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Weak Controllability and the New Choice of Actuators

Ines Kaarer
University of Constantine 1, Algeria.

Abdelhamid Ayadi and Imad Rezzoug
Laboratory of dynamical system and control.
University of Larbi Ben M'hidi, Oum El Bouaghi, Algeria.

Abstract

The weak controllability of a parabolic system has been developed by many researchers, in particular, the notion of actuators to ensure this weak controllability. In this paper, our objective is to choose new actuators that allow us to simplify the conditions of the weak controllability of Al Jai. The developing idea consists in a reduction of conditions introduced in [4] in such a way that weak controllability properties are reserved.

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1. Introduction

The controlled system arising in applied engineering, physics, medicine, etc., must often be considered with distributed parameters, being governed typically by partial differential equations. Tools used for the optimal control of distributed parameter systems vary from the purely theoretical to mathematical analysis and the theory of partial differential equations. A fundamental class of optimal controls and its mathematical approaches can be found in Lions [10].

In [[1], [2], [3], [4], [5], [8]], they study the linear optimal control problem for systems described by different types of partial differential operator ($n \times n$ matrix operators) defined on spaces of functions of an infinite number of variables. To obtain the weak controllability conditions, the arguments of El Jai [4] have been applied.

Using the condition in [4], they obtained necessary and sufficient conditions of optimality for the parabolic type equation. The questions treated in this paper related to the above results, but in a different direction, by a choice of actuators that allow us to simplify the hypotheses of El Jai theorem.

The outline of this paper is as follows:
In Section 2, we formulate the parabolic problem and its solution which is defined by the semi-group. In Section 3, the El Jai's basic theorem [4] is cited to ensure the weak controllability. Our choice of actuators that lead to the simplification of El Jai's [4] hypotheses is given in Section 4.

2. Mathematical Formulation of the Problem

Let Ω be a non-empty open bounded set of $\mathbb{R}^n$ ($n = 1, 2, 3$ in applications) with sufficiently smooth boundary Γ.

Denote by $\mathcal{Q} = \Omega\times \,]0, T[$, $\Sigma = \Gamma \times \,]0, T[$ and $\langle ., . \rangle$ the scalar product in L^2. Let $y(x, t; u)$ be the solution of the system

$$\begin{cases} \dfrac{\partial y}{\partial t} + Ay &= Bu \quad \text{in} \quad \mathcal{Q} \\ y\,(0) &= 0 \quad \text{in} \quad \Omega \\ y &= 0 \quad \text{on} \quad \Sigma \end{cases} \tag{1}$$

where A generates a strongly continuous semi-group $(\mathcal{S}\,(t))_{t\geq 0}$ on the state space $X = L^2\,(\Omega)$, $B \in \mathcal{L}(U, X)$ and $u \in L^2\,(0, T; U)$, $U = \mathbb{R}^p$ the controls space. The operator A provides the dynamics of the system whereas the operator B informs us about the nature of the actuators that excites the system and their localizations.

Denote by $y_u(t)$ the solution of the system (1).

If A is self-adjoint with compact resolving, then (1) admits a unique weak solution on $[0, T]$ which is given by (see [5]):

$$y_u(t) = \mathcal{S}(t)y(0) + \int_0^t \mathcal{S}(t-s)Bu(s)ds = \int_0^t \mathcal{S}(t-s)Bu(s)ds.$$

.

Consider $H : L^2\,(0, T; \mathbb{R}^p) \to X$ the operator defined by:

$$Hu = \int_0^t \mathcal{S}(t-s)Bu(s)ds, \;\; \forall u \in L^2\,(0, T; \mathbb{R}^p)\,.$$

And the adjoint operator H^* of H defined by:

$$H^*z^* = B^*\mathcal{S}^*(T - .)z^*, \forall z^* \in X^*.$$

Now, we are interested in choosing the operator B.

3. WEAK CONTROLLABILITY AND STRATEGIC ACTUATORS

Let Ω_i, $(i = \overline{1,p})$ be a closed set contained in Ω and $g_i \in L^2(\Omega_i)$, the couple (Ω_i, g_i) called actuator.

The system (1) is called weakly controllable in X on $[0,T]$ if for all y_d in X, $\forall\varepsilon > 0, \exists u \in L^2(0,T;U)$ we have

$$\|y_u(T) - y_d\|_X \leq \varepsilon.$$

A sequence of actuators $(\Omega_i, g_i)_{i=\overline{1,p}}$ is called strategic if the existing system is weakly controllable on Ω.

We suppose that the system (1) possesses a complete basis of Eigen functions

$$(\varphi_{nj})_{n\geq 1, j=\overline{1,r_n}} \text{ of } L^2(\Omega).$$

And (λ_n) the associated eigenvalues with multiplicity of r_n.

Thus, the semi-group $(\mathcal{S}(t))_{t\geq 0}$ is presented as follows (see [11]):

$$\mathcal{S}(t) = \sum_{n=1}^{\infty} e^{\lambda_n t} \sum_{j=1}^{r_n} \langle ., \varphi_{nj} \rangle \varphi_{nj}. \tag{2}$$

Under these hypotheses, we have the following characterization result proved by El Jai (see [4]).

Suppose that $r = \sup r_i < \infty$. Then, the system (1) is weakly controllable on Ω if and only if

1- $p \geq r$,

2- $rang\ G_n = r_n, \forall n \geq 1$,

Where

$$G_n = (G_n)_{i,j}, 1 \leq i \leq p, 1 \leq j \leq r_n$$

Is defined by:

$$(G_n)_{i,j} = \langle \varphi_{nj}, g_i \rangle_{\Omega_i}.$$

4. NEW CHOOSE OF ACTUATORS

In this section, we are interested in a particular choice of actuators, (g_k) in such a way that the second hypotheses of the precedent theorem is verified, given by:

$$g_k = \sum_{n=1}^{\infty} \sum_{j=1}^{r_n} \alpha_{nj}^{k-1} \varphi_{nj}, k = \overline{1,p}, 0 \leq \alpha_{nj} \leq 1,$$

This leads to the following result:

Suppose that $\sup r_i = r < \infty$, then the system (1) is weakly controllable on Ω if and only if

$$p \geq r$$

where $G_n = (G_n)_{k,j}$, $1 \leq k \leq p, 1 \leq j \leq r_n$ is defined by:

$$(G_n)_{k,j} = \langle \varphi_{nj}, g_k \rangle_{\Omega_k}.$$

.

Proof. The system (1) is weakly controllable if and only if $\ker H^* z^* = \{0\}$
We have

$$\ker H^* z^* = \{0\} \Leftrightarrow \sum_{n=1}^{\infty} e^{\lambda_n (T-t)} \sum_{j=1}^{r_n} \langle \varphi_{nj}, g_k \rangle_{\Omega_k} \langle \varphi_{nj}, z^* \rangle = 0$$

Then

$$\sum_{j=1}^{r_n} \langle \varphi_{nj}, g_k \rangle_{\Omega_k} \langle \varphi_{nj}, z^* \rangle = 0 \;\; k = \overline{1,p} \; , \; \forall n.$$

Since $\{e^{\lambda_n}\}$ forms a basis in $L^2(\Omega)$
For

$$g_k = \sum_{n=1}^{\infty} \sum_{j=1}^{r_n} \alpha_{nj}^{k-1} \varphi_{nj} \; , k = \overline{1,p} \, , 0 \leq \alpha_{nj} \leq 1$$

We get

$$\sum_{j=1}^{r_n} \langle \varphi_{nj}, \sum_{n=1}^{\infty} \sum_{j=1}^{r_n} \alpha_{nj}^{k-1} \varphi_{nj} \rangle_{\Omega_k} \langle \varphi_{nj}, z^* \rangle = 0 \; , \; k = \overline{1,p} \, , 0 \leq \alpha_{nj} \leq 1, \; \forall n.$$

Therefore

$$\sum_{j=1}^{r_n} \alpha_{nj}^{k-1} \langle \varphi_{nj}, z^* \rangle = 0 \;\; , k = \overline{1,p} \; , \; 0 \leq \alpha_{nj} \leq 1, \; \forall n.$$

For $k = 1$

$$\langle \varphi_{n1}, z^* \rangle + \langle \varphi_{n2}, z^* \rangle + ... + \langle \varphi_{nr_n}, z^* \rangle = 0$$

For $k = 2$

$$\alpha_{n1}^1 \langle \varphi_{n1}, z^* \rangle + \alpha_{n2}^1 \langle \varphi_{n2}, z^* \rangle + ... + \alpha_{nr_n}^1 \langle \varphi_{nr_n}, z^* \rangle = 0$$

For $k = p$

$$\alpha_{n1}^{p-1} \langle \varphi_{n1}, z^* \rangle + \alpha_{n2}^{p-1} \langle \varphi_{n2}, z^* \rangle + ... + \alpha_{nr_n}^{p-1} \langle \varphi_{nr_n}, z^* \rangle = 0$$

Which leads to the following system

$$\begin{pmatrix} 1 & 1 & 1 \\ \alpha_{n1} & \alpha_{n2} & \alpha_{nr_n} \\ . & & \\ . & & \\ \alpha_{n1}^{p-1} & \alpha_{n2}^{p-1} & \alpha_{nr_n}^{p-1} \end{pmatrix} \begin{pmatrix} \langle \varphi_{n1}, z^* \rangle \\ \langle \varphi_{n2}, z^* \rangle \\ \\ \\ \langle \varphi_{nr_n}, z^* \rangle \end{pmatrix} = 0$$

In terms of matrices, we have

$$AX = 0$$

Where A is a $P \times r_n$ matrix containing polynôms of degree P. We know that the matrix vectors are linearly independent, then

$$X = 0 \Longleftrightarrow \begin{pmatrix} \langle \varphi_{n1}, z^* \rangle \\ \langle \varphi_{n2}, z^* \rangle \\ \\ \\ \langle \varphi_{nr_n}, z^* \rangle \end{pmatrix} = 0, \ \forall n$$

So, z^* is orthogonal to the basis (φ_{nr_n}). This requires

$$z^* = 0$$

Therefore

$$\ker H^* z^* = \{0\}.$$

Which completes the proof. □

5. Conclusion

In this paper, a new weak controllability result is obtained by simplifying the conditions of the weak controllability of Al Jai the developing idea consists in a reduction of conditions introduced in [4] in such a way that weak controllability properties are reserved.

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