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Capacity Analysis based on Graph Theory for VANETs

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Abstract

Vehicular ad hoc networks (VANETs) provide an efficient and safe traffic system which are organized along the roads. In this paper, we propose an innovative method which gives a clear guidance in analyzing the capacity on comparing with the existing theoretical results. The geometrical structure of an urban area is constructed from any real map of a metropolitan zone. An Euclidean planar graph is constructed from the map which extracts an interference link graph. This graph considers the transmission interference relation between the nodes that are connected in the network. The asymptotic capacity of the metropolitan zone VANETs are calculated on comparison with the proximity of vehicles.

Keywords: Euclidean, interference, VANET.

I. INTRODUCTION

Vehicular Ad hoc Networks (VANETs) technology is an important research area over the past few years. They are a special type of Mobile Ad hoc Networks (MANETs) in which the vehicles that are connected through a wireless network travelling on the road immediately forms a network [1]. This mechanism provides a safe and an efficient transportation system. VANETs have many challenges such as security, transmission range, capacity and privacy. Among the others factors we consider capacity as an important and basic property of VANETs. It is really a challenging task to regulate the capacity of distributed wireless networks. We have proposed some statistical and probabilistic methods to calculate and control the capacity of VANETs [2].

Software based VANETs (S-VANETs) have been introduced to improve the performance of capacity analysis [3]. It works in a centralized manner and is more efficient than VANETs. Though it has many advantages, it has some issues to consider and they are as follows.

- Since all the vehicles move along the roads, determining the real map of a metropolitan zone affects the capacity of S-VANETs.
- As vehicles can be either completely regular or random, mobility of vehicles can be characterized in a statistical way alone.
- Since different roads have different geometrical structure, a unique model cannot be used for all the metropolitan zones.

Pishro et al. [4] proposed a unique grid like structure as shown in Figure.1 to depict all the roads of metropolitan zones. It has x vertical lines intersected with y horizontal lines and has a grid like view. Lu et.al [5] extended this work by providing a real map of a metropolitan zone which has different shapes and densities of roads.

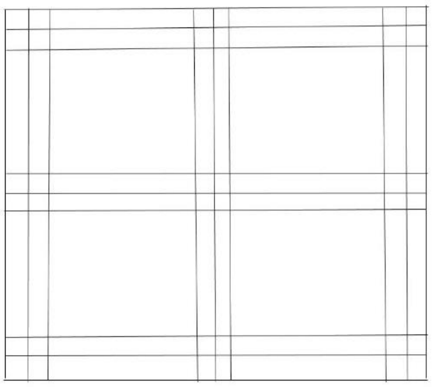

Figure 1: Grid like structure of roads in a metropolitan zone

Figure 2. Real map of a metropolitan zone

We propose a new framework named WVM (Wireless Vehicular Model) that is created by using a Euclidean Planar (EP) graph and an Interference Link (IL) graph [6]. WVM is based on the real world map and has all the geometrical structures and properties present in a real map. We use graph theory to analyze WVM in an efficient manner. Using the WVM, we estimate the throughput capacity of the proximity of vehicles in VANET to attain Θ (1/v) in lightly loaded areas of vehicles and a constant capacity in heavily loaded area of vehicles. Our contribution is summarized as follows:

- We propose a new WVM by using a Euclidean planar graph and an interference link graph. The EP graph can be extracted from the real map of a metropolitan zone. As the normal grid like structure do not provide good results due to the non-uniform nature of roads and vehicles, our approach provides accurate results in estimating the asymptotic capacity.
- The IL graph is extracted from the EP graph based on the interference between nodes in the network. We use graph theory to determine the interference links for calculating the transmission flows which is needed for analyzing the asymptotic capacity in the network.
- We use a two-hop method for calculating the asymptotic capacity and prove that a constant capacity could be achieved in highly loaded area of vehicles and can attain $\Theta\ (1/v)$ in lightly loaded areas of vehicles. The rest of the paper is organized as follows. Section II reviews the related works. Section III introduces the network model, capacity related definitions and some known theorems. Section IV analyzes the capacity of VANETs. Section V concludes the paper with some future works.

II. RELATED WORK

The throughput capacity of each node in wireless networks was observed to be $\Theta\ \left(\frac{T}{\sqrt{d\ log\ d}}\right)$ bits per second for any destination that is chosen randomly. This work was done by Gupta *et al.* in 2000 [7]. It was extended for the unicast as well as multicast broadcast. Grossglauser *et al.* found that the throughput of each node will increase when it is mobile on comparing with fixed nodes [8]. The main drawback is the large end to end delay experienced in networks. Works were extended for the analysis of capacity in energy constrained networks also. It was also investigated for the network capacity of randomly deployed networks and non-homogeneous networks for improvement.

Pishro – Nik analyzed the capacity of VANETs by using a grid like construction in which l horizontal and l vertical lines intersect with each other to form a grid like structure. As there are different road structures each differ from the other road in calculating the capacity bounds. Lu $et.al.$ used the geometrical structure of roads of a metropolitan zone. Initially they focused on a fixed density of vehicles with a grid like streets and vehicles. As the number of roads increase based on the vehicle count it was observed that the average throughput of each vehicle is $\Omega\left(\frac{1}{log(d)}\right)$ and experienced a fixed delay of $O\ (log^2(d))$ with maximum probability.

Alfano $et\ al.$ made his research work by considering each node in a restricted mobile zone from its starting point and found that the spatial distribution of nodes have an exponential decay ∂ [9]. For different values of ∂, the delay experienced in throughput was observed and concluded that when $\partial = 2$, the delay and throughput remains constant.

III SYSTEM MODEL

A. Definitions of Capacity

We define capacity as the possible throughput obtained in VANETs and is defined as follows.

$Definition$ 1 $(capacity\ in\ terms\ of\ a\ vehicular\ network)$[10]:

The average capacity of VANET is in the order of $\theta\ (r(d))$ bits / second if there are deterministic constants $e > 0$ and $e < e' < +\infty$ such that

$$\lim_{n\to\infty} P(\alpha(n) = c(g(n))\ = 1\ is\ possible)$$

$$\lim_{n\to\infty} inf\ P(\alpha(n) = c(g(n)) < 1\ is\ possible)$$

$Definition$ 2 $(capacity\ in\ terms\ of\ throughput)$ [10]

Let the number of packets received by all the vehicles at time t be $C\ (t)$. Capacity throughput in a vehicular network is possible if the vehicles are scheduled in a proper order[11]. It should hold the following condition:

$$\lim_{t\to\infty} P\left(\frac{C(t)}{t} \geq \alpha\right) = 1$$

B. Network Model

The grid based network is appropriate because of its restricted normalized structure, where we use a novel network model constructed by an EP graph and IL graph. For constructing the model we are using the real map of a metropolitan zone as shown in Figure 1. Each intersection in the map is considered to be a vertex with diameter m as a component with a transmission range g of 300 meters. When vehicles are away from this transmission range, they are obviously out of coverage area and the wireless communication medium cannot communicate with the vehicles. So, when any two adjacent vertices that are 300 meters away they are covered by components in the graph.

These components are represented by vertices in the EP graph in accordance with the coordinate position in the real map. An edge is placed if there is a road between any two components. The entire area is considered to be A with the perimeter E. We understand that all the components are arbitrarily distributed and the connection between any two adjacent components are also random for obtaining an arbitrary WVM. In Figure 2, we consider every crossing point to be a center and draw a sphere with a diameter m. A component is one that has a road covered by a sphere.

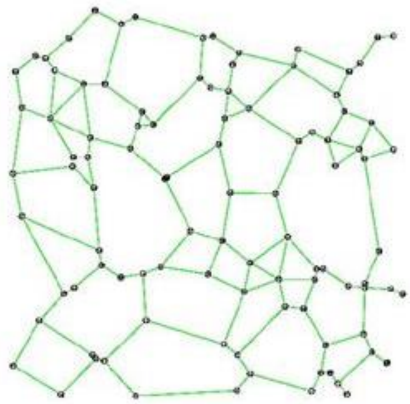

Figure 3. Euclidean planar graph

All the components in a component set $= \{c_1, c_2, \ldots., c_d\}$. Vertices in the EP graph are the components in a real map according to its position. When an edge is introduced between any two components in the real map, we can derive the EP graph. We choose any region EP_R from the acquired EP graph as an arbitrary Euclidean planar graph as shown in Figure3. All the components represented by the vertices constitute the set $C_R = \{c_1, c_2, \ldots., c_{N_C}\}$ where N_C represents the number of components in C_R.

C. Mobile Model

We use the probability density function to denote the non-uniform nature of the density of vehicles [12]. It may not be uniform due to their movement in confined regions. Since VANETs have social vicinity properties, we use the constrained mobility model to indicate its social vicinity traffic. Each vehicle chooses a component in C_R in a uniform manner which is centered at an initial point. This is called as partial area that does not overlay with one another.

Let $L_v(t)$ denote the location of a vehicle v at time t and $L_v^f(t)$ denote the the location of the initial point of a vehicle v at time t. The Euclidean distance between vehicle v and its initial point at time t is defined by $\varepsilon_i = \| L_v(t) - L_v^f(t) \|$. The spatial distribution of nodes can be represented by using $\Omega(s)$ in terms of distance s from the initial point and assume that $\Omega(s)$ decays exponentially. i.e., $\Omega(s) \sim s^{-\partial}$ with $\partial > 0$. To derive the probability density function, we introduce a function $x(s) = \min\left(1, s^{-\partial}\right)$. Therefore, $\Omega(s) = \frac{(x(s))}{\iint (x(s))}$, where $\partial > 0$ denotes a uniform spatial distribution.

D. Interference Model

A vehicle cannot transmit packets to more than a vehicle at the same time slot because of the intervention of wireless communication medium. We use the protocol interference model to denote the nature of MAC protocol. The model is defined as follows:

The transmission from vehicle a to b will be successful in a time slot if:

i) $\| L_a(t) - L_b(t) \| \leq g$

If any other vehicle z tries to transmit at the same time slot,

ii) $\| L_z(t) - L_b(t) \| \geq (1 + \rho) g$

in which ρ is a sentinel for defining a secure zone around the receivers.

E. Transmission Model

There are f transmission flows in the network simultaneously because each vehicle will be the source of one transmission flow and the destination of another transmission flow. A source vehicle can relay packets to the destination vehicle directly if the transmission flow between them belong to the same initial point. If they do not belong to a dissimilar initial point, the source vehicle will relay packets through an intermediary vehicle which in turn transmits to the destination vehicle.

F. Known Results

We use the Groemer Inequality and Borel's law of large numbers to analyze capacity in an efficient manner. The results are as follows.

Lemma 1 (*Borel's law of large numbers*) [13]: Let $N(v)$ represent the number of times an event v occurs in x number of trials and p is the probability that v occurs. For any positive integer i we have,

$$\lim_{x \to \infty} P\left\{\left|\frac{N(v)}{x} - p\right| < i\right\} = 1$$

Lemma 2 (Groemer Inequality) [14]: Let X be a convex set and C is a set of points with distance between them to be at least one. Then,

$$|C \cap X| \leq \frac{area\ (X)}{\sqrt{3}/2} + \frac{peri\ (X)}{2} + 1$$

where $area\ (X)$ and $peri\ (X)$ denote the area and perimeter of X respectively.

IV. ANALYSIS OF CAPACITY IN VANETs

A. $\boldsymbol{Maximum\ Number\ of\ Simultaneous\ Flows}$

To analyze the wireless transmission under IL graph, we introduce the maximum independent set and maximum independent number. They are defined as follows.

$Definition$ 3 (maximum autonomous set): An autonomous set of a IL graph is a set of non-contiguous vertices and a maximum autonomous set is the largest autonomous set for a given graph. [15]

$Definition$ 4 (maximum autonomous number): The maximum autonomous number of a graph is the maximum size of a maximum autonomous set. [15]

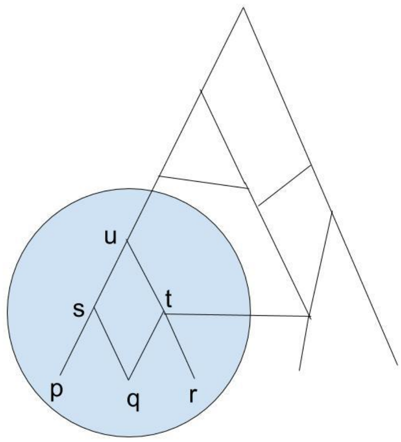

Figure 4. Interference Link Graph

An interference link graph has vertices, each to be considered as a distinctive component. We say that two vertices y and z are adjacent and have interference if there is an edge between y and z. Consider the IL graph in Figure 4. Vertices p, q, r, s, t, u are the vertices in the graph and two vertices cannot transmit packets at the same time. According to definition 4, vertices $p, q, r\ , u$ constitute an autonomous set S_1 and vertices s, t constitute an autonomous set S_2.

Thus, vertices $p, q, r\ , u$ cannot transmit packets at the same time and vertices s, t as well. Also, when vertices in the set S_1 cannot transmit when the vertices in the set S_2 is transmitting. The IL graph shows that in the maximum autonomous set S_1 , at most 4 components can transmit without interference with the other vertices. Using a greedy algorithm, we can easily attain a maximum autonomous number [16]. From Lemma 1, we introduce the following corollary for a random IL graph.

$Corollary$ 1: In a square with area A and perimeter E, assume that X is a compact convex set and C is a set of points with mutual distances at least $(1+\partial)m$. Then,

$$|C \cap X| \leq 1+\frac{E}{2(1+\partial)m}+\frac{A}{\sqrt{3}/2[(1+\partial)m]^2}$$

$Proof$: We scale down the EP graph with the proportion $(1+\partial)m$. The distance between each pair of elements of autonomous sets is greater than $(1+\partial)m$ in the original Euclidean planar graph EP. In the scaled down Euclidean planar graph EP', the distance between each pair of elements of autonomous sets is greater than 1. This scales down for the area and perimeter of the EP graph too which is denoted by A' and L' respectively. Therefore,

$$A' = \frac{A}{[(1+\partial)m]^2}$$

$$L' = \frac{A}{(1+\partial)m}$$

This shows that the scaled down Euclidean planar graph EP' fulfills Lemma 1 and the original Euclidean planar graph EP fulfills Corollary 1. Another Lemma can be derived based on Corollary 1.

$Lemma$ 3: In a rectangular area with length of the side as L, the number of simultaneous flow of packet transmissions F fulfills the following.

$$1+\frac{E}{2(1+\partial)m}+\frac{A}{\sqrt{3}/2[(1+\partial)m]^2} \geq F \geq 1$$

$B. Capacity\ Bounds$

Based on Lemma 3, we derive the upper bound [17] of the throughput capacity of VANETs using the protocol interference model.

$Theorem$ 1: The average throughput of VANETs with the two – hop transmission scheme cannot be enhanced than

$$\frac{1+\frac{E}{2(1+\partial)m}+\frac{A}{\sqrt{3}/2[(1+\partial)m]^2}}{n} \geq \alpha(n)$$

$Proof$ 2: Let $N_d(t)$ be the total number of packets transmitted from source to destination vehicle through direct mode of transmission in the time interval $[0,t]$ and $N_r(t)$ be the total number of packets transmitted from source to destination vehicle through relay mode of transmission in the time interval $[0,t]$. As per Definition 2, throughput $\alpha(n)$ satisfies the following:

$$\frac{N_d(t) + N_r(t)}{t} \geq n\alpha(n) - i \tag{1}$$

where $i > 0$ and is a fixed arbitrary number, $i \to 0$ as $t \to \infty$. Let $O(t)$ denote the number of packet transmitting opportunities during the time interval $[0, t]$. $O(t)$ should be greater than the total number of packets transmitted for a maximum time. As the relay mode of transmission needs double the time of packet transmitting opportunities, we have

$$\frac{1}{t} O(t) \geq \frac{1}{t} N_d(t) + \frac{2}{t} N_r(t) \tag{2}$$

When $i \to 0$ and $t \to \infty$ and on substituting (1) in (2), we get

$$\alpha(n) \leq \frac{\frac{1}{t} O(t) + \frac{1}{t} N_d(t)}{2n} \tag{3}$$

The number of simultaneous transmissions must be greater than the total number of packet transmissions during the time interval$[0, t]$. As per Lemma 1, we have

$$\lim_{x \to t} \frac{1}{t} O(t) \leq F \tag{4}$$

We also have

$$\lim_{x \to t} \frac{1}{t} N_d(t) \leq F \tag{5}$$

On substituting (4) and (5) in (3), we derive

$$\alpha(n) \leq \frac{F}{n} \tag{6}$$

Substitute the value of F in (6), we get

$$\frac{1 + \frac{E}{2(1 + \partial)m} + \frac{A}{\sqrt{3}/2[(1 + \partial)m]^2}}{n} \geq \alpha(n)$$

Therefore, from Theorem 1, we are able to prove that the throughput of each vehicle is feasible to attain $\Theta\ (1/v)$. The traffic cannot go boundless with the increase in the number of vehicles and it increases based on the asymptotic bound of $\Theta\ (1/v)$ [18].

$Lemma$ 4: Let N_s denote the number of vehicles that belong to the same zone. It increases with high probability of $\Theta\ (v)$. To prove this Lemma we use the Borel's law of large numbers.

$Proof$ 3: Let $\frac{1}{N_v}$ denote the probability that a vehicle belong to the same zone. As per Lemma 2 with e as a positive integer, we have

$$\lim_{x\to\infty} P\left\{\left|\frac{N_s}{x} - \frac{1}{N_v}\right| < e\right\} = 1$$

Therefore, $$\lim_{n\to\infty}\left\{N_s < x\left(e + \tfrac{1}{N_v}\right)\right\} = 1$$

As per the above Lemma, we conclude that the number of vehicles that belong to a specified zone cannot exceed $\Theta\ (v)$ and transmission of packets between vehicles will be shared by at most $\Theta\ (v)$ vehicles.

$Theorem$ 2: The most probable throughput capacity $\alpha(n)$ can be within $\Theta\ (1/v)$ and cannot increase above this range. Thus the capacity of VANET is constant according to the derived capacity.

V.CONCLUSION

In this paper, we have analyzed the capacity of the proximity of metropolitan zone vehicular networks. A new method was proposed with the Euclidean planar graph representing the components and the interference link graph represents the link between components. The autonomous set is used in order to find the interference link in the IL graph. We proved that the asymptotic capacity of zones with lightly loaded vehicles is limited by $\Theta\ (1/v)$ and a constant throughput capacity can be attained at zones with heavily loaded vehicles. When inference of vehicles is complex, we can use a model that could give us accurate results such as Gauss model. Also, delay is a major feature to be analyzed which is not considered in this paper. It can be extended as our future work in analyzing the delay which could be experienced in the throughput of VANETs. Thus, our paper proves that VANETs can be scaled up to be deployed in metropolitan zones.

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