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Decompositions of (μ_m, λ)-Continuity

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Abstract

In this paper, we introduce and study the notions of $D(\mu, \alpha)$-sets, $D(\mu, \sigma)$-sets, $D(\alpha, \sigma)$-sets, $D_m(\mu, \alpha)$-sets, $D_m(\mu, \sigma)$-sets and $D_m(\alpha, \sigma)$-sets. Also we obtain decompositions of (μ_m, λ)-continuity.

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1. Introduction

In 2002, Csaszar introduced the notions of generalized topology and generalized continuity. Let X be a nonempty set and μ be a collection of subsets of X. Then μ is called a *generalized topology* [1] (briefly GT) on X, if $\emptyset \in \mu$ and an arbitrary union of elements of μ belongs to μ. The μ-interior of a subset A of (X, μ) is denoted by $i_\mu(A)$, is the union of μ-open sets contained in A. The μ-closure $c_\mu(A)$ is the smallest μ-closed subset of X containing A. A subset A of a space (X,μ) is *μ-α-open* [2] (resp. *μ-semi-open [2], μ-pre-open [2], μ-β-open [2]*), if $A \subset i_\mu c_\mu i_\mu(A)$ (resp. $A \subset c_\mu i_\mu(A)$, $A \subset i_\mu c_\mu(A)$, $A \subset c_\mu i_\mu c_\mu(A)$). Let us denote by $\alpha(\mu)$ the class of all μ-α-open sets, $\sigma(\mu)$ by that of all μ-semi-open sets, $\pi(\mu)$ by that of all μ-pre-open sets, $\beta(\mu)$ by that of all μ-β-open sets. The *μ-α-interior* [2] (resp. *μ-semi-interior* [2]) of a subset A of a generalized topological space (X,μ) denote by $i_\alpha A$ (resp. $i_\sigma(A)$) is defined by the union of all μ-α-open (resp. μ-semi-open) sets of (X,μ) contained A. Let (X,μ) and (Y,λ) be GTS's. Then a function $f:(X,\mu)\to(Y,\lambda)$ is *(μ,λ)-continuous* [1] (resp. *(α,λ)-continuous* [3], *(σ,λ)-continuous* [3]), if for each λ-open set U in Y, $f^{-1}(U)$ is μ-open (resp. μ-α-open, μ-semi-open) in (X,μ). Throughout this paper (X,μ) and (Y,λ) denote generalized topological spaces. Let X be a non empty set and g_X be a generalized topology and m_X a minimal structure on X. A triple (X,g_X,m_X) is called a *generalized topology and minimal structure space (briefly $GTMS$ space).* Let (X,g_X,m_X) be a generalized topology and minimal structure and A a subset of X. The closure and interior of A in g_X are denote by g_X-$Cl(A)$ and g_X-$Int(A)$, respectively. And the closure and interior of A in m_X are denoted by $m_X - Cl(A)$ and $m_X - Int(A)$, respectively. In this paper (X,μ,m) (resp. $i_m(A)$) denote generalized topology and minimal structure space (resp. m_X-$Int(A)$). The element of (X,μ,m) are called m_μ-open sets. $A \subset X$ is m_μ-α-open [7] (resp. m_μ-semi-open [7], m_μ-pre-open, [7] m_μ-β-open) [7], iff $A \subset i_m(c_\mu(i_m(A)))$ (resp. $A \subset c_\mu(i_m(A))$, $A \subset i_m(c_\mu(A))$, $A \subset c_\mu(i_m(c_\mu(A)))$). A function $f:(X,\mu,m)\to(Y,\lambda)$ is said to be (μ_m,λ) [7] (resp. (α_m,λ)-continuous [7],(σ_m,λ)-continuous [7], (π_m,λ)-continuous [7], (β_m,λ)-continuous) [7], if for each λ-open set U in (Y,λ), $f^{-1}(U)$ is m_μ-open (resp. m_μ-α-open, m_μ-semi-open, m_μ-pre-open, m_μ-β-open) set in (X,μ,m). In this paper the minimal structure m is closed under arbitrary union.

Lemma 1.1. [2], Lemma 2.2 Let (X,μ) be a generalized topological spaces. For any $A \subset X$, we have

1. $i_\sigma(A) = A \cap c_\mu i_\mu(A)$
2. $i_\alpha(A) = A \cap i_\mu c_\mu i_\mu(A)$.

Lemma 1.2. [2], Theorem 2.1 For a generalized topology μ on X, we have $\mu \subset \alpha(\mu) \subset \sigma(\mu) \subset \beta(\mu)$.

Proposition 1.3. Let (X,μ,m) be generalized topology and minimal structure space, where m is closed under union. Then

1. $A \subset B \subset X$ implies $i_m(A) \subset i_m(B)$,
2. $i_m(A) \subset A$,
3. $i_m(i_m(A)) = i_m(A)$,
4. $i_m(A) = A$, if $A \in m$,

5. A is m-closed if and only if $c_m(A) = A$.

Proof.

1. (1) and (2) are evident.
3. By (2) $i_m i_m(A) \subset i_m(A)$. On the other hand if $L \in m$ and $L \subset A$ then $L \subset i_m(A)$ by definition so that $L \subset i_m i_m(A)$ by definition again, consequently $i_m(A) \subset i_m i_m(A)$ and $i_m(A) = i_m i_m A$.
4. If $A \in m$, then $i_m(A) = \bigcup\{F : F \subset m, F \subseteq A\} = A$. Conversely, since m is closed under arbitrary union, then $i_m(A) \in m$. It follows that $A \in m$.
5. If A is m-closed set, then $X - A \in m$. By definition of i_m, $i_m(X - A) = X - A$, $i_m(X - A) = X - c_m(A)$. In consequence $c_m(A) = A$.

2. The class of $D(\mu, \alpha)$-sets and related sets

Definition 2.1. For a generalized topological space (X, μ), we denote:

1. $D(\mu, \sigma)$= $\{A \subseteq X : i_\mu(A) = i_\sigma(A)\}$,
2. $D(\mu, \alpha)$=$\{A \subseteq X : i_\mu(A) = i_\alpha(A)\}$,
3. $D(\alpha, \sigma)$= $\{A \subseteq X : i_\alpha(A) = i_\sigma(A)\}$.

Theorem 2.2. In (X, μ), the following holds:

1. Every $D(\mu, \sigma)$-set is $D(\mu, \alpha)$-set, but not conversely.
2. Every μ-α-open set is $D(\alpha, \sigma)$-set but not conversely.

Proof.

1. Let A be a $D(\mu, \sigma)$-set. Then $i_\mu(A) = i_\sigma(A) = A \cap c_\mu i_\mu(A)$. Now, $i_\alpha(A) = A \cap i_\mu c_\mu i_\mu(A) \subseteq A \cap c_\mu i_\mu(A) = i_\sigma(A) = i_\mu(A)$. Also, $i_\mu(A) \subseteq A \cap i_\mu c_\mu i_\mu(A) = i_\alpha(A)$. Hence $i_\mu(A) = i_\alpha(A)$ and A is a $D(\mu, \alpha)$-set.
2. Let A be a μ-α-open set. Then $A = i_\alpha(A)$ and $A = i_\sigma(A)$, since $\alpha(\mu) \subset \sigma(\mu)$ by Lemma 1.2. Therefore $i_\alpha(A) = i_\sigma(A)$. Hence A is $D(\alpha, \sigma)$-set.

Example 2.3. Let $X = \{a, b, c\}$, $\mu = \{\emptyset, \{a\}, \{a, b\}\}$. Then $A = \{c\}$ is $D(\mu, \alpha)$-set but not $D(\mu, \sigma)$-set.

Example 2.4. Let $X = \{a, b, c\}$, $\mu = \{\emptyset, \{a, b\}, \{b, c\}, X\}$. Then $A = \{a, c\}$ is $D(\alpha, \sigma)$-set but not μ-α-open set.

Remark 2.5. The following example show that in (X, μ), the intersection (resp. union) of two $D(\mu, \alpha)$-sets (resp. $D(\mu, \sigma)$-sets, $D(\alpha, \sigma)$-sets) need not be $D(\mu, \alpha)$-set (resp. $D(\mu, \sigma)$-set, $D(\alpha, \sigma)$-set).

Example 2.6. Let $X = \{a, b, c, d, e\}$, $\mu = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}, \{a, c, d\}, \{b, c, d\}, \{a, b, c, d\}\}$. Then $A = \{a, c, d\}$ and $B = \{b, c, d\}$ are $D(\mu, \sigma)$-sets and $D(\alpha, \sigma)$-sets but $A \cap B = \{c, d\}$ is neither $D(\mu, \sigma)$-set nor $D(\alpha, \sigma)$-set.

Example 2.7. Let $X = \{a, b, c, d, e\}$, $\mu = \{\emptyset, \{a\}, \{d\}, \{a, d\}, \{d, e\}, \{b, c, e\}, \{a, c, e\}, \{a, d, e\}, \{a, b, c, e\}, \{b, c, d, e\}, \{a, c, d, e\}, X\}$. Then $A = \{a, c, d, e\}$ and $B = \{b, c, d, e\}$ are $D(\mu, \alpha)$-sets but $A \cap B = \{c, d, e\}$ is not a $D(\mu, \alpha)$-set.

Example 2.8. Let $X = \{a, b, c, d\}$, $\mu = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{b, c, d\}, \{a, c, d\}, X\}$. Then $A = \{a, b\}$ and $B = \{d\}$ are $D(\mu, \alpha)$-set and $D(\mu, \sigma)$-set but $A \cup B = \{a, b, d\}$ is neither $D(\mu, \alpha)$-set nor $D(\mu, \sigma)$-set.

Example 2.9. Let $X = \{a, b, c, d\}$, $\mu = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}, \{a, c, d\}, X\}$. Then $A = \{a, b\}$ and $B = \{d\}$ are $D(\alpha, \sigma)$-sets but $A \cup B = \{a, b, d\}$ is not a $D(\alpha, \sigma)$-set.

Theorem 2.10. Let (X, μ) be a generalized topological space. Then $\alpha(\mu) \cap D(\mu, \alpha) = \mu$.

Proof. Let $A \in \alpha(\mu) \cap D(\mu, \alpha)$. Then $A = i_\alpha(A)$, $i_\mu(A) = i_\alpha(A)$ and consequently $A \in \mu$. Conversely, if $A \in \mu$, then $A = i_\mu(A)$ and $i_\alpha(A)$ since $\mu \subset \alpha(\mu)$. Therefore $A \in \alpha(\mu) \cap D(\mu, \alpha)$. Hence $\alpha(\mu) \cap D(\mu, \alpha) = \mu$. ■

Theorem 2.11. Let A be a subset of (X, μ). Then the following conditions are equivalent:

1. A is μ-open,
2. A is μ-α-open and a $D(\mu, \sigma)$-set,
3. A is μ-semi-open and a $D(\mu, \sigma)$-set.

Proof. (1) $\Rightarrow$ (2) Let A be a μ-open set. Every μ-open set is μ-α-open set by Lemma 1.1. Since $A = i_\mu(A)$, then $A \cap c_\mu i_\mu(A) = A \cap c_\mu(A) = A = i_\mu(A)$. Therefore A is $D(\mu, \sigma)$-set.
(2) $\Rightarrow$ (3) is trivial by Lemma 1.2.
(3) $\Rightarrow$ (1) Let A is μ-semi-open and $D(\mu, \sigma)$-set. Then $A = i_\sigma(A)$ and $i_\mu(A) = i_\sigma(A)$. Therefore $A = i_\mu(A)$. Hence A is μ-open. ■

Theorem 2.12. Let (X, μ) be a generalized topological space. Then $\sigma(\mu) \cap D(\alpha, \sigma) = \alpha(\mu)$.

Proof. Let $A \in \sigma(\mu) \cap D(\alpha, \sigma)$. Then $A = i_\sigma(A)$, $i_\alpha(A) = i_\sigma(A)$ and consequently $A \in \alpha(\mu)$. Conversely, if $A \in \alpha(\mu)$, then $A = i_\alpha(A)$ and $A = i_\sigma(A)$, since $\alpha(\mu) \subset \sigma(\mu)$. Therefore $A \in \sigma(\mu) \cap D(\alpha, \sigma)$. Hence $\sigma(\mu) \cap D(\alpha, \sigma) = \alpha(\mu)$. ■

Remark 2.13.

1. The notions of μ-α-open sets and $D(\mu, \alpha)$-sets are independent,
2. The notions of μ-α-open sets and $D(\mu, \sigma)$-sets are independent,
3. The notions of μ-semi-open sets and $D(\mu, \sigma)$-sets are independent,
4. The notions of μ-semi-open sets and $D(\alpha, \sigma)$-sets are independent,
5. The notions of $D(\mu, \alpha)$-sets and $D(\alpha, \sigma)$-sets are independent.

Example 2.14. Let $X = \{a, b, c, d\}$, $\mu = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, b, c\}, \{b, c, d\}, X\}$. Then $A = \{a, b, d\}$ is μ-α-open but neither $D(\mu, \alpha)$-set nor $D(\mu, \sigma)$-set.

Example 2.15. Let $X = \{a, b, c\}$, $\mu = \{\emptyset, \{a\}, \{a, b\}\}$. Then $A = \{a, c\}$ is $D(\mu, \alpha)$-set but not μ-α-open, $B = \{b\}$ is $D(\mu, \sigma)$-set but not μ-α-open and $C = \{c\}$ is μ-semi-open but neither $D(\mu, \sigma)$-set nor $D(\alpha, \sigma)$-set.

Example 2.16. Let $X = \{a, b, c\}$, $\mu = \{\emptyset, \{a, b\}, \{b, c\}, X\}$. Then $A = \{a\}$ is $D(\mu, \sigma)$-set and $D(\alpha, \sigma)$-set but not μ-semi-open.

Example 2.17. Let $X = \{a, b, c\}$, $\mu = \{\emptyset, \{a\}, \{a, b\}\}$. Then $A = \{a, c\}$ is $D(\mu, \alpha)$-set but not $D(\alpha, \sigma)$-set.

Example 2.18. Let $X = \{a, b, c, d\}$, $\mu = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{b, c, d\}, \{a, c, d\}, X\}$. Then $A = \{a, b, c\}$ is $D(\alpha, \sigma)$-set but not $D(\mu, \alpha)$-set.

3. The class of $D_m(\mu, \alpha)$-sets and related sets

Definition 3.1. For a generalized topology and minimal structure space (X, μ, m), we denote:

1. $D_m(\mu, \alpha)$=$\{A \subseteq X : i_m(A) = A \cap i_m(c_\mu(i_m(A)))\}$,
2. $D_m(\mu, \sigma)$= $\{A \subseteq X : i_m(A) = A \cap c_\mu(i_m(A))\}$,
3. $D_m(\alpha, \sigma)$= $\{A \subseteq X : A \cap i_m(c_\mu(i_m(A))) = A \cap c_\mu(i_m(A))\}$.

Theorem 3.2. In (X, μ, m), every $D_m(\mu, \sigma)$-set is $D_m(\mu, \alpha)$-set, but not conversely.

Example 3.3. Let Let $X = \mathbb{R}$ be the set of all real numbers, $\mu = \{\emptyset, \mathbb{R} - \mathbb{Q} - \sqrt{2}\}$ and $m = \{\emptyset, \mathbb{Q}, \mathbb{R}\}$, where $\mathbb{Q}$ is set of all rational numbers and $\mathbb{R} - \mathbb{Q}$ is set of all irrational numbers. Then $A = \mathbb{Q} \cup \sqrt{2}$ is $D_m(\mu, \alpha)$-set but not $D_m(\mu, \sigma)$-set.

Remark 3.4. The following example show that in (X, μ, m), intersection (resp. union) of two $D_m(\mu, \alpha)$-sets (resp. $D_m(\mu, \sigma)$-sets, $D_m(\alpha, \sigma)$-sets) need not be $D_m(\mu, \alpha)$-set (resp. $D_m(\mu, \sigma)$-set, $D_m(\alpha, \sigma)$-set).

Example 3.5. Let $X = \{a,b,c,d,e\}$, $\mu = \{\emptyset, \{a\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{c,d\}, \{a,b,c\}, \{a,c,d\}, \{b,c,d\}, \{b,d,e\}, \{a,b,c,d\}, \{a,b,d,e\}, \{b,c,d,e\}, X\}$ and $m = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}, \{b,c,d\}, \{a,b,c,d\}, \{b,c,d,e\}, X\}$. Then $A = \{a,b,d,e\}$, $B = \{b,c,d,e\}$ are $D_m(\mu,\alpha)$-sets (resp. $D_m(\mu,\sigma)$-sets) but $A \cap B = \{b,d,e\}$ is not $D_m(\mu,\alpha)$-set (resp. $D_m(\mu,\sigma)$-set) and $C = \{b,d\}$, $D = \{e\}$ are $D_m(\mu,\alpha)$-sets (resp. $D_m(\mu,\sigma)$-sets) but $C \cup D = \{b,d,e\}$ is not $D_m(\mu,\alpha)$-set (resp. $D_m(\mu,\sigma)$-set).

Example 3.6. Let $X = \{a,b,c,d,e\}$, $\mu = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}, \{a,c,d\}, \{b,c,d\}, \{a,b,d\}, \{a,b,c,d\}\}$ and $m = \{\emptyset, \{a\}, \{b\}, \{a,b\}, \{a,c\}, \{a,b,c\}, \{b,c,d,e\}, X\}$. Then $A = \{a,b,c,d\}$, $B = \{a,b,d,e\}$ are $D_m(\alpha,\sigma)$-sets but $A \cap B$ is not a $D_m(\alpha,\sigma)$-set and $C = \{c\}$, $D = \{e\}$ are $D_m(\alpha,\sigma)$-sets but $D \cup C = \{c,e\}$ is not $D_m(\alpha,\sigma)$-set.

Theorem 3.7. A subset A of (X,μ,m) is m_μ-open if and only if m_μ-α-open and $D_m(\mu,\alpha)$-set.

Proof. Lat A be m_μ-α-open and $D_m(\mu,\alpha)$-set. Then $A \subset i_m(c_\mu(i_m(A)))$ and $i_m(A) = i_m(c_\mu(i_m(A)))$. Which implies $i_m(A) \subset A \subset A \cap i_m(c_\mu(i_m(A)))$. Therefore $i_m(A) \subset A \subset i_m(A)$. Hence A is m_μ-open. Conversely, if A is m_μ-open, then $i_m(A) \subset c_\mu(i_m(A))$, $i_m(A) \subset i_m(c_\mu(i_m(A)))$ implies $A \cap i_m(A) \subset A \cap i_m(c_\mu(i_m(A)))$ thus $i_m(A) \subset A \cap i_m(c_\mu(i_m(A)))$. $i_m(c_\mu(i_m(A))) \subset c_\mu(i_m(A)) = c_\mu(A))$ implies $A \cap i_m(c_\mu(i_m(A))) \subset A \cap c_\mu(A) = A = i_m(A)$. Hence A is $D_m(\mu,\alpha)$-set.

Theorem 3.8. Let A be a subset of (X,μ,m). Then the following conditions are equivalent:

1. A is m_μ-open,
2. A is m_μ-α-open and $D_m(\mu,\sigma)$-set,
3. A is m_μ-semi-open and $D_m(\mu,\sigma)$-set.

Proof. (1) $\Rightarrow$ (2). Let A is m_μ-open. Then $i_m(A) \subset c_\mu(i_m(A))$, $i_m(A) \subset i_m(c_\mu(i_m(A)))$. Therefore $A \subset i_m(c_\mu(i_m(A)))$ and $A \cap c_\mu(i_m(A)) = A \cap c_\mu(A) = A = i_m(A)$. Hence A m_μ-α-open and $D_m(\mu,\sigma)$-set.
(2) $\Rightarrow$ (3). Obvious.
(3) $\Rightarrow$ (1). Let A be m_μ-semi-open and $D_m(\mu,\sigma)$-set. Then $A \subset c_\mu(i_m(A))$ and $i_m(A) = A \cap c_\mu(i_\mu(A))$, $A \subset c_\mu(i_m(A)) = A \cap c_\mu(i_m(A)) = i_m(A)$. Hence A is m_μ-open. ■

Theorem 3.9. A subset A of (X,μ,m) is m_μ-α-open if and only if m_μ-semi-open and $D_m(\alpha,\sigma)$-set.

Proof. Let A be m_μ-α-open set. Then $A \subset i_m(c_\mu(i_m(A))) \subset c_\mu(i_m(A))$, thus A is m_μ-semi-open and $A \cap i_m(c_\mu(i_m(A))) = A = A \cap c_\mu(i_m(A))$. Hence A is $D_m(\alpha,\sigma)$-set. Conversely A be m_μ-semi-open and $D_m(\alpha,\sigma)$-set. Then $A \cap i_m(c_\mu(i_m(A))) = A \cap c_\mu(i_m(A))$

and $A \subset c_\mu(i_m(A))$ which implies $A \subset A \cap (c_\mu(i_m(A))) = A \cap i_m(c_\mu(i_m(A))) \subset i_m(c_\mu(i_m(A)))$. Hence A is m_μ-α-open set. ■

4. Decompositions of (μ_m, λ)-continuity

Definition 4.1. A function $f : (X, \mu) \to (Y, \lambda)$ is said to be $(D(\mu, \alpha), \lambda)$-continuous (resp.$(D(\mu, \sigma), \lambda)$-continuous and $(D(\alpha, \sigma), \lambda)$-continuous) if $f^{-1}(U)$ is a $D(\mu, \alpha)$-set (resp. $D(\mu, \sigma)$-set, $D(\alpha, \sigma)$-set) for each $U \in \lambda$.

Definition 4.2. A function $f : (X, \mu) \to (Y, \lambda)$ is said to be $(D_m(\mu, \alpha), \lambda)$-continuous (resp.$(D_m(\mu, \sigma), \lambda)$-continuous and $(D_m(\alpha, \sigma), \lambda)$-continuous) if $f^{-1}(U)$ is a $D_m(\mu, \alpha)$-set (resp. $D_m(\mu, \sigma)$-set, $D_m(\alpha, \sigma)$-set) for each $U \in \lambda$.

Remark 4.3.

1. The notions of m_μ-α-open sets and $D_m(\mu, \alpha)$-sets are independent,
2. The notions of m_μ-α-open sets and $D_m(\mu, \sigma)$-sets are independent,
3. The notions of m_μ-semi-open sets and $D_m(\mu, \sigma)$-sets are independent,
4. The notions of m_μ-semi-open sets and $D_m(\alpha, \sigma)$-sets are independent,
5. The notions of $D_m(\mu, \alpha)$-sets and $D_m(\alpha, \sigma)$-sets are independent.

Example 4.4. Let $X = \{a, b, c, d, e\}$, $\mu = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}, \{a, c, d\}, \{b, c, d\}, \{a, b, d\}, \{a, b, c, d\}\}$ and $m = \{\emptyset, \{a, b\}, \{a, c\}, \{a, b, c\}, \{b, c, d, e\}, X\}$. Then $A = \{a, b, c, d\}$ is m_μ-α-open (resp. m_μ-semi-open) but not $D_m(\mu, \alpha)$-set (resp. $D_m(\mu, \sigma)$-set), $B = \{c\}$ is $D_m(\mu, \alpha)$-set (resp. $D_m(\mu, \sigma)$-set) but not m_μ-α-open (resp. m_μ-semi-open), $C = \{e\}$ is m_μ-semi-open (resp. $D_m(\mu, \alpha)$-set) but not $D_m(\alpha, \sigma)$-set.

Theorem 4.5. A function $f : (X, \mu) \to (Y, \lambda)$ is (μ, λ)-continuous if and only if it is (α, λ)-continuous and $(D(\mu, \alpha), \lambda)$-continuous.

Proof. This is an immediate consequence of Theorem 2.10. ■

Theorem 4.6. Let $f : (X, \mu) \to (Y, \lambda)$. Then the following conditions are equivalent:

1. f is (μ, λ)-continuous,
2. f is (α, λ)-continuous and $(D(\mu, \sigma), \lambda)$-continuous,
3. f is (σ, λ)-continuous and $(D(\mu, \sigma), \lambda)$-continuous.

Proof. The proof is follows from Theorem 2.11. ■

Theorem 4.7. A function $f : (X, \mu) \to (Y, \lambda)$ is (α, λ)-continuous if and only if it is (σ, λ)-continuous and $(D(\alpha, \sigma), \lambda)$-continuous.

Proof. This is an immediate consequence of Theorem 2.12. ■

Theorem 4.8. A function $f : (X, \mu, m) \to (Y, \lambda)$ is (μ_m, λ)-continuous if and only if it is (α_m, λ)-continuous and $(D_m(\mu, \alpha), \lambda)$-continuous.

Proof. This is an immediate consequence of Theorem 3.7. ■

Theorem 4.9. Let $f : (X, \mu, m) \to (Y, \lambda)$. Then the following conditions are equivalent:

1. f is (μ_m, λ)-continuous,
2. f is (α_m, λ)-continuous and $(D_m(\mu, \sigma), \lambda)$-continuous,
3. f is (σ_m, λ)-continuous and $(D_m(\mu, \sigma), \lambda)$-continuous.

Proof. The proof is follows from Theorem 3.8. ■

Theorem 4.10. A function $f : (X, \mu, m) \to (Y, \lambda)$ is (α_m, λ)-continuous if and only if it is (σ_m, λ)-continuous and $(D_m(\alpha, \sigma), \lambda)$-continuous.

Proof. The proof is follows from Theorem 3.9. ■

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