $\pi g\beta$-closed.

Example 3.2. Let $U = \{a, b, c\}$ with $U/R = \{\{a\}, \{b, c\}\}$ and $X = \{a\}$. Then the nano topology $\tau_R(X) = \{\phi, \{a\}, U\}$.

1. then $\{a, b\}$ is nano $\pi g\beta$-closed set.
2. then $\{c\}$ is nano $\pi g\beta$-open set.

Theorem 3.3. In a space $(U, \tau_R(X))$, the following properties are equivalent:

1. If H is nano $g\beta$-closed, then H is nano $\pi g\beta$-closed.
2. If H is nano πg-closed, then H is nano $\pi g\beta$-closed.

Proof. Obvious. ∎

Remark 3.4. For a subset of a space $(U, \tau_R(X))$, we have the following implications:

$$
\begin{array}{ccccc}
 & & \textbf{nano } gp\textbf{-closed} & \Leftarrow & \textbf{nano } gs\textbf{-closed} \\
 & & \Downarrow & \Rsh\ \textbf{nano } \pi g\textbf{-closed}\ \Lsh & \Downarrow \\
 & & \textbf{nano } \pi gp\textbf{-closed} & \Leftarrow & \textbf{nano } \pi gs\textbf{-closed} \\
 & & \Downarrow & & \\
\textbf{nano } g\beta\textbf{-closed} & \Rightarrow & \textbf{nano } \pi g\beta\textbf{-closed} & &
\end{array}
$$

None of the above implications are reversible as shown by the following Examples.

Example 3.5.

1. Let $U = \{a, b, c\}$ with $U/R = \{\{a, c\}, \{b\}\}$ and $X = \{c\}$. Then the nano topology $\tau_R(X) = \{\phi, \{a, c\}, U\}$. Then $\{a, c\}$ is nano $\pi g\beta$-closed set but not nano $g\beta$-closed.

2. Let $U = \{a, b, c, d\}$ with $U/R = \{\{a, b\}, \{c\}, \{d\}\}$ and $X = \{a, d\}$. Then the nano topology $\tau_R(X) = \{\phi, \{d\}, \{a, b\}, \{a, b, d\}, U\}$. Then $\{d\}$ is nano $\pi g\beta$-closed set but not nano πgp-closed.

Lemma 3.6. In a space $(U, \tau_R(X))$,

1. every nano open set is nano $\pi g\beta$-closed.

2. every nano closed set is nano $\pi g\beta$-closed.

Remark 3.7. The converses of statements in Lemma 3.6 are not necessarily true as seen from the following Examples.

Example 3.8. In Example 3.2,

1. then $\{a, c\}$ is nano $\pi g\beta$-closed set but not nano open.

2. then $\{a\}$ is nano $\pi g\beta$-closed set but not nano closed.

Theorem 3.9. In a space $(U, \tau_R(X))$, the following properties are equivalent:

1. H is H is nano π-open and nano $\pi g\beta$-closed.

2. H is nano regular-open.

Proof. (1) $\Rightarrow$ (2). By (1) $N\beta cl(H) \subseteq H$, since H is nano π-open and nano $\pi g\beta$-closed. Thus $Nint(Ncl(Nint(H))) \subseteq H$, Since $N\beta cl(H) = H \cup Nint(Ncl(Nint(H)))$. As H is nano open, then H is clearly nano α-open and so $H \subseteq Nint(Ncl(Nint(H)))$. Therefore $Nint(Ncl(Nint(H))) \subseteq H \subseteq Nint(Ncl(Nint(H)))$ or equivalently $H = Nint(Ncl(H))$, which shows that H is nano regular-open.

(2) $\Rightarrow$ (1). Every nano regular-open set is nano π-open and it is every nano β-closed. ■

Corollary 3.10. If H is nano open and nano $\pi g\beta$-closed, then H is nano β-closed and hence nano $g\beta$-closed.

Proof. By assumption and Theorem 3.9, H is nano regular-open. Thus H is nano β-closed and hence nano $g\beta$-closed. ■

Remark 3.11. In a space $(U, \tau_R(X))$, $N\beta cl(U - H) = U - \beta int(H)$, for any subset H of a space U.

Theorem 3.12. In a space $(U, \tau_R(X))$, $H \subseteq U$ is nano $\pi g\beta$-open $\iff$ $F \subseteq N\beta int(H)$ whenever K is nano π-closed and $K \subseteq H$.

Proof. Necessity. Let H be nano $\pi g\beta$-open. Let K be nano π-closed and $K \subseteq H$. Then $U - H \subseteq U - K$ where $U - K$ is nano π-open. nano $\pi g\beta$-closedness of $U - H$ implies $N\beta cl(U - H) \subseteq U - K$. By Remark 3.11, $N\beta cl(U - H) = U - N\beta int(H)$. So $K \subseteq N\beta int(H)$.

Su±ciency. Suppose K is nano π-closed and $K \subseteq H$ imply $K \subseteq N\beta int(H)$. Let $U - H \subseteq G$ where G is nano π-open. Then $U - G \subseteq H$ where $U - G$ is nano π-closed. By hypothesis $U - G \subseteq N\beta int(H)$. That is $U - N\beta int(H) \subseteq G$. By Remark 3.11, $N\beta cl(U - H) \subseteq G$. So, $U - H$ is nano πgs-closed and H is nano $\pi g\beta$-open. ■

Theorem 3.13. In a space $(U, \tau_R(X))$, the following properties are equivalent:

1. H is nano π-clopen.
2. H is nano π-open, a nano strong $\mathcal{B}_Q$-set and nano $\pi g\beta$-closed.

Proof. (1) $\Rightarrow$ (2) is Obvious.

(1) $\Rightarrow$ (2). By Theorem 3.9, H is nano regular-open. Since H is a nano strong $\mathcal{B}_Q$-set, $H = Nint(Ncl(H)) = Ncl(Nint(H))$. So H is nano regular-closed. This shows that H is nano π-closed and hence H is nano π-clopen. ■

Theorem 3.14. In a space $(U, \tau_R(X))$, the union of two nano $\pi g\beta$-closed sets is nano $\pi g\beta$-closed.

Proof. Let $H \cup Q \subseteq G$, then $H \subseteq G$ and $Q \subseteq G$ where G is nano π-open. As H and Q are $\pi g\beta$-closed, $Ncl(H) \subseteq G$ and $Ncl(Q) \subseteq G$. Hence $Ncl(H \cup Q) = Ncl(H) \cup Ncl(Q) \subseteq G$. ■

Example 3.15. In Example 3.2, then $H = \{1\}$ and $Q = \{2\}$ is nano $\pi g\beta$-closed. Clearly $H \cup Q = \{1, 2\}$ is nano $\pi g\beta$-closed.

Theorem 3.16. In a space $(U, \tau_R(X))$, the intersection of two nano $\pi g\beta$-open sets are nano $\pi g\beta$-open.

Proof. Obvious by Theorem 3.14. ■

Example 3.17. In Example 3.2, then $H = \{2, 3\}$ and $Q = \{1, 3\}$ is nano $\pi g\beta$-open. Clearly $H \cap Q = \{3\}$ is nano $\pi g\beta$-open.

Remark 3.18. In a space $(U, \tau_R(X))$, the union of two nano $\pi g\beta$-closed sets but not nano $\pi g\beta$-closed.

Example 3.19. In Example 3.5 (2), then $H = \{a, b\}$ and $Q = \{a, d\}$ is nano πgs-closed sets. Clearly $H \cup Q = \{a, b, d\}$ is but not nano πgs-closed.

Remark 3.20. In a space $(U, \tau_R(X))$, the intersection of two nano $\pi g\beta$-open sets but not nano $\pi g\beta$-open.

Example 3.21. In Example 3.5(2), then $H = \{a, c\}$ and $Q = \{b, c\}$ is nano πgs-open sets. Clearly $H \cap Q = \{c\}$ is but not nano πgs-open.

Theorem 3.22. Let H be nano $\pi g\beta$-closed. Then $N\beta cl(H) - H$ does not contain any non-empty nano π-closed set.

Proof. Let K be a nano π-closed set such that $K \subseteq N\beta cl(H) - H$. Then $K \subseteq U - H$ implies $H \subseteq U - K$. Therefore $N\beta cl(H) \subseteq U - K$. That is $K \subseteq U - N\beta cl(H)$. Hence $K \subseteq N\beta cl(H) \cap (U \cap N\beta cl(H)) = \phi$. This shows $K = \phi$. ■

Theorem 3.23. If H is nano $\pi g\beta$-closed and $H \subseteq P \subseteq N\beta cl(H)$, then P is nano $\pi g\beta$-closed.

Proof. Let H be nano $\pi g\beta$-closed and $P \subseteq G$, where G is nano π-open. Then $H \subseteq P$ implies $H \subseteq G$. Since H is nano $\pi g\beta$-closed, $N\beta cl(H) \subseteq G$. Also $P \subseteq N\beta cl(H)$ implies $N\beta cl(P) \subseteq N\beta cl(H)$. Therefore $N\beta cl(P) \subseteq G$ and hence P is nano $\pi g\beta$-closed. ■

References

[1] Adinatha C. Upadhya, *On quasi nano p-normal spaces*, International Journal of Recent Scientific Research, 8(6)2017, 17748–17751.

[2] M. Lellis Thivagar and Carmel Richard, *On Nano forms of weakly open sets*, International Journal of Mathematics and Statistics Invention, 1(1)2013, 31–37.

[3] Z. Pawlak, *Rough sets*, International journal of computer and Information Sciences, 11(5)(1982), 341–356.

[4] A. Revathy and G. Ilango, *On nano β-open sets*, Int. Jr. of Engineering, Contemporary Mathematics and Sciences, 1(2)2015, 1–6.

[5] I. Rajasekaran and O. Nethaji, *On some new subsets of nano topological spaces*, Journal of New Theory, 16(2017), 52–58.

[6] I. Rajasekaran T. Siva Subramania Raja and O. Nethaji, *On nano πgp-closed sets*, Journal of New Theory, 19 (2017), 20–26.

[7] I. Rajasekaran and O. Nethaji, *On nano πgs-closed sets*, Journal of New Theory, 19 (2017), 56–62.

[8] I. Rajasekaran, M. Meharin and O. Nethaji, *On nano $g\beta$-closed sets*, International Journal of Mathematics And its Applications, 5(4-C) (2017), 377–382.