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Study of heat transfer on an unsteady elastic stretching surface under the magnetic and ohmic effect

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Abstract

In this paper we study the heat transfer of boundary layer flow of an incompressible viscous fluid over hyperbolic stretching cylinder. The governing nonlinear partial differential equations are converted into ordinary differential equations by using suitable transformations, which are then tackled the homotopy perturbation method (HPM), which is a standard technique to handle such problems that are difficult to study analytically.The homotopy perturbation method gives us solutions in the form of series. The influence of the Magnetic and Ohmic effect on velocity, temperature and concentration profiles are investigated and results can be seen visually in graphs. The computational results without these effects agree excellently with the previous results by [9,10].

AMS subject classification:
Keywords: Ohmic effect, Magnetohydrodynamic, stretching sheet.

1. Introduction

The analysis of heat transfer in fluid flow across liquid films is essential in order to comprehend the design of various heat exchangers and chemical processing equipment. It has various applications in the polymer and metallurgical industries including wire and fiber coating, reactor fluidization, transpiration cooling, polymer processing, etc. Different authors have previously investigated various aspects of heat transfer conducted in an infinite fluid medium surrounding a stretching sheet. For example, G. S. Seth and M. K. Mishra in [1] analysed an unsteady Magnetohydrodynamic nanofluid flow past stretching sheet with velocity slip and obtained solutions numerically using finite element technique. Swati Mukhopadhyay in [2], also considered the boundary layer flow of a viscous incompressible fluid along a porous nonlinearly stretching sheet by converting the partial differential equation corresponding to the momentum equation into nonlinear ordinary differential equation by carrying out similarity transformations. A Numerical solution of this was attained using the shooting method. On parallel lines, C. Sulochana et al. in [5] had studied the the effects of thermal radiation and slip effects on magneto hydrodynamic forced convective flow of a nano fluid over a slendering stretching sheet in porous medium. Self-similarity transformation were used to reduce the governing partial differential equations into nonlinear ordinary differential equations which were solved numerically using Matlab. Reddy in [6] investigated the effect of magnetic field, non-uniform heat source/sink and slip effects on chemically reacting nano fluid flow over a slendering stretching sheet by converting the governing partial differential equations into a set of non linear ordinary differential equations using suitable similarity transformations, solutions of which were obtained using Runge-Kutta Fehlberg method. Reddy et al. in [8] also studied the effect of thermophoresis and Brownian moment on hydro-magnetic motion of a nanofluid over a slendering stretching sheet by reaching a similarity solution. On the other hand, Anjali Devi in [7] identified the characteristics of hydromagnetic flow over a slendering stretching sheet in slip flow regime by studying the effect of variable magnetic field and slip flow regime on a steady, two dimensional, nonlinear, hydro-magnetic laminar flow of an incompressible, viscous and electrically conducting fluid over a stretching sheet with variable thickness. Governing equations of the problem were reduced into non-linear ordinary differential equations utilizing similarity transformations which were in turn solved numerically by utilizing Nachtsheim – Swigert shooting iterative scheme. Whereas, Babu et al. in [4] investigated the cross-diffusion effects on the magneto hydrodynamic Williamson fluid flow across a stretching sheet with variable thickness by observing its velocity slip. Runge-Kutta based shooting process was applied to solve the transformed differential equations numerically. Hang Xu et al. however in [3] investigated the unsteady flow and heat transfer of nanofluids caused by a linear stretching velocity over a horizonal elastic sheet using the homotopy analysis method (HAM). Here, the purpose of this present paper is to explore the characteristics of Ohmic effects,the effects of magnetic field and rate of chemical reaction of the problem considered by I. C. Mandal et al. in [9] where the flow is extended and heat transfer in boundary layer over an exponentially stretching sheet embedded in porous medium with variable surface heat flux is analyzed.

2. Problem formulation

In the system studied by I. C. Mandal et al. in [9] where the stretching sheet is subjected to a variable heat flux (VHF) $q_w(x)$. The fluid flow is confined to $y > 0$. Two equal and opposite forces are applied along the x-axis so that the wall is stretched keeping the origin fixed. The effect of these two equal and opposite forces cause a symmetric boundary at the center of the porous medium. Equations governing such type of flow (with the application of Darcy's law) are written as:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0,\;\; u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\partial^2 u}{\partial y^2}-\frac{\gamma}{k}u-\mu\sigma B_0 u,$$
$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\kappa}{\rho c_p}\frac{\partial^2 T}{\partial y^2}-\frac{\sigma B_0^2}{\rho_0 c_p}u^2, \tag{2.1}$$
$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_m\frac{\partial^2 C}{\partial y^2}-K_0(x)(C-C_\infty)^n,$$

with boundary conditions,at $y=0$

$$u=U, v=-V(x),$$
$$C=C_w(x), \frac{\partial T}{\partial y}=\frac{-q_w(x)}{K}, \tag{2.2}$$

$u\to 0, T\to 0, C\to C_\infty$, as $y\to\infty$. where

$$q_w(x)=q_{w_0}T_0\sqrt{\frac{U_0}{2\gamma L}}e^{\frac{Nx}{L}},$$
$$V(x)=V_0 e^{\frac{Nx}{L}}, k=k_0 e^{\frac{Nx}{L}},$$
$$C_w(x)=C_\infty+C_0(x+c)^{\frac{1-n}{2}}$$

κ is the thermal conductivity of the fluid. where u, v are velocity components along x and r directions, T represents temperature of the fluid. We introduce the stream function $u=\frac{1}{r}\frac{\partial\psi}{\partial r}, v=-\frac{1}{r}\frac{\partial\psi}{\partial x}$ by introducing the dimensionless variables f,θ,ϕ and the following similarity transformations,

$$\eta=\sqrt{\frac{U_0}{2\gamma L}}e^{\frac{Nx}{L}}y, u=U_0 e^{\frac{Nx}{L}}\frac{\partial f(\eta)}{\partial\eta},$$
$$v=-N\sqrt{\frac{U_0\gamma}{2L}}\frac{Nx}{L}(f(\eta)+\eta\frac{\partial f(\eta)}{\partial\eta}), T=\frac{q_{w_0}}{K}T_0 e^{\frac{Nx}{L}}\theta(\eta), \tag{2.3}$$
$$C=C_0(x+c)^{\frac{1-n}{2}},$$

that convert (2.1) to the system of ODE,

$$\frac{d^3 f(\eta)}{d\eta^3}-K_{12}\frac{df(\eta)}{d\eta}+K_{13}\frac{d^2 f(\eta)}{d\eta^2}f(\eta)+K_{14}\frac{df(\eta)}{d\eta}=0,$$
$$\frac{d^3\theta(\eta)}{d\eta^3}-K_{15}\frac{d\theta(\eta)}{d\eta}f(\eta)+K_{15}\frac{df(\eta)}{d\eta}\theta(\eta)+K_{14}(\frac{df(\eta)}{d\eta})^2=0, \tag{2.4}$$

For $n = 1$, it becomes,

$$\frac{d^2\phi(\eta)}{d\eta} + \tilde{K_{13}}\frac{d\phi(\eta)}{d\eta} + \tilde{K_{14}} + \tilde{K_{15}}\frac{d\phi(\eta)}{d\eta}f(\eta) + \tilde{K_{16}}^{\phi}(\eta)\frac{df(\eta)}{d\eta} = 0, \tag{2.5}$$

For $n = 2$, it becomes,

$$\frac{d^2\phi(\eta)}{d\eta} + K_{23}\frac{d\phi(\eta)}{d\eta} + K_{24} + K_{25}\frac{d\phi(\eta)}{d\eta}f(\eta) + \tilde{K_{26}}\phi(\eta)\frac{df(\eta)}{d\eta} + K_{27}(\phi(\eta))^2 = 0, \tag{2.6}$$

with corresponding boundary conditions (2.2) reducing to,

$$\begin{gathered}\frac{df(0)}{d\eta} = 1, f(0) = S, \frac{d\theta(0)}{d\eta} = -1, \phi(0) = 1,\\ \frac{d\theta(0)}{d\eta} = -1, \phi(0) = 1,\end{gathered} \tag{2.7}$$

$\theta(\eta) \to -1$ as $\eta \to \infty$, $\phi(\eta) \to 0$ as $\eta \to \infty$ where

$$S = \frac{V}{N\sqrt{\frac{U_0\gamma}{2L}}} \tag{2.8}$$

$S > 0$ is the suction parameter whereas $S < 0$ is the blowing parameter.

3. Solution methodology

First we establish the following homotopy equations for (2.4), (2.5) and (2.6) with boundary conditions (2.7)

$$\begin{aligned}&H(f,p) : (1-p)\left(\frac{d^3f(\eta)}{d\eta^3} - K_{12}\frac{df(\eta)}{d\eta}\right) + p\left(\frac{d^3f(\eta)}{d\eta^3} - K_{12}\frac{df(\eta)}{d\eta}\right) + K_{13}\frac{d^2f(\eta)}{d\eta^2}f(\eta)\\ &\quad + K_{14}\left(\frac{df(\eta)}{d\eta}\right)^2 = 0,\\ &H(\theta,p) : (1-p)\left(\frac{d^3\theta(\eta)}{d\eta^3}\right) + p\left(\frac{d^3\theta(\eta)}{d\eta^3} - K_{15}\frac{d\theta(\eta)}{d\eta}f(\eta) + K_{15}\frac{df(\eta)}{d\eta}\theta(\eta)\right.\\ &\quad \left. + K_{14}\left(\frac{df(\eta)}{d\eta}\right)^2\right) = 0.\end{aligned} \tag{3.1}$$

For $n = 1$,

$$\begin{aligned}H(\phi,p) : (1-p)\left(\frac{d^2\phi}{d\eta^2} + \tilde{K_{13}}\frac{d\phi(\eta)}{d\eta}\right) + p\left(\frac{d^2\phi(\eta)}{d\eta} - \tilde{K_{13}}\frac{d\phi(\eta)}{d\eta}\right.\\ \left. + \tilde{K_{14}} + \tilde{K_{15}}\frac{d\phi(\eta)}{d\eta}f(\eta) + \tilde{K_{16}}\phi(\eta)\frac{df}{d\eta}\right) = 0,\end{aligned} \tag{3.2}$$

For $n = 2$,

$$H(\phi, p) : (1-p)\left(\frac{d^2\phi(\eta)}{d\eta} - K_{23}\frac{d\phi(\eta)}{d\eta}\right) + p\left(\frac{d^2\phi(\eta)}{d\eta} - K_{23}\frac{d\phi(\eta)}{d\eta} + K_{24} + K_{25}\frac{d\phi(\eta)}{d\eta}f(\eta) + \tilde{K_{26}}\phi(\eta)\frac{df(\eta)}{d\eta} + K_{27}(\phi(\eta))^2\right) = 0, \tag{3.3}$$

According to the generalized homotopy method, assume the solution for (3.1), (3.2) and (3.3) in the form

$$\begin{aligned} f_p &= p^0 f_0 + f_1 p + f_2 p^2 + f_3 p^3, \\ \theta_p &= p^0\theta_0 + \theta_1 p + \theta_2 p^2 + \theta_3 p^3, \\ \phi_p &= p^0\phi_0 + \phi_1 p + \phi_2 p^2 + \phi_3 p^3, \end{aligned} \tag{3.4}$$

Substituting (3.5) into (3.1) – (3.3) and rearranging the terms of order p, we attain the following simpilfied ordinary differential equations. We have concerning f,

$$\begin{aligned} \frac{d^3 f_0(\eta)}{d\eta^3} - K_{12}\frac{df_0(\eta)}{d\eta} &= 0, \\ \frac{d^3 f_1(\eta)}{d\eta^3} - K_{12}\frac{df_1(\eta)}{d\eta} + K_{13} + \frac{d^2 f_0(\eta)}{d\eta^2}f_0(\eta) + K_{14}\left(\frac{df_0(\eta)}{d\eta}\right)^2 &= 0 \\ \frac{d^3 f_2(\eta)}{d\eta^3} - K_{12}\frac{df_2(\eta)}{d\eta} + \frac{d^2 f_1(\eta)}{d\eta^2}f_0(\eta) + \frac{d^2 f_0(\eta)}{d\eta^2}f_1(\eta) + 2K_{14}\left(\frac{df_0(\eta)}{d\eta}\frac{df_1(\eta)}{d\eta}\right) &= 0. \end{aligned} \tag{3.5}$$

Concerning θ,

$$\begin{aligned} \frac{d^2\theta_0(\eta)}{d\eta^2} &= 0, \\ \frac{d^2\theta_1(\eta)}{d\eta^2} - K_{15}\frac{d\theta_0(\eta)}{d\eta}f_0(\eta) + K_{15}\frac{df_0(\eta)}{d\eta}\theta_0(\eta) + K_{16}\left(\frac{df_0(\eta)}{d\eta}\right)^2 &= 0, \\ \frac{d^2\theta_2(\eta)}{d\eta^2} - K_{15}\frac{d\theta_1(\eta)}{d\eta}f_0(\eta) - K_{15}\frac{d\theta_0(\eta)}{d\eta}f_1(\eta) + K_{15}\frac{df_1(\eta)}{d\eta}\theta_0(\eta) & \\ + K_{15}\frac{df_0(\eta)}{d\eta}\theta_1(\eta) + K_{16}\frac{df_0(\eta)}{d\eta}\frac{df_1(\eta)}{d\eta} &= 0. \end{aligned} \tag{3.6}$$

Concerning ϕ, For $n = 1$,

$$\frac{d^2\phi_0}{d\eta^2} - \tilde{K_{13}}\frac{d\phi_0}{d\eta} = 0,$$
$$\frac{d^2\phi_1}{d\eta^2} - \tilde{K_{13}}\frac{d\phi_1}{d\eta} + \tilde{K_{14}} + \tilde{K_{15}}\frac{d\phi_0}{d\eta}f_0(\eta) + \tilde{K_{16}}\phi_0(\eta)\frac{df_0}{d\eta} = 0, \tag{3.7}$$
$$\frac{d^2\phi_2}{d\eta^2} - \tilde{K_{13}}\frac{d\phi_2}{d\eta} + \tilde{K_{14}} + \tilde{K_{15}}\frac{d\phi_0}{d\eta}f_1(\eta) + \tilde{K_{15}}\frac{d\phi_1}{d\eta}f_0(\eta) + \tilde{K_{16}}\phi_1(\eta)\frac{df_0}{d\eta} + \tilde{K_{16}}\phi_0(\eta)\frac{df_1}{d\eta} = 0.$$

For $n = 2$,

$$\frac{d^2\phi_0(\eta)}{d\eta} - K_{23}\frac{d\phi_0(\eta)}{d\eta} = 0$$
$$\frac{d^2\phi_1}{d\eta} - K_{23}\frac{d\phi_1}{d\eta} + K_{24} + K_{25}\frac{d\phi_0}{d\eta}f_0(\eta) + \tilde{K_{26}}\phi_0(\eta)\frac{df_0}{d\eta} + K_{27}(\phi_0(\eta))^2 = 0,$$
$$\frac{d^2\phi_2(\eta)}{d\eta} - K_{23}\frac{d\phi_2(\eta)}{d\eta} + K_{25}\frac{d\phi_1(\eta)}{d\eta}f_0(\eta) + K_{25}\frac{d\phi_0(\eta)}{d\eta}f_1(\eta) + \tilde{K_{26}}\phi_0(\eta)\frac{df_1(\eta)}{d\eta} + \tilde{K_{26}}\phi_1(\eta)\frac{df_0(\eta)}{d\eta} + K_{27}\phi_0(\eta)\phi_1(\eta) = 0. \tag{3.8}$$

The boundary conditions (2.7) reduce to,

$$\frac{df_0(0)}{d\eta} = 1, \frac{df_i(0)}{d\eta} = 0, \tag{3.9}$$

for $i = 1, 2, 3, \ldots$

$$f_0(0) = S, f_i(0) = 0, \tag{3.10}$$

for $i = 1, 2, 3, \ldots$

$$\frac{df_i(\infty)}{d\eta} = 0, \tag{3.11}$$

for $i = 0, 1, 2, 3, \ldots$

$$\frac{d\theta_0(0)}{d\eta} = -1, \frac{d\theta_i(0)}{d\eta} = 0, \tag{3.12}$$

for $i = 1, 2, 3, \ldots$

$$\theta_i(\infty) = 0, \tag{3.13}$$

for $i = 0, 1, 2, 3, \ldots$

$$\phi_0(0) = 1, \phi_i(0) = 0, \tag{3.14}$$

for $i = 1, 2, 3, \ldots$

$$\phi_0(\infty) = 1, \phi_i(\infty) = 0. \tag{3.15}$$

for $i = 1, 2, 3, \ldots$
The solutions to (3.5) satisfying boundary conditions (3.9)–(3.11) are,

$$\begin{aligned}
f_0 &= (S + \frac{1}{\sqrt{K_{12}}}) - \frac{1}{\sqrt{K_{12}}} e^{-\sqrt{K_{12}}\eta}, \\
f_1 &= L^* + d_1 + (d_3 + L_3) e^{-\sqrt{K_{12}}\eta} + L_4 e^{-2\sqrt{K_{12}}\eta} \\
&\quad + L_3 \eta e^{-\sqrt{K_{12}}\eta}, \\
f_2 &= \tilde{d_1} + R_{11}{}^* + (R_5 + \tilde{d_2}) e^{-\sqrt{K_{12}}\eta} + R_6 e^{-2\sqrt{K_{12}}\eta} \\
&\quad + R_7 e^{-3\sqrt{K_{12}}\eta} + R_8 \eta e^{-2\sqrt{K_{12}}\eta} + R_9 \eta e^{-\sqrt{K_{12}}\eta} + R_{11} \eta^2 e^{-\sqrt{K_{12}}\eta}.
\end{aligned} \tag{3.16}$$

The solutions to (3.6) satisfying boundary conditions (3.12), (3.13) are,

$$\begin{aligned}
\theta_0 &= 0, \\
\theta_1 &= \frac{K_{16}}{4K_{12}} e^{-2\sqrt{K_{12}}\eta}, \\
\theta_2 &= L_{14} e^{-2\sqrt{K_{12}}\eta} + L_{15} e^{-3\sqrt{K_{12}}\eta} + L_{16} \eta e^{-2\sqrt{K_{12}}\eta}.
\end{aligned} \tag{3.17}$$

For $n = 1$:
The solutions to (3.7) satisfying boundary conditions (3.14), (3.15) are,

$$\begin{aligned}
\phi_0 &= e^{-\sqrt{\tilde{K_{13}}}\eta}, \\
\phi_1 &= -L_{21} e^{-\sqrt{\tilde{K_{13}}}\eta} + L_{21} e^{-(\sqrt{\tilde{K_{13}}} + \sqrt{K_{12}})\eta} + L_{23} \eta e^{-\sqrt{\tilde{K_{13}}}\eta}, \\
\phi_2 &= P_6 e^{-\sqrt{2\tilde{K_{13}}}\eta} + P_8 e^{-(\sqrt{\tilde{K_{13}}} + 2\sqrt{K_{12}})\eta} + P_9 e^{-\sqrt{\tilde{K_{13}}}\eta} + P_{10} e^{-(\sqrt{\tilde{K_{13}}} + \sqrt{K_{12}})\eta} \\
&\quad + P_{11} \eta e^{-\sqrt{2\tilde{K_{13}}\eta}} + P_{12} \eta e^{-(\sqrt{\tilde{K_{13}}} + \sqrt{K_{12}})\eta} + P_{14} \eta^2 e^{-\sqrt{\tilde{K_{13}}}\eta} + P_{15} e^{-\sqrt{K_{12}}\eta}.
\end{aligned} \tag{3.18}$$

For $n = 2$:

The solutions to (3.8) satisfying boundary conditions (3.9)–(3.11) are,

$$\begin{aligned}\phi_0 &= e^{-\sqrt{\tilde{K_{13}}}\eta},\\ \phi_1 &= L_{31} - (L_{31} + L_{27} + L_{29})e^{-\sqrt{K_{23}}\eta} + L_{27}e^{-(\sqrt{K_{23}}+\sqrt{K_{12}})\eta} + L_{29}e^{-2\sqrt{K_{23}}\eta}\\ &\quad + L_{30}\eta e^{-\sqrt{K_{23}}\eta},\\ \phi_2 &= Q_{16} + Q_{15}\eta + (c_{16} + Q_9)e^{-\sqrt{K_{23}}\eta} + Q_{10}e^{-(\sqrt{K_{23}}+\sqrt{K_{12}})\eta} + Q_{11}e^{-2\sqrt{K_{23}}\eta}\\ &\quad + Q_{12}e^{-(2\sqrt{K_{23}}+\sqrt{K_{12}})\eta} + Q_{13}e^{-(\sqrt{K_{23}}+2sqrtK_{12})\eta} + Q_{14}e^{-2\sqrt{K_{23}}\eta}\\ &\quad + Q_{17}\eta e^{-\sqrt{K_{12}}\eta} + Q_{18}e^{-\sqrt{K_{12}}\eta} + Q_{19}\eta^2 e^{-\sqrt{K_{23}}\eta} + Q_{21}e^{-3\sqrt{K_{12}}\eta}\\ &\quad + Q_{22}\eta e^{-(\sqrt{K_{23}}+\sqrt{K_{12}})\eta} + L_{30}\eta e^{-\sqrt{K_{23}}\eta},\end{aligned} \tag{3.19}$$

Now the solutions to (2.4)–(2.6) satisfying boundary conditions (2.7) is seen below.

So we can write the first and second approximations to f, θ, ϕ respectively as, First approximation to f

$$\begin{aligned}f &= f_0 + f_1\\ &= L^* + d_1 + \left(S + \frac{1}{\sqrt{K_{12}}}\right)\left(d_3 + L_3 - \frac{1}{\sqrt{K_{12}}}\right)e^{-\sqrt{K_{12}}\eta}\\ &\quad + L_4 e^{-2\sqrt{K_{12}}\eta} + L_3\eta e^{-\sqrt{K_{12}}\eta}\end{aligned} \tag{3.20}$$

Second approximation to f

$$\begin{aligned}f &= f_0 + f_1 + f_2\\ &= \tilde{d_1} + R_1 1^* + L^* + d_1 + \left(S + \frac{1}{\sqrt{K_{12}}}\right)\left(d_3 + L_3 + R_5 + \tilde{d_2} - \frac{1}{\sqrt{K_{12}}}\right)e^{-\sqrt{K_{12}}\eta}\\ &\quad + (L_4 + R_6)e^{-2\sqrt{K_{12}}\eta} + R_7 e^{-3\sqrt{K_{12}}\eta} + R_8\eta e^{-2\sqrt{K_{12}}\eta} + (L_3 + R_9)\eta e^{-\sqrt{K_{12}}\eta}\\ &\quad + R_{11}\eta^2 e^{-\sqrt{K_{12}}\eta},\end{aligned} \tag{3.21}$$

First approximation to θ

$$\begin{aligned}\theta &= \theta_0 + \theta_1\\ &= \frac{K_{16}}{4K_{12}}e^{-2\sqrt{K_{12}}\eta},\end{aligned} \tag{3.22}$$

Second approximation to θ

$$\begin{aligned}\theta &= \theta_0 + \theta_1 + \theta_2\\ &= \left(\frac{K_{16}}{4K_{12}} + L_{14}\right)e^{-2\sqrt{K_{12}}\eta} + L_{15}e^{-3\sqrt{K_{12}}\eta} + L_{16}\eta e^{-2\sqrt{K_{12}}\eta}.\end{aligned} \tag{3.23}$$

For $n = 1$:
First approximation to ϕ

$$\begin{aligned}\phi &= \phi_0 + \phi_1 \\ &= (1 - L_{21})e^{-\sqrt{\tilde{K_{13}}}\eta} + L_{21}e^{-2\sqrt{(\tilde{K_{13}}+K_{12})}\eta} + L_{23}\eta e^{-\sqrt{\tilde{K_{13}}}\eta},\end{aligned} \tag{3.24}$$

Second approximation to ϕ

$$\begin{aligned}\phi &= \phi_0 + \phi_1 + \phi_2 \\ &= (1 - L_{21})e^{-\sqrt{\tilde{K_{13}}}\eta} + L_{21}e^{-(2\sqrt{\tilde{K_{13}}}+\sqrt{K_{12}})\eta} + L_{23}\eta e^{-\sqrt{\tilde{K_{13}}}\eta} \\ &+ (P_6 + P_9)e^{-\sqrt{2\tilde{K_{13}}}\eta} + P_8 e^{-(2\sqrt{\tilde{K_{13}}}+2\sqrt{K_{12}})\eta} + P_{11}\eta e^{-\sqrt{2\tilde{K_{13}}}\eta} \\ &+ P_{12}e^{-(\sqrt{\tilde{K_{13}}}+\sqrt{K_{12}})\eta} + P_{14}\eta^2 e^{-\sqrt{\tilde{K_{13}}}\eta} + P_{15}e^{-\sqrt{K_{12}}\eta}.\end{aligned} \tag{3.25}$$

For $n = 2$:
First approximation to ϕ

$$\begin{aligned}\phi &= \phi_0 + \phi_1 \\ &= L_{31} + (1 - (L_{31} + L_{27} + L_{29}) + c_{16} + Q_9)e^{-\sqrt{K_{23}}\eta} + (L_{27} + Q_{10})e^{-(\sqrt{K_{23}}+\sqrt{K_{12}})\eta} \\ &+ (L_{29} + Q_{11} + Q_{14})e^{-2\sqrt{K_{23}}\eta} + L_{30}\eta e^{-\sqrt{K_{23}}\eta},\end{aligned} \tag{3.26}$$

Second approximation to ϕ:

$$\begin{aligned}\phi &= \phi_0 + \phi_1 + \phi_2 \\ &= L_{31} + Q_{16} + Q_{15}\eta + (1 - (L_{31} + L_{27} + L_{29}))e^{-\sqrt{K_{23}}\eta} + L_{27}e^{-(\sqrt{(K_{23}+K_{12})}\eta} \\ &+ L_{29}e^{-2\sqrt{K_{23}}\eta} + L_{30}\eta e^{-\sqrt{K_{23}}\eta} + Q_{12}e^{-(2\sqrt{(K_{23}+K_{12})}\eta} + Q_{13}e^{-(\sqrt{K_{23}}+2sqrtK_{12})\eta} \\ &+ Q_{17}\eta e^{-\sqrt{K_{12}}\eta} + Q_{18}e^{-\sqrt{K_{12}}\eta} + Q_{19}\eta^2 e^{-\sqrt{K_{23}}\eta} + Q_{21}e^{-3\sqrt{K_{12}}\eta} \\ &+ Q_{22}\eta e^{-(\sqrt{K_{23}}+\sqrt{K_{12}})\eta},\end{aligned} \tag{3.27}$$

Where the evaluated constants K_i, L_i, P_i, Q_i as they occupy immense space are not mentioned in this paper. As the series is convergent, we ignore terms f_3, θ_3, ϕ_3 onwards, as their effects are negligible.

4. Results and discussion

The system of partial differential equations with the boundary conditions, are converted to the system of ordinary differential equations (2.5), (2.6), (2.7) using similarity transformations. These ODE are solved using the homotopy technique. First we calculate

f^p, θ^p, ϕ^p and then taking p'1 we get the solution to f, θ, ϕ . The 1st approximations to f, θ, ϕ are (3.20), (3.22), (3.24), (3.26) respectively and the 2nd approximations to f, θ, ϕ are (3.21), (3.23), (3.25), (3.27) respectively. We analyse the profiles of velocity, temperature through graphs, for impacts of the Ohmic effect, the effect of a magnetic field as well as that of chemical reaction on them. K_{12} gives the magnetic effect and K_{16} gives the ohmic effect, whereas the rate of chemical reaction is given by $\tilde{K_{13}}$, $\tilde{K_{14}}$ for $n = 1$ and by K_{23}, K_{24} for $n = 2$
Iterations for f:

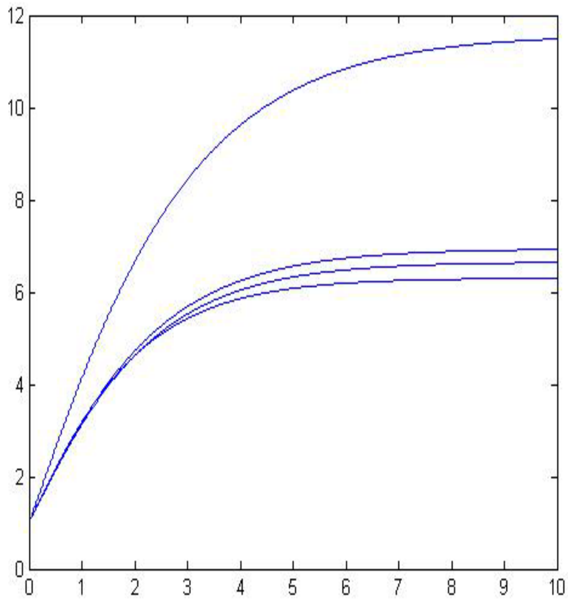

Figure 1: First Iteration for f under Magnetic effect nearly 0, Ohmic effect nearly 0 , $S = 1$

Refering figures 1 to 5, the x-axis denotes values of η and the$y$-axis denotes values of velocity. When S is 1, under the absence or taking very insignificant values of of magnetic effect as well as ohmic effects (i.e magnetic effect taking values between 0 and 0.9, ohmic effect taking values between -0.2 and 0) velocities initially taking value 1 with increase in η till a particular point of η after which velocity tends to remain constant for all increasing values of η. When S taking value -1 after initially taking value -1, velocity behaves the same way as that for $S = 1$ as long as magnetic effect and ohmic effect are absent. When magnetic effects are significant like taking values between 1 and 2 with $S = 1$, and ohmic effects taking values between -2 and -0.2 velocity initially takes value 1 and increases more steeply as η increases more steeply as η increases, in comparison to its behaviour when magnetic effect and ohmic effect were absent. It increases steeply until a particular value of η after which it tends to remain constant with further increase in η. At $S = -1$ velocity behaves exactly in the same manner as that

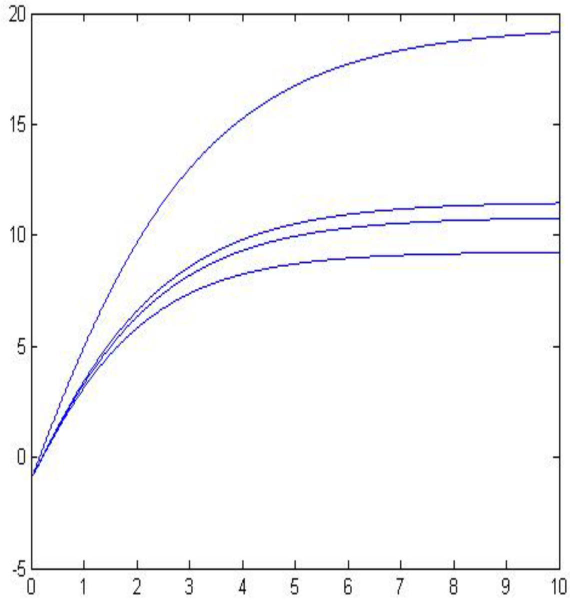

Figure 2: First Iteration for f under Magnetic effect nearly 0,Ohmic effect nearly 0, $S = -1$

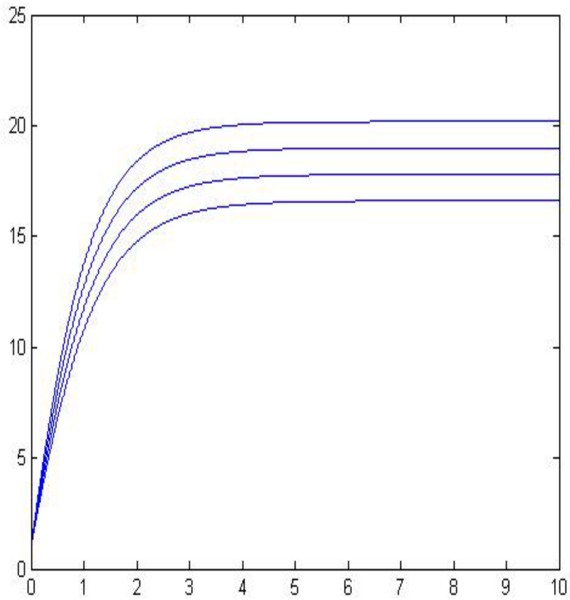

Figure 3: First Iteration for f under Magnetic effect between 1 and 2 , Ohmic effect between -2 and -0.2 , $S = 1$

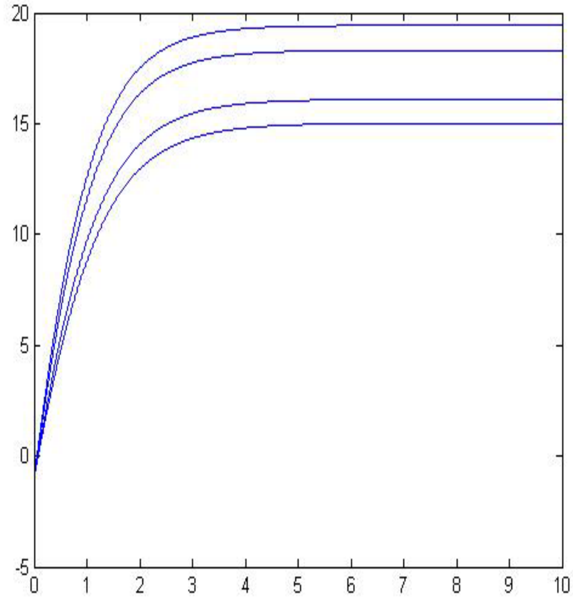

Figure 4: First Iteration for f under Magnetic effect between 1 and 2 , Ohmic effect between -2 and -0.2 , $S = -1$

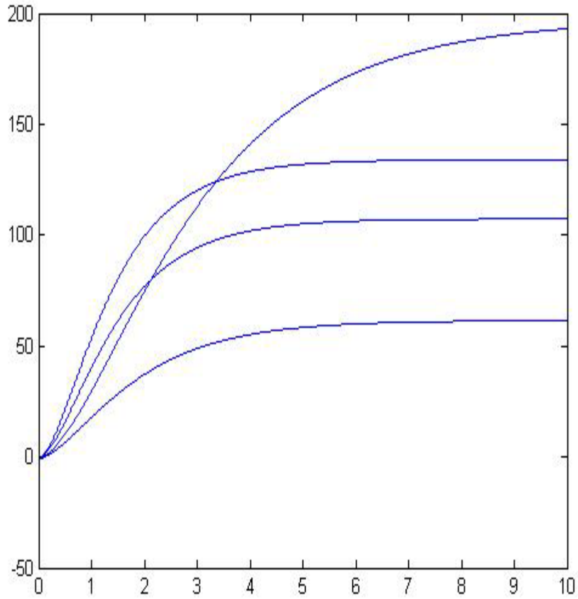

Figure 5: Second Iteration for f under 0 Magnetic effect, 0 Ohmic effect , $S = -1$

for $S = -1$ except that velocity initially takes the value -1.

Iterations of θ:

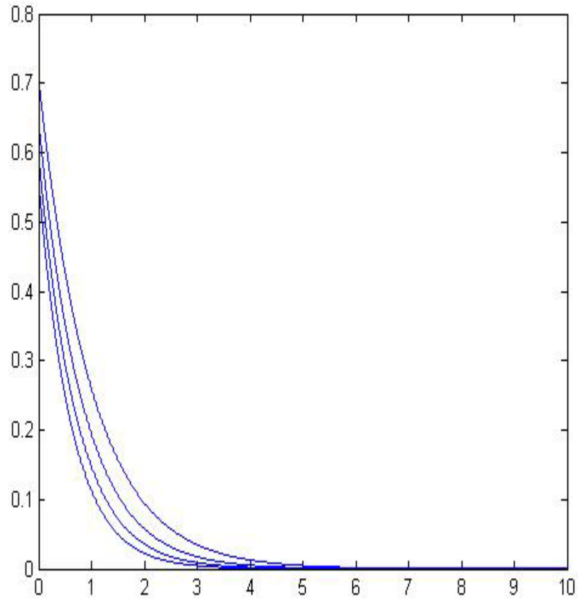

Figure 6: First Iteration for θ under Magnetic effect nearly 0, Ohmic effect nearly 0, $S = 1$

Refering figures 6 to 9, the x-axis denotes values of η and the$y$-axis denotes values of temperature.At $S = 1$ as well as when $S = -1$ when magnetic effect and ohmic effects are nearly absent ((i.e magnetic effect taking values between 0 and 0.9, ohmic effect taking values between -0.2 and 0) temperature reduces steeply with increase in η till after some particular η it takes value 0 with further increase in eta. As we increase the values of magnetic and ohmic effect, (i.e magnetic effect taking values between 1 and 2, ohmic effect taking values between -2 and -0.2) temperature behaves in a similar matter except that temperature decrease at a steeper rate as eta increases till some particular η after which it takes value 0 with further increase in η.

Refering figures 10 to 12, the x-axis denotes values of η and the$y$-axis denotes values of concentration.When $n = 1$ under the absence of magnetic, ohmic effects as well as the effects of chemical reaction, concentration initially takes value 1, after which it at first increases with inbcrease in η, after which it tends to reduce till it eventually takes value 0 for some particular value of eta after which it tends to remain constant at 0, with further increase in η.

Refering figures 10 to 12, it is easy to see that for $S = -1$, in the absence of the magnetic and ohmic effects as well as the effects of Chemical reaction, concentration behaves behaves in a similar manner initially by taking initial value 1 and then increasing with increase in η until some particular η after which it reducers and then tends to increase

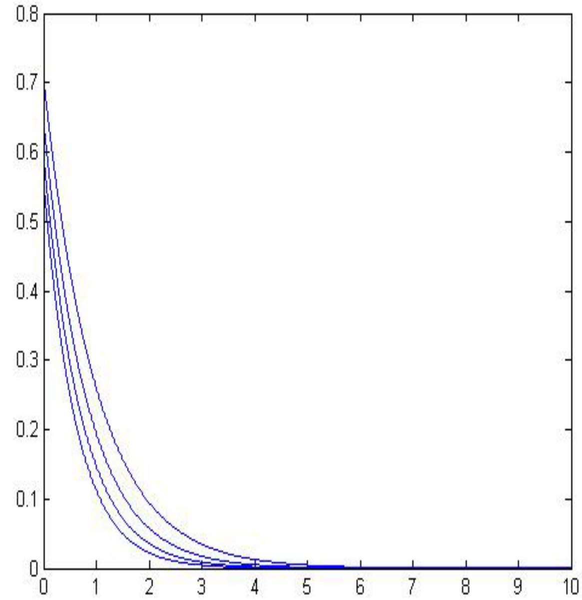

Figure 7: First Iteration for θ under Magnetic effect nearly 0, Ohmic effect nearly 0, $S = -1$

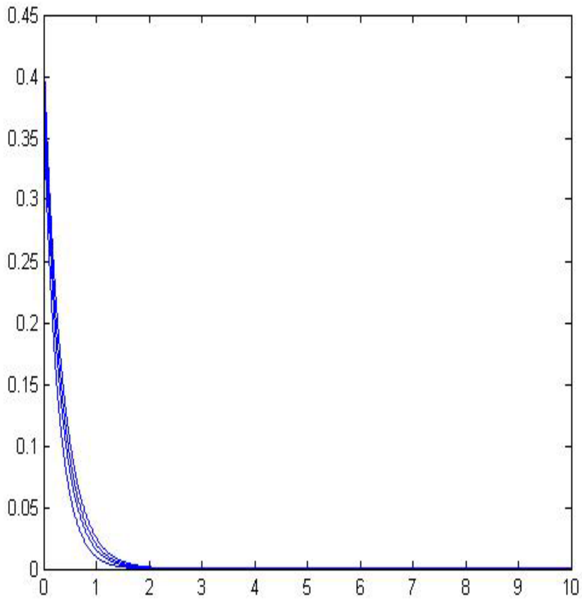

Figure 8: First Iteration for θ under Magnetic effect between 1 and 2, Ohmic effect between -2 and -0.2 , $S = 1$

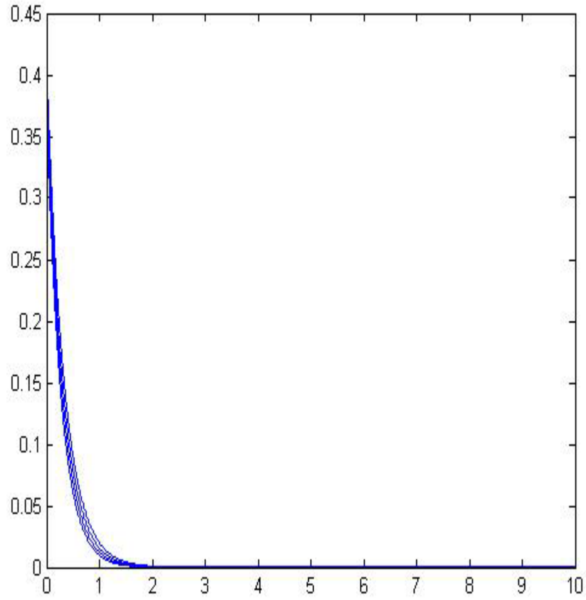

Figure 9: First Iteration for θ under Magnetic effect between 1 and 2, Ohmic effect between -2 and -0.2 , $S = -1$

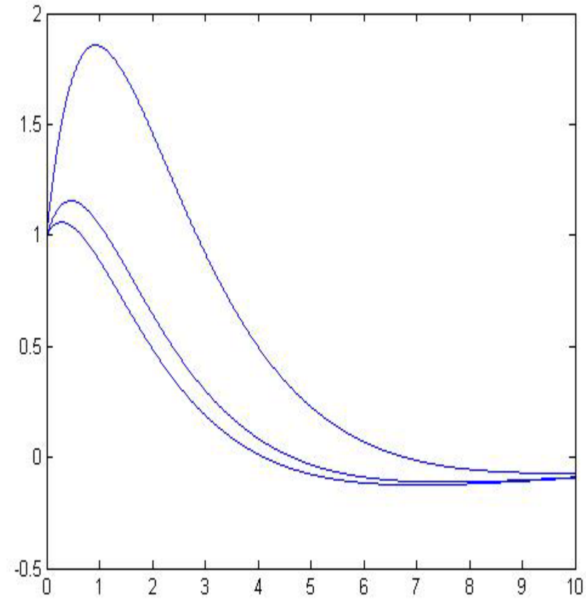

Figure 10: First Iteration for ϕ for $n = 1$ under Magnetic effect nearly 0, Ohmic effect nearly 0 ,$S = 1$

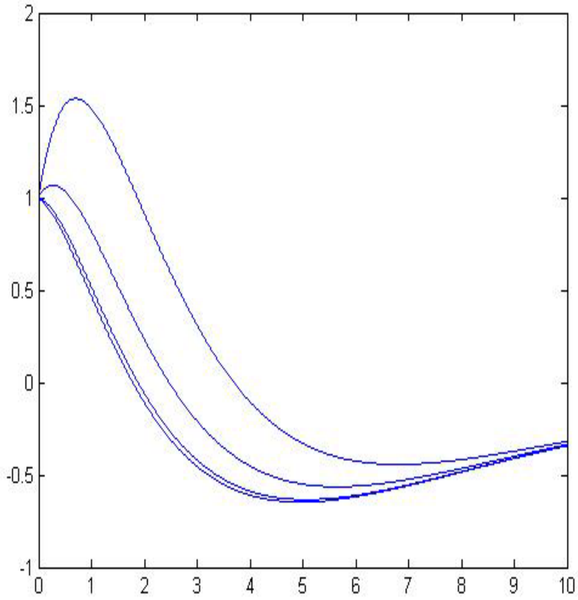

Figure 11: First Iteration for ϕ for $n = 1$ under Magnetic effect nearly 0, Ohmic effect nearly 0 , $S = -1$

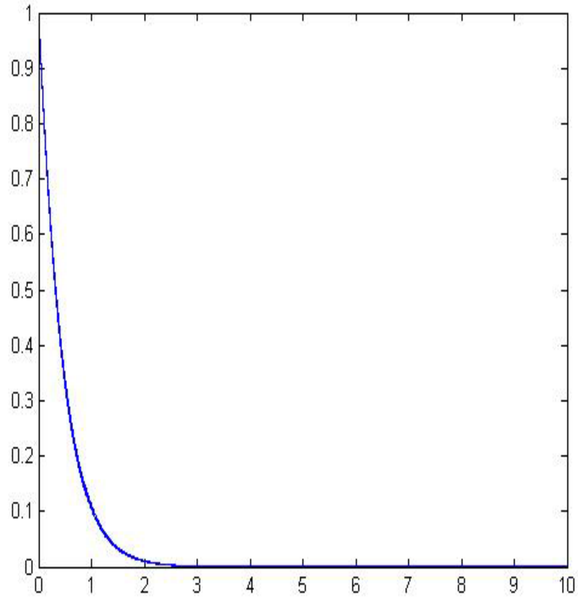

Figure 12: First Iteration for ϕ for $n = 1$ under Magnetic effect between 1 and 2, Ohmic effect between -2 and -0.2, $S = -1$

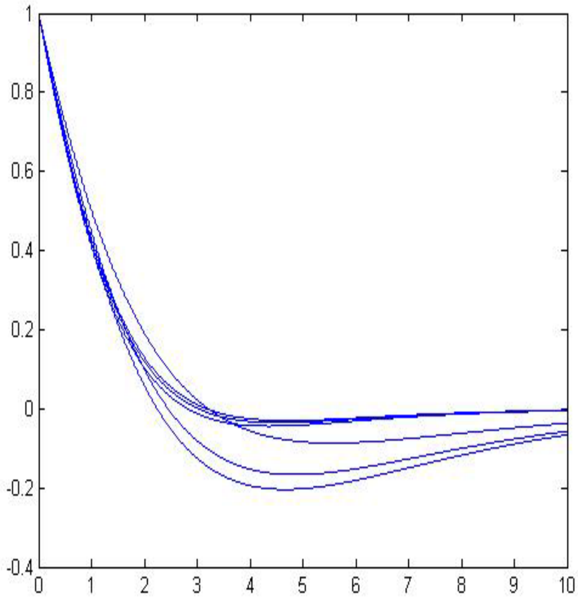

Figure 13: First Iteration for ϕ for $n = 2$ under Magnetic effect nearly 0 , Ohmic effect nearly 0 , $S = 1$

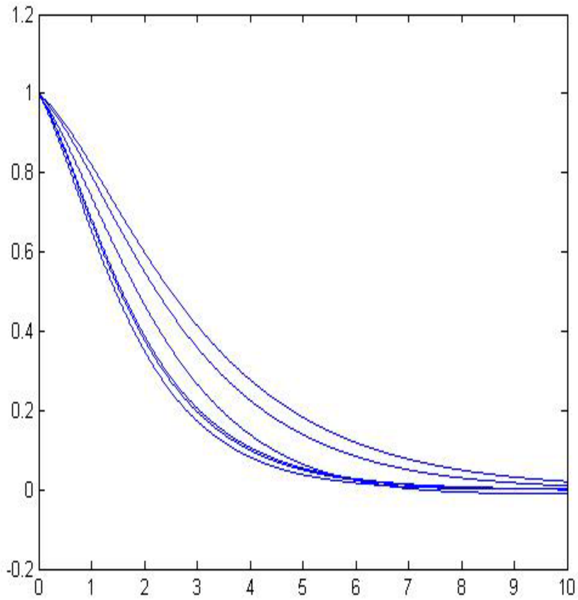

Figure 14: First Iteration for ϕ for $n = 2$ under Magnetic effect nearly 0, Ohmic effect nearly 0 , $S = -1$

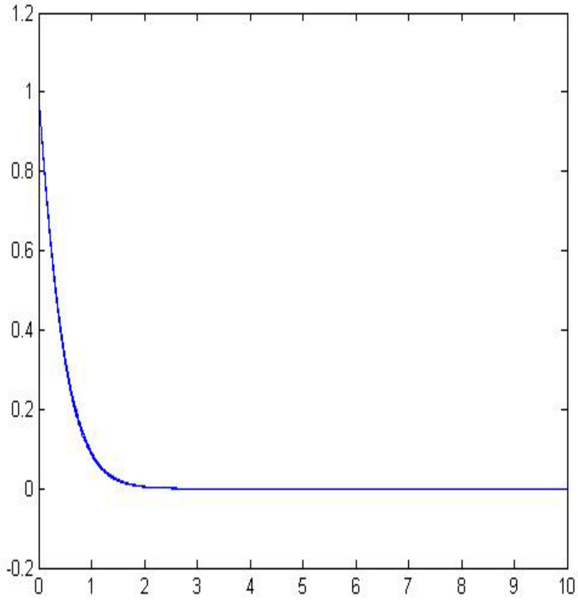

Figure 15: First Iteration for ϕ for $n = 2$ under Magnetic effect between 1 and 2, Ohmic effect between -2 and -0.2 , $S = 1$

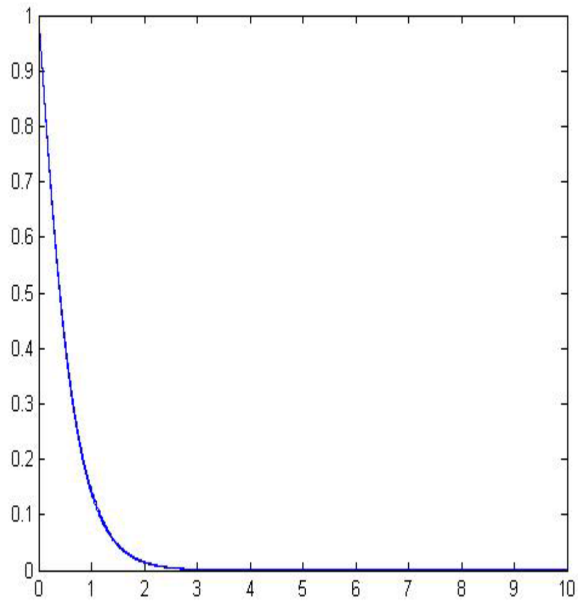

Figure 16: First Iteration for ϕ for $n = 2$ under Magnetic effect between 1 and 2, Ohmic effect between -2 and -0.2 , $S = -1$

after it hits a particular value of η until it reaches value 0, after which it remains constant with further increase in values of η, when $n = 1$. However when $n = 2$ at $S = 1$ at nearly absent values of magnetic and ohmic effect,chemical reaction taking insignificant values too, (magnetic effect and effect of chemical reaction taking values between 0 and 0.9, ohmic effect taking values -0.2 to 0), concentration reduces steeply from 1 at $\eta = 0$ to 0 after a particular value of η after which the steepness of the reduction in concentration slows down with further increase in η, concentration tends to be constant at 0, as seen in figures 13 to 16 .At $S = -1$ the behaviour of conc entration is similar except that as we take higher values of magnetic and ohmic effects as well as effects of chemical reaction (magnetic effect taking values between 1 and 2, ohmic effect taking values -2 to -0.2 and effect of chemical reaction taking values between 5 and 6), the concentration reduces to values less than 0 and then tends to increase with increase in η until for some particular value of η, it takes value 0, and after this particular η it remains constant at 0 with further increase in η. At $S = -1$ under the absence of magnetic, ohmic effects as well as the effects of chemical reaction concentration tends to reduce from taking initial value 1 at a slow rate to the value 0, after which it tends to remain constant with further increase in η. Even as the effects of magnetic, ohmic effects as well as the effects of chemical reaction are increased (magnetic effect taking values between 1 and 2, ohmic effect taking values -2 to -0.2 and effect of chemical reaction taking values between 5 and 6), the behaviour of concentration at $S = 1$ is similar to that when thesew are absent, but it reduces steeply with increase in η taking initial value 1 until it reaches concentration value 0, for some paricular η, after which it tends to remain constant at 0 with further increase in η. S.P. Anjali Devi and D. Vasantha kumari, have studied in [9], the heat transfer on an exponentially stretching porous sheet embedded in a porous medium with variable surface heat flux, in the absence of effects of magnetic field as well as that of the rate of chemical reaction, where they have compared the different values of $-\theta'(0)$ for various values of Prandtl number Pr using numerical approaches. The comparison of the table obtained by them Table 1 and by us Table 2, using the homotopy method, is drawn below,

Table 1: Values of $-\theta^{'}(0)$ for several values of Prandtl number Pr. attained in [9]

Pr	Magyari and Keller [21]	Bidin and Nazar [28]	El-Aziz [29]	Ishak [32]	Present study with N = 1, k_1 = 0, S = 0
1	0.9548	0.9547	0.9548	0.9548	0.9547
2		1.4714		1.4715	1.4714
3	1.8691	1.8691	1.8691	1.8691	1.8691
5	2.5001		2.5001	2.5001	2.5001
10	3.6604		3.6604	3.6604	3.6603

5. Conclusions

The boundary layer flow of an incompressible viscous fluid over hyperbolic stretching cylinder has invoked appreciable curiosity due to its applications in the polymer and met-

Table 2: Non-dimensional rate of heat transfer for various values of Magnetic Effect K_{12}, Ohmic effect K_{16}, Rate of chemical reaction $\tilde{K}_{13}$, $\tilde{K}_{14}$ for $n = 1$ and that for $n = 2$, K_{23}, K_{24}, K_{27} attained by the homotopy method.

Non-dimensional rate of heat transfer for various values of Magnetic Effect K_{12}, Ohmic effect K_{16}, Rate of chemical reaction $K_{13}^{\sim}$, $K_{14}^{\sim}$ for $n = 1$ and that for $n = 2$, K_{23}, K_{24}, K_{27}.

K_{12}	K_{16}	$K_{13}^{\sim}$	$K_{14}^{\sim}$	K_{23}	K_{24}	K_{27}	$-\Theta'(0)$
0	0	0	0	0	0	0	0
0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.7071
0.5	0	0.5	0.5	0.5	0.5	0.5	0.7071
1	0.5	0.5	0.5	0.5	0.5	0.5	1
1	1	5	5	5	5	5	1

allurgical industries including wire and fiber coating, reactor fluidization, transpiration cooling, polymer processing and many other areas. Noticing this we have examined the two-dimensional magnetohydrodynamic boundary layer viscous flow across the slendering sheet under the Ohmic effect as well as the effects of magnetic field and chemical reaction. The major discoveries can be seen as follows. Firstly we see that the chemical reaction rate and thermal conductivity κ has very insignificant effects on the concentration of the fluid flow at any point of the flow.Secondly we notice that the velocity of the fluid flow and its temperature behave in an exactly opposite manner both in the absence of the ohmic effects and effects of magnetic field as well as when these effects are reasonably present. We also observe that the concentration of the fluid flow with $n = 1$ also behave in an exactly opposite manner in comparison to the concentration with $n = 2$, also both in the absence of the ohmic effects, effects of magnetic field and the rate of chemical reaction as well as when these effects are fairly present.

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